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These syllabuses for K-6 were written, evaluated, and revised by a team of writers from the Orange County Science Education Improvement Project (OCSEIP). OCSEIP is a cooperative enterprise undertaken by the University of California (Irvine), California State College at Fullerton, the Orange County Schools Office, and local districts throughout Orange County. These syllabuses were written to help teachers teach the best aspects of recent mathematics programs. Presented are some methods of approach, intuitive examples, suggestions for additions and deletions, and applications in mathematics. The mathematical content for these syllabuses includes materials from geometry, sets, numbers and numerations, order and relations, addition and subtraction, problem solving, and measurement. (RP)

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O.C.S.E.I.P. SYLLABUS

Kindergarten

SE 006 069

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The first draft of this syllabus was written during an 8 week session at University of California, Irvine during the summer of 1966 by:

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PREFACE

Kindergarten

These units were written to provide teachers with instructional materials that would implement modern approaches to teaching mathematics. Stress was placed on developing an articulated program from kindergarten through college.

In determining the units to be developed, the writing team agreed to give an "in-depth" treatment to the areas considered weak in the present mathematics curriculum in Orange County schools.

The "strands" of mathematics, as presented by the Advisory Committee on Mathematics, were used as the basis for evaluating those topics in elementary school mathematics to be considered for inclusion in this project. Implementing Mathematics Programs in California, A Guide K-8 was used as a guide in the examination of the scope and sequence of topics included in the current state-adopted texts.

As you use these materials, you are urged to be creative in your teaching and to not restrict your instruction to the suggestions, examples, and ideas given here. In some areas there are more suggestions than you will need or can use effectively. When this is the case, select the material that will be most appropriate for your class.

The degree to which these or any other materials will improve mathematics instruction in your room depends on your enthusiasm and desire to provide stimulating math experiences for your students.

The writing team hopes you will find these materials helpful and that, through the discovery approach, your students will be challenged to develop their math potential.

Mathematics in Kindergarten should be informal and flexible, but it must also be carefully planned to capitalize upon the natural curiosity and eagerness for learning that most kindergartners possess. The productive readiness period cannot be left to chance, but must be nurtured by a well informed teacher. Both planned and incidental math lessons are necessary--neither alone is adequate.

The length of the kindergarten day, the other than math curriculum and the natural short interest span of a five year old will determine the amount of time spent each day on mathematics. Game and aids at a "Math" table for free choice time provide intuitive learning.

The content of the program is presented in three natural phases. Phase one: Pre-number; phase two: matching sets, number and numeration; and phase three: the operations.

Each phase is filled with experiences involving manipulation of aids that the child can see, touch, move about, group, regroup and discuss. The skillful teacher "gives" no answers but motivates questions from the children and answers the questions with yet another question making it possible for the children to use known facts to discover the answers alone. The teacher supplies the new language only when the child needs it to verbalize his thoughts.

It is strongly recommended that individual worksheets or workbooks do not become a part of the work done by children in Kindergarten. Experience provided by workbooks and worksheets too often makes little contribution for effective learning. The mechanics of finding and marking the correct place, when eye and hand coordination may not be adequately developed, interferes with the mathematical ideas being developed. Furthermore, visual perception is often not developed enough for some children to understand the math concepts that pictures are to convey. The manipulation of objects makes a more lasting contribution to deeper understanding of the ideas under study and are obtained when children are active participants and not merely passive watchers and listeners.

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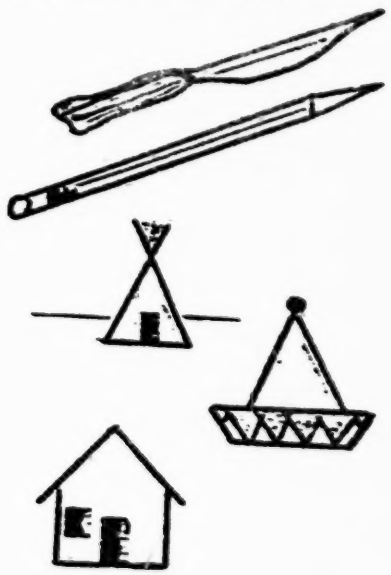
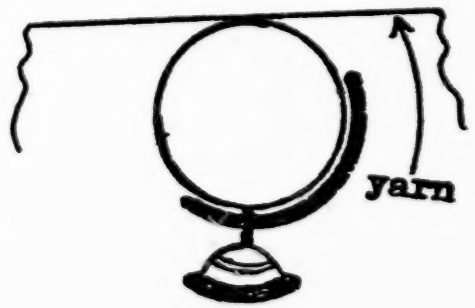
GEOMETRY

KINDERGARTEN

Points and Lines

Points of pencils--chalk dots, holes in paper made by pin.





Ask a student to think of a place on the chalk board and mark it. Ask another student to make the chalk dot smaller. Discuss the smallest mark we can make to show a place of which we're thinking. 8:357

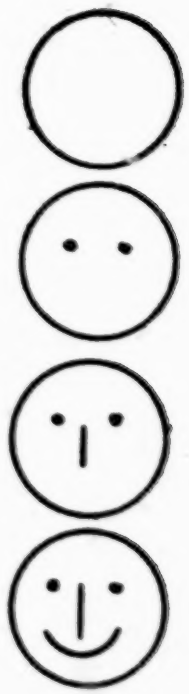
Show and talk about objects that remind us of points.

Prick a paper with a straight pin. Compare the mark with the chalk dot. Draw two chalk dots. Have student connect the dots with many paths. "Which is the longest path? Which is the shortest path? Which paths are curved? Which path is straight?"

Connect points in the classroom with rope or colored yard.

Connect two adjacent corners of a large box with a piece of red yard. What other lines in the classroom can they find which connect "corners"? Guide students to see lines formed by ceiling and walls and floor and walls.

Connect two points above the eye level of the class such as the pencil sharpener and flag holder. Ask "Through what objects or places would this line pass if it went on and on through the walls?" Expect answers such as: classroom next door, a tree, the swings, cars, etc. Keep stretching the line on and on naming things that would be intersected. Some sharp youngster will realize the line may leave the earth and "go out" into space. If this level is reached, use globe showing how the line might appear in



relation to earth. Don't force concept if students aren't able to suggest this possibility themselves.

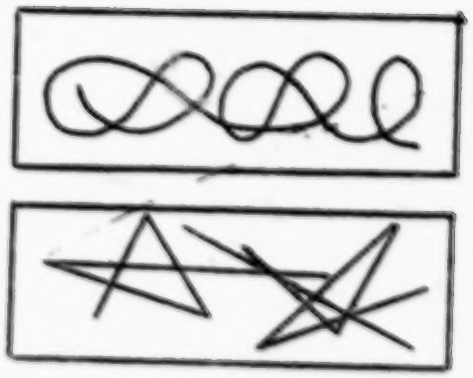
When many points are placed close together, they can form a line. Explain to the children that there are two kinds of lines: curved and straight. Show the children a piece of yarn. Have two children hold the ends of the yarn and pull it tight. Explain to the children that the yarn is now forming (making) a straight line. Ask the two children holding yarn to each take one step forward. The yarn now forms a curved line. Then extend (stretch) the curved line around and form a circle. Place two points inside the circle with you chalk which makes eyes. Then use a straight line and curved line for nose and mouth of face. Your face has been made from points and lines (curved and straight).

Give each child a piece of paper and let him make a face using points and lines.

Read to the children the story "The Dot" by Cliff Roberts (New York: Franklin Watts, Inc., 1960). Call the dot the point and it will fit right in with the lesson.

As correlating activities, yarn or string can be used to discover the various kinds of lines that can be made; between two points such as curved and straight. Discover also which is the longest and shortest.

Introduce the "magic finger." Have the children form the same line in the air with their finger as is being illustrated with the yarn or string.



Have the children make line drawings--curved and straight with crayons.

They can also dip yarn or string in starch and make line designs.

For Halloween suggest that the children place a curved line (string) in a ghost shape. Place a piece of black construction paper over the string and use the side of a piece of white chalk to gently cover the paper. A spooky ghost will result.

Some songs that could possible be used are "In a Line," "Marching to My Drum," and "Puppy's Tail." With these songs see how many kinds of lines can be made.

Kindergarten Book, pg. 25 and pg. 112.

Remind them that they form lines to come into the room.

Games that can be played are "Muffin Man" and "Follow the Leader."

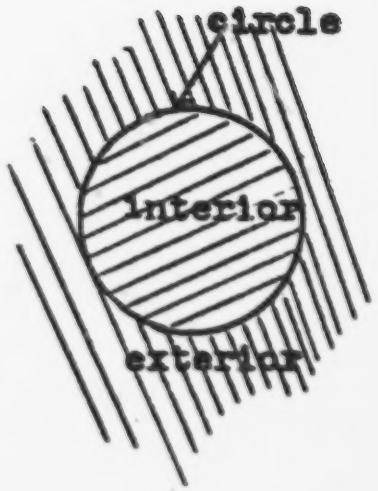
Transition to the circle to form shape: use a "Magic Finger" line. Have the child form a curved line or a half moon. If we continue this line what shape do we have? We form a circle.

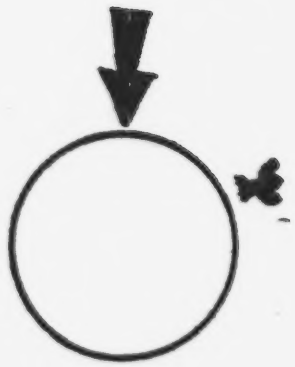
Awareness of geometric shapes in the environment and

Recognition of two-dimensional shapes and their interior and exterior regions.

Show the children circles which are as nearly perfect as possible. Use hula hoops, embroidery hoops, bracelets, or make your own circles from wire. (A guitar wire is excellent since it returns to its original position after bending.)

3:1-20





Is the fly
inside the circle,
on the circle,
or outside the
circle?

Students often confuse the circle with the disk. Do not ask children to color circles--(an error in some texts). We, in fact, can color only the interior of the circle. Say, "Color the circle shapes." Play games with children which require children to form circles, standing on the circle, inside the circle, and outside the circle, e.g., "Dodge Ball," "Hot Potato," "In and Out the Window," "Brownies and Fairies," and "Cat and Mouse."

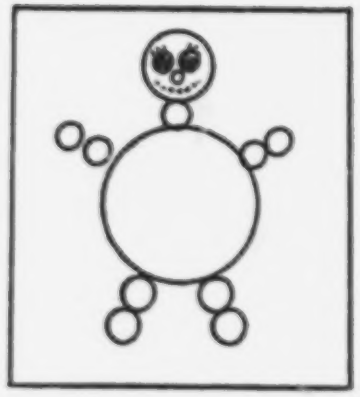
13:9-10

6:32-38

Poem - Author Unknown

ROUND

My name is Little Miss Round.
No corners on me are found.
I don't even have an end;
but you always see me bend.
How many things do you see
That Roly Poly like me?



Teacher can make a large circle man to have up in the room. If you use various colors of tissue paper for the circles and then mount on chipboard, you will have a colorful, permanent teaching aid. Also when the colored tissue circles overlap it adds an interesting contrast. The children will grasp the idea of circles quickly.

THINGS ROUND
By Elizabeth Landeweer

So many things I like are round!
The shiny penny that I found,
The round sun sinking low at night,
The moon a round balloon of light,
The doughnut Mother gave to me,
The round earth, and the round blue sea,
But what I like best in the world
Is my white kitty lying curled
Just like a soft, round ball of wool,
Asleep with his round tummy full!

Read these stories: "A Kiss is Round"
by Blossom Budney
"Round is a Pancake"
by Joan Sullivan

Have the children look for circles in the classroom, playground and home. (Clocks, coins, glasses, wheels, cans, etc.) Compare the objects they suggest with models of a circle. 9:1-53

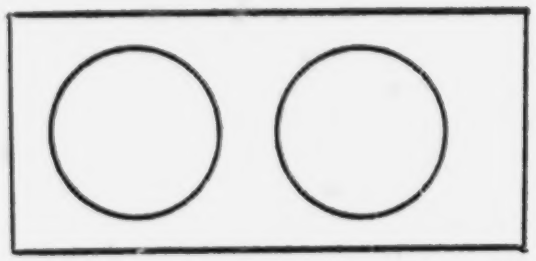
Encourage students to make comparisons between circles. One circle may be <u>larger than</u> another, <u>smaller than</u> another, or the <u>same</u> size. This is an important step in the development of congruent figures.

Possible books to use to help develop the concepts of larger than and smaller than are:

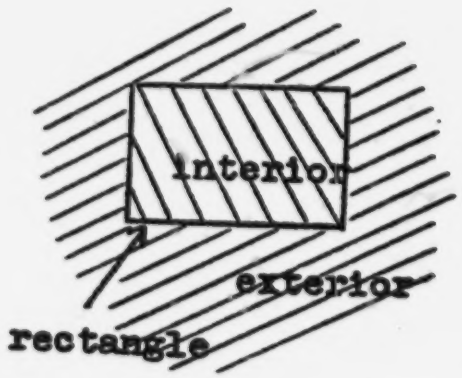
<u>Let's Find Out What's Big and Small</u>
By Charles and Martha Shapp

<u>The Very Little Girl</u>
By Phyllis Krasilousky

<u>The Growing Boy</u>
By Ruth Krauss

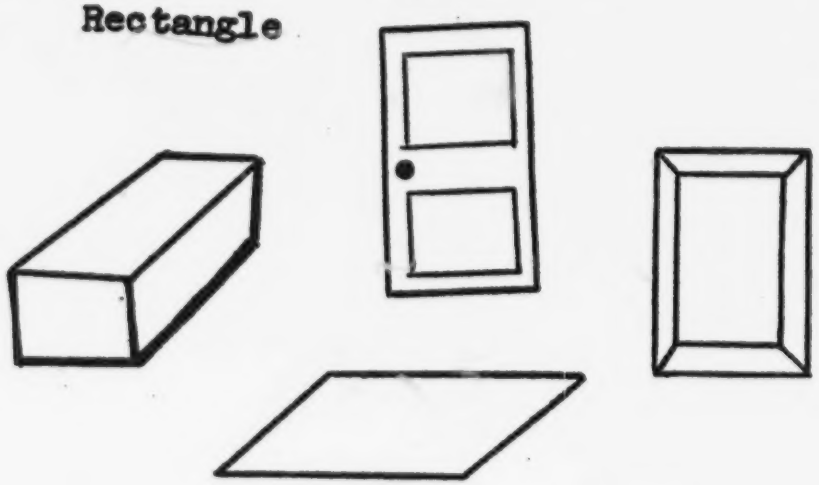


Place two circle shapes (cut from orange construction paper) on flannel or bulletin board. Have the children identify the shapes. Ask a child to come to the board and change one circle shape to a happy jack-o-lantern by drawing a happy face on it with a black crayon or tint-ink. Ask another child to come to the board and change the other circle shape to a sad jack-o-lantern by drawing a sad face on it with a black crayon or tint-ink. Pass out work sheets to children. Give oral directions. "Color the circle shapes orange. Put a happy face on one circle shape. Put a sad face on the other circle shape."



Use the top edges of a box or the sides of a picture frame to show the rectangle--avoid the square now. Develop the square later as "a special kind of rectangle."

Rectangle

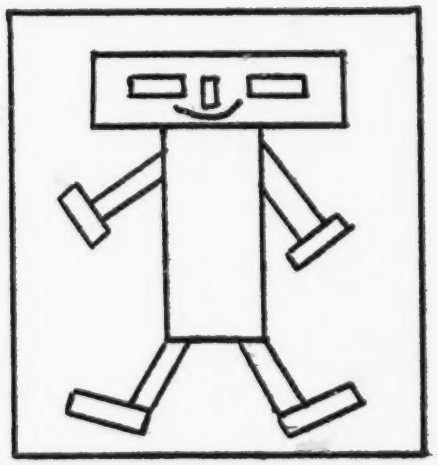


Show the rectangle in many positions.

Ask students to name rectangles they can see in class, playground and home. Let students <u>verbalize</u> about what they see, (doors, boxes, walls, books) as follows:

Encourage students to talk about the ways in which circles and rectangles are different. Have students draw circles and rectangles in the air with their fingers. Let others guess what figure they are drawing.

Draw rectangles on the classroom floor using chalk or masking tape. Play games which require children to be <u>inside</u>, <u>on</u>, or <u>outside</u> the rectangle.



Square

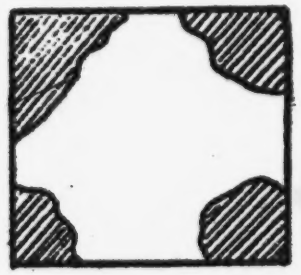
Poem and teaching aid for rectangle shape:

MR. RECTANGLE
By Georgia Bray

Mr. Rectangle you do see,
Here are some clues to find me.
I am a funny sort,
For two of my sides are short.
The other two sides are tall,
That makes me have four sides in all.

Use a box with equal edges to show the square or make a model of wood, wire or starched string. Rotate the square resting it on each of its four sides. Have children close eyes while you change the square's position. "Can you tell whether the square has been moved? Why not? Why can you tell when the position of a rectangle is changed?"

Use a nest of square boxes to compare the sizes of squares. Guide children to talk about squares--"Which square is <u>larger than</u> this one? Which square is <u>smaller than</u> this one? Which square is the <u>same size</u> as this one?"



Prepare students for concept of right angles and similarity of angles by having students cut or tear off corners of sheets of colored newsprint and fit these pieces on a white construction paper square. Paste corners in place. Repeat the activity with rectangles. Suggest the activity with circles. "Do circles have corners?"

Find squares in the classroom and play yard, e.g., tiles in the ceiling and floor, boxes, cabinet doors, calendar grids, etc.

Lead children to generalize that the sides and corners of a square are the same size respectively.

Poem - Arthur Unknown

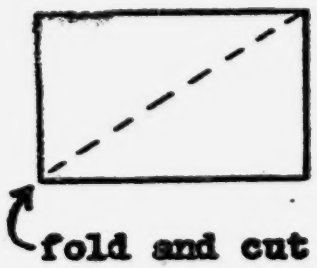
SQUARE

My name is Mr. Square;
You see I have no hair.
I have but corners four,
And even sides once more.
Please come and bring to me
Squares for us to see.

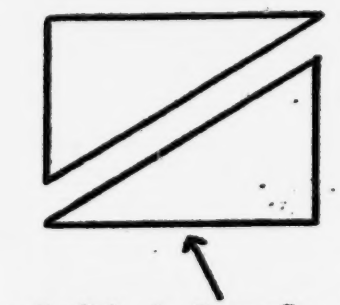
Place square man on a table and let the children find square objects at home and in the classroom which, as the poem suggests, are brought to him. Square objects can be left on the table as reinforcement during the study of the square.

Triangle

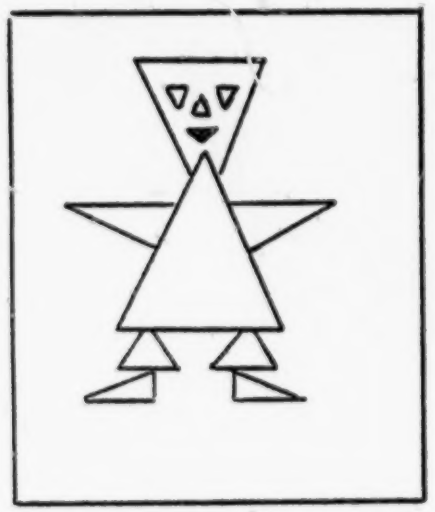
Show all types of triangles using wire or wood models. Tagboard strips connected with paper fasteners are suitable for student handling. Make large triangles using rope or colored yarn for the sides and student fingers for the vertices. Have students move about showing various sized triangles. Find triangles in the classroom. Rotate the positions of a triangle. "Can you tell if the triangle has been moved?" Have the pupils close eyes while a classmate places the triangle on a different base in the chalk tray or on a flannel board. Ask if someone can place the triangle in its original position.

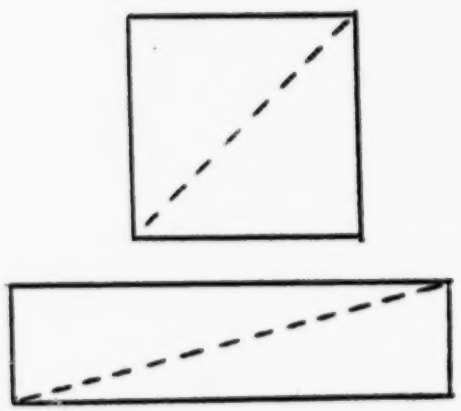


Ask students to fold squares and rectangles in half along a diagonal and cut the halves apart. Paste the resulting triangular shapes together on a larger piece of white paper. Paste some colored corners on the right triangles. "Do all triangles have corners like the corners of squares and rectangles?"



Students should learn to discriminate between squares, rectangles, triangles and circles rather quickly. Place models on chalk board showing a sequential pattern, arranged left to right a circle, a square, and a triangle. Have students repeat pattern at their desks with smaller models cut from construction paper. Have them continue the pattern.





Congruent circles, triangles, squares and rectangles.

Poem - Author Unknown

TRIANGLE

Mr. Triangle is my name.
Sometimes my sides are all the same.
Sometimes one side is very small;
And other sides are oh so tall.
Together I have sides of three.
How many triangles do you see?

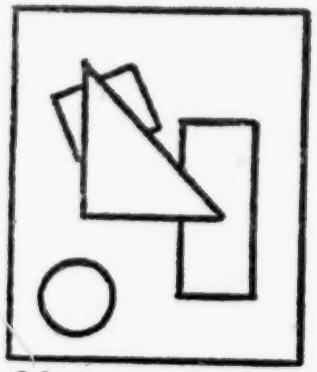
The teacher can take a piece of paper that is square and another that is a rectangle and cut each from point to point forming triangular shapes. Give children precut squares and rectangles. Let them cut from point to point to make triangles; after which they may create their own triangle pictures.

Read story, The Wing on a Flea
By Ed Emberley

The children could make pictures of triangular shapes which may be pre-cut or cut by the children. Paste shapes on a larger piece of paper.

Have the students match shapes on a flannel board or chalk board. Pin precut triangles, circles, squares and rectangles on children. Have students mingle, and then on a given signal, pair up by finding a partner with a matching shape. Variations can be made calling for matching of unlike figures (squares find circles) or colors (blue triangles find yellow triangles). 13:11 9:xu

Geometric designs.



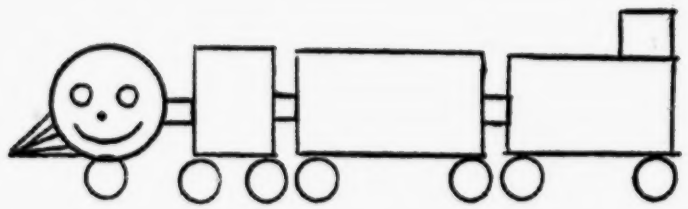
8½"x 11" black

Guide students to cut out squares, rectangles, triangles, and circles by tracing around objects, e.g., boxes, cans, etc. Make triangles by folding and cutting rectangles along diagonals. Let students choose both colors and shapes they wish. Arrange the paste on black (or white) construction paper. Display geometry designs. Discuss what designs they like best.

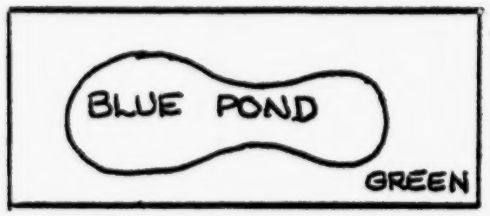
Review the various shapes to make sure all children are aware which could be used in this project.

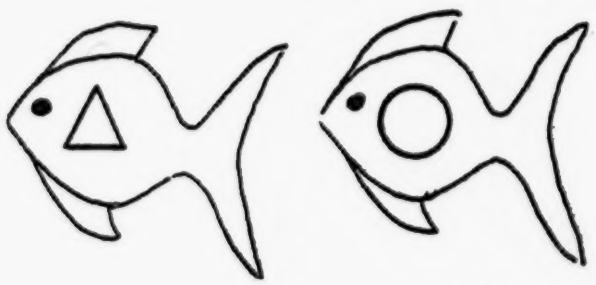
If your children are having problems cutting out shapes, have pre-cut shapes for each group of children during the geometry shape units.

Make a train using the four geometric shapes.



Children cut many shapes to combine together on a large 12" by 18" sheet of construction paper. The connecting together may be done with the line shape.

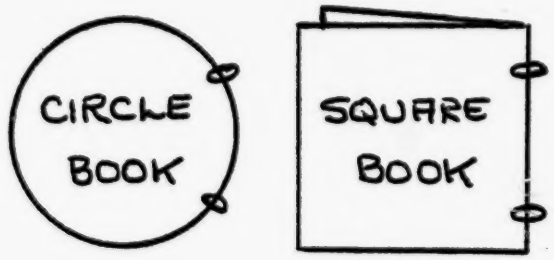




Fish Game.

Have a pond made from cardboard with a section in the middle for a pond. The surrounding area is green. The fish are made of cardboard in many different shapes and sizes. The Geometry shapes are attached with clips.

Make a fishing pole with a small magnet on the end of the string. Place fish on pond with the geometric shapes fishing down. When a child catches a fish he tells the shape and possibly the color too and then may pass poles on to another child. As a variation you may prepare "frying pans" from frozen pie tins, with foil-wrapped cardboard handles

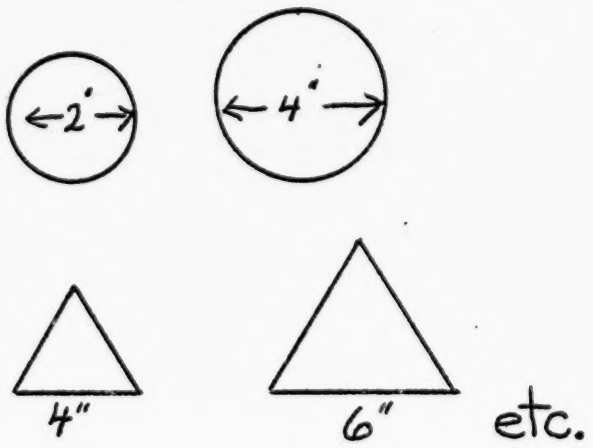


and a different shape on each one. The child must fry his fish in the right pan.

Very good for independent quiet game. Can be used for spelling, phonics, colors, etc.

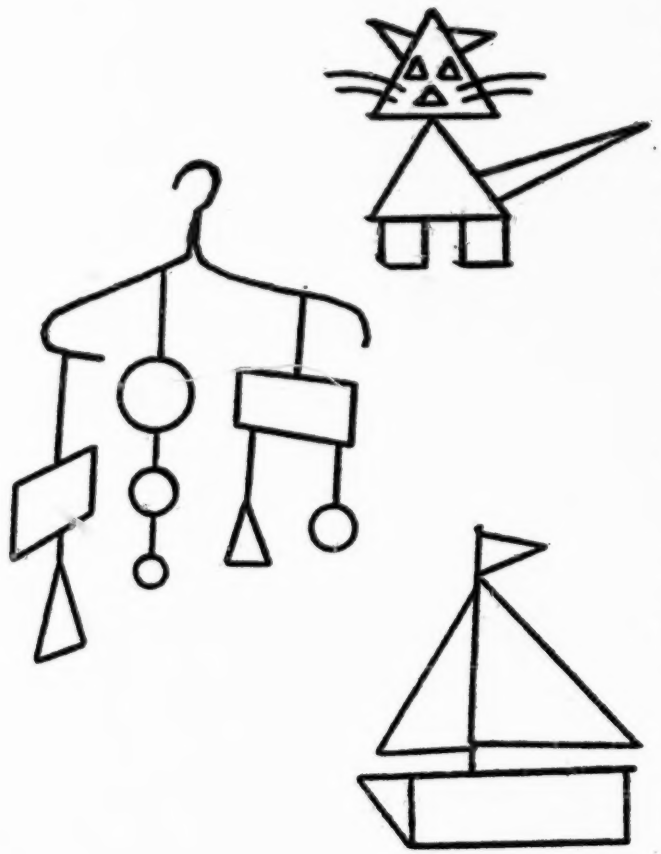
Make a booklet for each geometric shape and have children bring from home (or make in class) pictures of the particular shape you are studying. As they bring them, they may paste them in during choosing time.

As each shape is discussed, children enjoy bringing in a great variety of objects which can be pinned on a bulletin board: jar-lids, pieces of toys, box-tops, drafting tools, and even crackers!



Prepare plywood shapes in varying sizes and colors. Make a drawstring "Magic Bag" from a small hand towel. As each shape is introduced and discussed and as the concept of comparison is introduced, use the bag for Oral English: "I found a red circle." "I found a blue square." "I found a small green circle." "Now we know that we can call the red circle large, because we have two circles and we can compare them." This is an interesting method of answering roll call; the children also like to work with the Magic Bag and a friend at Choosing Time.

Books concerning comparisons:
The Size of It
By Berkley
What is Big?
By Henry R. Wing



With colored construction paper cut strips of paper and paste on background paper to form Geometric shapes. Use various colors for overlapping.

Begin this lesson with line shapes and continue through various shapes and for the final project, combine each of the Geometric shapes to form art projects. (If children have trouble making these shapes, make cardboard samples for each group.)

Encourage students to experiment, making their own designs for place mats, book marks and covers, plaques, calendars, and wall posters.

Make mobiles from wire clothes hangers and geometric shapes suspended by yarn or thread.

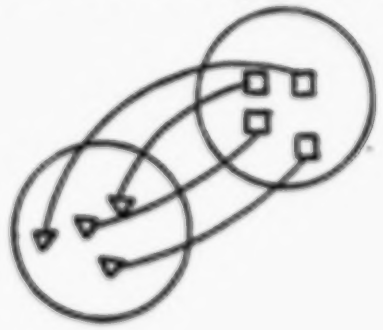
Modernistic pictures of toys and animals can be made using geometric shapes.

KINDERGARTEN

SETS

Kindergarten

Sets that are equal in number.



Vocabulary development

Set--a collection of objects. Members of the sets.

SETS

Equivalent sets contain the same number of members but the members are not necessarily alike.

Use the idea of sets in normal conversation in the classroom, i.e., set of boys present, set of cups in the playhouse, set of blocks in the pattern board.

Many kindergarten experiences lend themselves to the development of a concept of set. We can speak of the set of boys present, set of cups in the playhouse, set of blocks in the pattern board. A variety of materials such as the Structural Math Kit will provide actual objects to manipulate at the right time to emphasize a concept. The flannelboard can be used to provide many experiences in one-to-one matching: "I see a set of witches on the flannel-board and a set of cats on the table. Let's see if there is a cat for every witch." At other times one could use Christmas trees and stars, snowmen and brooms, bunnies and carrots. 4:23

Teachers are familiar with the situation when six children wish to paint, but we have only four easels. The sophisticated child will realize at once that

there is not room for all. This can be followed up with flannelboard experiences in which we do not have a carrot for every bunny--i.e., we do not have equivalent sets.

An understanding of one-to-one correspondence is basic to an understanding of number; therefore, it is important for every child to have many opportunities to make this concept an integral part of his thinking. 4:23

A variety of materials such as the Structural Math Kit will provide actual objects to manipulate at just the right time to emphasize a concept.

Materials for flannel board or magnetic board. They include animals, fruit, stars, storybook characters, stick figures, and geometric shapes.

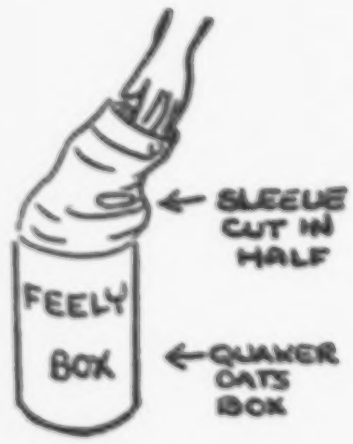
A variety of objects such as books, cars, blocks, pencils, crayons, bottle tops, beads, paper clips, pegs and balls provide manipulative devices.

Pictures of collections make a bridge from the actual objects to the semi-concrete.

When introducing sets to the children, the teacher beforehand can prepare pictures for the flannel board that show sets--including sets of one, two and three. Place these pictures into a treasure box (any small box in which pictures fit). After a little introduction as to what a set is, go to the treasure box and have one child put a picture (set of one) on the flannel board. If they come upon a set that is other than one, they must remove this picture to a chair rather than placing it on the board.

Sets

A Game with Sets



With a feely box or bag the teacher introduces the concept of matching set to set.

The child is to find in the feely box marbles to match the teacher's drawing on the chalk board or objects on flannel board.

Example: The teacher draws a set of whatever numeral is discussed and the child is to put his hand in the feely box--without looking (arrange bag or box so child cannot look in) and try to feel and pull out the same amount as indicated on the teacher's drawing.

Equivalent Sets.
Matching one-to-one

Have the children bring in collections of shells, leaves, pine cones, rocks. Discuss the idea of set. SRA 4:21

Place six cups and six saucers on a table. Ask the children if there are enough cups to put one on each saucer. Ask them if they think there are as many cups as saucers. Lead the children to discover that if they put one cup on one saucer, they will have as many as they need. SRA 4:21

Explain that if it is possible to match the objects of two sets so that for each object in one set there is exactly one object in the other and vice versa, then the sets have the same number and are equivalent.

Show the children that they found it possible to tell whether the sets had the same number without using number names and without counting. All they needed was the method of one to one matching.

Sets

Equivalent Sets.
Matching one-to-one

Reinforce the concept of equivalent sets by having the children match paint jars and brushes, dolls and dresses, two kinds of blocks, or boys and girls.

Give the children opportunities throughout the day to practice one-to-one matching while you are passing out papers, crayons, or scissors.

Have a child demonstrate to the class that he has the same number of fingers on each hand. Tell him he must do this without speaking or counting.

Have the children "act-out" one-to-one matching. Let one set of boys be birds. Let the same number of girls be trees. As each bird flies to a particular tree, one-to-one correspondence is demonstrated. Vary the activity by asking the children to pretend to be flowers and bees, or engines and engineers. In the same vein, let chairs stand for doghouses and the children for puppies; each puppy must find a doghouse. SRA 4:22

Compose jingles to reinforce understanding of equivalent sets. Have the children cut out pictures of cookies from magazines and paste them on pieces of cardboard. Let one child hand out a given number of cookies to the same number of children while the other children recite an appropriate jingle: SRA 4:22

Cookies, cookies.
Cookies, I see.
Cookies, cookies.
Match them with me.
One is for Tommy,
One is for Betty,
One is for sister Sue;
One is for Jerry
One is for Alice,
And here is one for you.

Sets

Equivalent Sets.
Matching one-to-one

Place a set of felt rabbits on the flannel board. 4:24-25
Pass out an equivalent set of felt carrots. Have each child place his carrot beside one of the rabbits on the flannel board, and then lead the class to conclue that the sets are equivalent. Have then explain how they know this.

Be sure that the children understand that the kind of objects in the sets does not matter when one trys to find out if the two sets are equivalent. Provide two equivalent sets of mixed pictures or felt cutouts and have them placed side by side on the flannel board. Repeat this activity with different mixed sets, reminding the children on each occasion that it is the number rather than the kind of objects in the sets that is important. The children will soon discover for themselves that the objects in the sets may be matched one-to-one even though the objects matched are unrelated.

Make sheets on which appear pictures of several boys, cups, or ice-cream cones. Have the children draw a set which is equivalent to the given set. They may wish to draw a balloon for each boy, a saucer for each cup, or ice cream for each cone.

Direct the children to draw a set on a piece of paper. Have them trade papers and then ask each to draw a set equivalent to the set on the paper he has now.

Some children may enjoy cutting pairs of equivalent sets of pictures from magazines. Have the children paste these on heavy paper and make into books--sets of one, sets of two, etc.

A story involving sets may heighten the children's interest. The theme may revolve around a holiday

Sets

Equivalent Sets.
Matching one-to-one

or a season. The following suggestion may be helpful.

Four witches went out one Halloween. (Place four felt witches on the flannel board.)
Each witch took along her jack-o'-lantern to light the way. (Have a child place the jack-o'-lantern beside each witch.
No self respecting witch would go out without her black cat on Halloween (Have another child place a cat under each jack-o'-lantern or each witch.)
Continue the story with an encounter between the witches and four ghosts.

One-to-one correspondence.

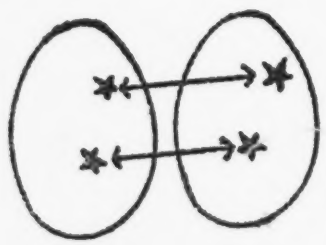
Members of two sets may be paired to find out whether one set has more or fewer members or the same number of like members. 1:44

The volcabulary used is: none, no, not any, set with no members, the empty set, zero.

Every member of a set may be indentified within a set. The vocabulary used is: set within a set, and subset.

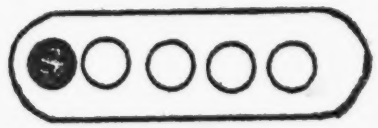
Two sets may be joined to form a new set. The order used in joining makes no difference.

Matching of single sets

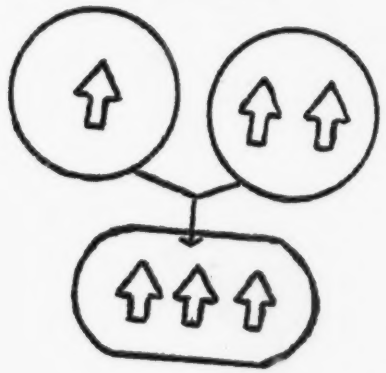


Empty set

Subsets



Set union



Sets
Union of sets

A child can be encouraged to discover that a new set is formed when one set is joined to another. He should be led to discover that the new set contains both of the sets he joined. The child's understanding of this operation of union of sets will form the foundation on which the development of the concept of addition of numbers will be based. 4:61

If at this time you wish to introduce the idea of addition, ask the child to determine the cardinal number of each of the two sets. Then have the child join one set with the other one set to form a new set. The number of the two sets should be determined.

Care must be taken to use precise language so that the concepts of union of sets and addition do not become confused in the mind of the child. When dealing with union of sets the term "join" is appropriate. "Add" and "plus" must be reserved for work with numbers. Avoid speaking about "adding sets." Instead, talk about "joining sets." Never use a plus sign between pictures of sets.

On the flannel board, arrange two sets, each containing one object. Have the children determine the number of each set.

Bring the set on the right close to the set on the left. Say,"A set of one has been joined to another set of one to make a set of two." 4:62

Sets
Union of Sets

Discuss with the children the new set that has been formed. Help them determine the number of the new set, the number of the union of the two sets. Say, "A set of one and a set of one can be put together to make a set of two." Repeat this activity with different combinations of sets.

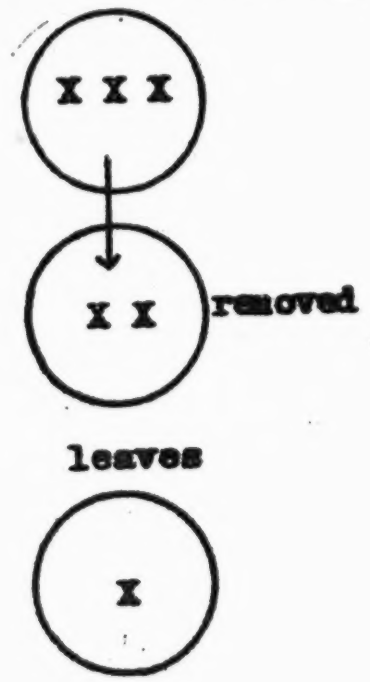
Do not write equations, but simply have children verbalize about the ideas involved with sets and with the numbers of the sets.

Removal of Members of a Set

When members of a set are removed, there is a remaining set.

Vocabulary used: take from, left, remove, remain, remaining.

The same activity could be used for set removal as in set union only start out with larger sets and work down.

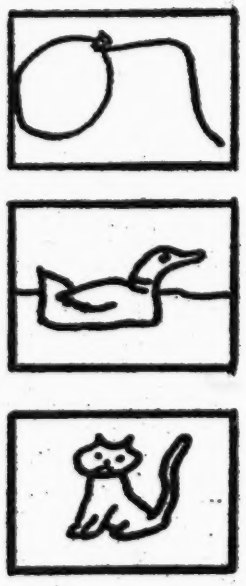


NUMBERS AND NUMERATION

KINDERGARTEN

Counting by saying number names in order in one-to-one correspondence with the members of a set. (The last number named is the cardinal number of the set.)

Sets of "one."

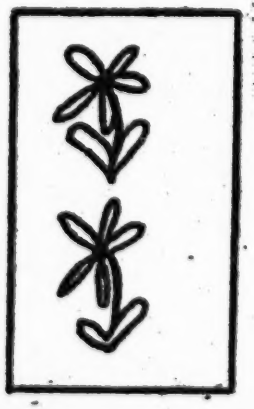


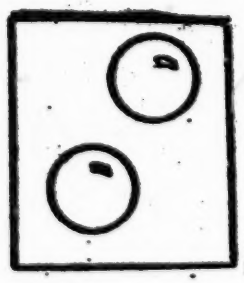
A numeral for the number one is "1."

Number line

A numeral for the number two is "2."

Sets of "two."





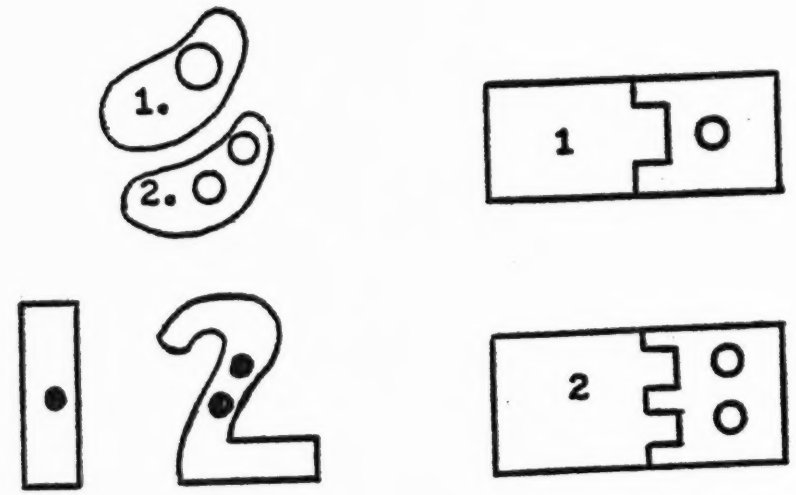
Choose activities and games for the entire class or for a group within the class. These activities involve such concrete, highly sensory experiences as matching objects to the number line, jumping on the number line, manipulating objects and geometric forms, and playing finger-counting games.: 13:1

Place a red disk on the flannel board and have the children tell how many objects they see. Then place another red disk on the board and separate it from the first with a vertical divider such as yarn. Ask the children to tell how many they see then. Place a green disk under the second red disk and guide the children to an awareness that this new set contains one and <u>one more</u>. 3:2

Lead the children to the understanding that this set is <u>two</u>.

Give children a page where they have pictures of "one" object and pictures of "two" objects. Have children mark the pictures of "one."

Supplemental puzzles which require children to match sets with numerals.



Plan a bulletin board on which may be displayed items that come in pairs, or twos, such as mittens, earmuffs, and shoes.

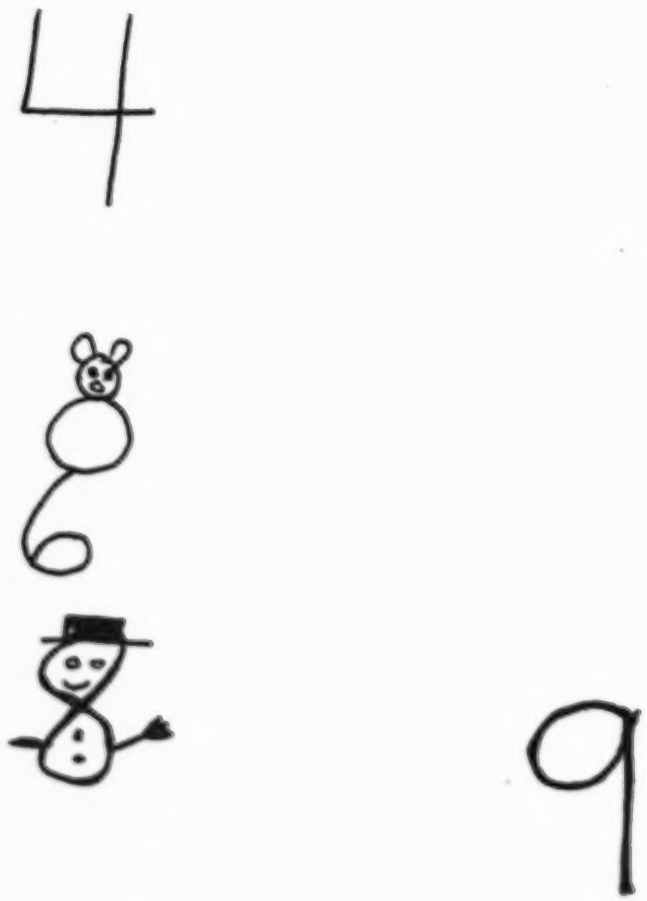
2
3

Before the numeration concept is introduced in detail, groundwork can be laid for correctly made numerals as the calendar is discussed.

"Two" can be thought of as half a valentine sitting on a line.

To make "three," a little man must drive his car around a curve. He turns on to a deadend street which is too narrow to turn around in, and has to come back up on the very same line and go around another curve.

One day the same little man drove down a short



street and turned to the right. Just after he got through an intersection, a reckless driver came whizzing down. This creates a picture of the numeral "four."

Number Men by Louise True contains some useful poems:

Curving down, around, and in,
A monkey's tail I see.
That is how I make a six,
As easy as can be.

Around and down and up we go
Like a snowman in the snow.
Standing round and tall and straight,
That is how I make an eight.

A round balloon upon a stick
Such a funny circus trick.
Close the circle and draw a line,
That is how I make a nine.

As one or two children make the numerals on the chalkboard, the rest gain valuable practice making them in the air, pretending to be skywriters.

"Three"

A numeral for the number three is 3

12" numerals can be cut from oaktag with a green disc for "GO" and a red disc for "stop." The child follows the path with his fingers. These numerals can be on a table for independent activities. Numerals made of sandpaper can be backed with tagboard to make them sturdy and can be "felt" by tracing with fingers.

The word "numeral" should be used only when there is a reference to the mark or written symbol. In all other cases the word "number" should be used.

Concepts of "Four" through "Ten" should be presented as one, two and three have been. Use appropriate numbers of objects.

Review--place a set of one and a set of two side by side. Ask the children to explain what "two" means. "One more" is the idea to keep uppermost.

Place a set of three next to a set of two. Have two of the elements in one color and the third in another color to show "two-and-one-more." Have the one child place the appropriate numerals under the set of one and the set of two.

When the children first discuss the fact that "1" is a symbol that can be used to describe a set of one, they will like telling the "Story of One." Provide a large piece of oaktag on a chart rack with a large number 1 in the center. They can cut pictures from magazines which show the "One" story to paste on the chart. Meanwhile, the Magic Bag has been used to locate the one colored bead and the numeral one has been added to the large wooden number line. When the chart is filled, say:

The chart is full--now what shall we do?
Let's see if we know the Story of Two!

As they verbalize the concept that Two is "one and one more," provide the magic bag with two beads, the large wooden numeral two, and a new chart for the Story of Two. In time,

The Story of Two is full as can be--
It's time to learn the Story of Three.

A child may indicate that Three is "one and one more and one more." Flannelboard discs which carry out the "one more" pattern will help them see that it would be easier to call it "Two and one more."

Who can tell the Story of Four?
Why, it's just three and then one more!

Sure as you're alive,
It's time to learn the Story of five.

In each case, the teacher of course, does not need to finish the poem. The children's knowledge of rote-counting, and their delight in rhymes, will take care of this.

There are many poems which can be used with the flannelboard to reinforce the concepts of number and numeration, helping the child to recognize sets at a glance. Patterns for some of these poems are included in the appendix pp. i through xv. The teacher will find it easy to lay a good medium weight pellon on top of the pictures and trace them with a felt pen. They can then be colored with ordinary crayons and will withstand many hours of use by teacher and children.

Three Balls

Here is a big, round bouncy ball; 4:31
I bounce it, 1, 2, 3.
Here is a ball for throwing;
I can catch it,
Watch and see.
Here is a ball for rolling;
Please roll it back to me.
Bouncing -
Throwing -
Rolling balls -
Let's count them; 1, 2, 3!

I See Three

I see three--one, two, three, 4:30
  Three little bunnies
  Reading the funnies.
I see three--one, two, three,
  Three little kittens
  All wearing mittens.
I see three--one, two, three,
  Three little frogs,
  Sitting on logs.
I see three--one, two, three,
  Three little bears
  Climbing upstairs.
I see three--one, two, three,
  Three little ducks
  Riding on trucks.

(for flannelboard patterns see appendix pp. i-v)

Three Little Nickels

Three little nickels in a pocketbook new; 15:50
One bought a peppermint, and then there were two.
Two little nickels, before the day's done,
One bought an ice cream cone, and then there was one.
One little nickel; I heard it plainly say,
"I'm going into the piggy bank for a rainy day!"

Five Golden Pennies 15:12

Five golden pennies in my purse;
This one's for some gum;
This one's for a lollipop;
This one's for a drum.
These I'll save inside my purse,
Until our birthdays come!

(How many will I save?)

Valentines

15:72

Five gay valentines
From the ten cent store;
I sent one to Mother,
Now there are four.
  Four gay valentines,
  Pretty ones to see;
  I gave one to Brother,
  Now there are three.
Three gay valentines
Yellow, red and blue;
I gave one to Sister,
Now there are two.
  Two gay valentines,
  My, we have fun;
  I gave one to Daddy,
  Now there is one.

One gay valentine,
The story is almost done;
I gave it to Baby, and
now there are none.

Fred and His Fishes 4:35

Fred had a fishbowl.
In it was a fish,
Swimming around with a swish, swish, swish!
Fred said, "I know what I will do.
I'll buy another and that will make _____."
(Have the children give the missing word.)

Fred said, "I am sure it would be
Very, very nice if I just had ____."

Fred said, "If I just had one more,
That would make one, two, three, _____."

Fred Said, "What fun to see them dive,
One, two, three, four, _____."

How many fishes to do you see?
How many fishes? Count them with me!

(for flannelboard pattern see appendix p. vi)

Purple Violets 4:35

One purple violet in our garden grew;
Up popped another, and that made two.

Two purple violets were all that I could see;
But Billy found another and that made three.

Three purple violets if I could find one more,
I'd make a wreath for Mother, and that would
make four.

Four purple violets -- sure as you're alive!
Why, here is another! And now there are five!

Five Little Goslings 15:28

One little gosling, yellow and new,
Had a fuzzy brother, and that made two.
Two little goslings now you can see;
They had a little sister, and that made three.
Four little goslings went to swin and dive;
They met a little neighbor, and that made five.
Five little goslings, watch them grow!
They'll turn into fine, big geese, you know!

Turtles

One little turtle feeling so blue; 15:94
Along came another. Now there are two.
Two little turtles on their way to tea;
Along came another. Now there are three.
Three little turtles going to the store;
Along came another. Now there are four.
Four little turtles going for a drive;
Along came another. Now there are five.

Five Little Seashells 4:37

Five little seashells lying on the shore;
Swish! went the waves, and then there were four.

Four little seashells cozy as could be;
Swish! went the waves, and then there were three.

Three little seashells all pearly new;
Swish! went the waves and then there were two.

Two little seashells sleeping in the sun;
Swish! went the waves and then there was one.

Sets of Six

One little seashell left all alone
Whispered "shhhhhh" as I took it home.

(for flannelboard patterns see appendix p. vii)

Counting Kittens

One little kitten with a furry tail; 4:46
Two little kittens lapping milk from a pail;
Three little kittens rolling on the floor;
Four little kittens running out the door;
Five little kittens roll a yellow ball;
Six litt[illegible] kittens and now that's all.

(for pattern see appendix p. viii)

One, two, three, four, five, six.
First they were eggs -
Now they are chicks!

Fold a paper in half. On one side have six children (one at a time) come up and color a big egg. On the other side of the paper have them make six chicks.

Waddle, Waddle, Waddle 15.
The baby ducks go
Waddling after mother duck
Seven in a row.

Have some of the children dramatize this poem while the other children are reciting it.

High in the sky, in the shape of a V 15.
How many geese can you see?
Hurry and count them as they fly by,
You will see nine geese and so will I.

Sets of nine

Sets of ten

See My Clothes 15:56

One, two, three, four, five, six, seven, eight,
nine;
See the dresses on the clothesline;
Some are Mother's and some are mine,
But all of them dry in the yellow sunshine.

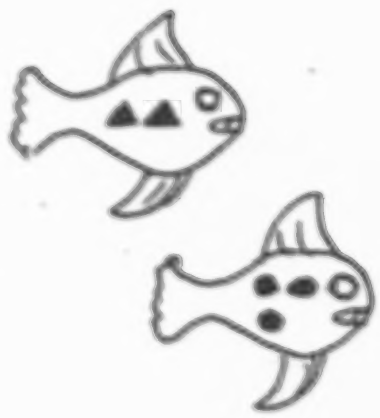
How Many? 15:57

Ten little socks hanging on a line;
If one blows away, that will leave _____.
Nine little flowers by a garden gate;
I picked a red one, and that leaves _____.
Eight pretty stars shining in the heaven;
A cloud hides one, and now there are ____.
Seven little boys playing with bricks
One boy runs away, and now there are_____.
Six little bees in a beehive;
One bee flips away, and now there are ____.
Five little chickens by the barn door;
One goes inside, and then there are _____.
Four little ducks in a pond you see;
One duck takes a dive, and so there are ____.
Three little kittens crying, "Mew, Mew!"
One finds a ball of yarn, and then there are ____.
Two little bunnies hopping in the sun;
One looked for a carrot, and now there is ____.
One little puppy having lots of fun;
He chased a cat, so now there are <u>none</u>.

One kitten with a furry tail;
Two kittens on the floor;
Three kittens in the apple tree;
Four kittens at the door;
Five kittens roll a yellow ball;
Six kittens gently purr;
Seven kittens watch a mouse;
Eight kittens wash their fur;
Nine kittens lap their morning milk;
Ten kittens chase a hen
Help me count the kittens:
1--2--3--4--5!
6--7--8--9--10!
(See appendix p. viii)

Easily made games help the children associate the appropriate numeral with any set whose number is less than five:

Make five boy dolls and five girl dolls from 4:33
pellon. Place numerals from 1 through 5 on the boys and sets from one through five buttons on the girls. If the dolls are all mixed up on the flannelboard, the children will delight in matching them correctly. (See appendix pp. ix and x)



Place sets of less than five on many different 4:35
fishes, attach a paper clip to the mouth of each fish, and place them in a large paper fish pond. Suspend a magnet from a fishing pole. Each child in turn catches a fish and "fries" it in the appropriate frying pan. (See appendix p. xi)
These are made from aluminum frozen pie tins with handles made of foil covered cardboard. A large oaktag numeral makes it easy to see which is the correct pan. Interest is maintained if not a word is said during the game (in order not to

Nonverbal instruction activities

Use number lines --

1.

2.

3.

disturb the fish); as the game is repeated, few children will find it necessary to count the spots but will recognize the number at a glance.

Pretend you have laryngitis--Then draw a line on the board. (See example 1) 13:2

Begin labeling the line, making no comment, (2) Stop. Don't label the fourth place but look at the class expectantly. If necessary, tap once at one, twice at two, etc. Label the fourth dot if you need to. After that you should have all the help you need.

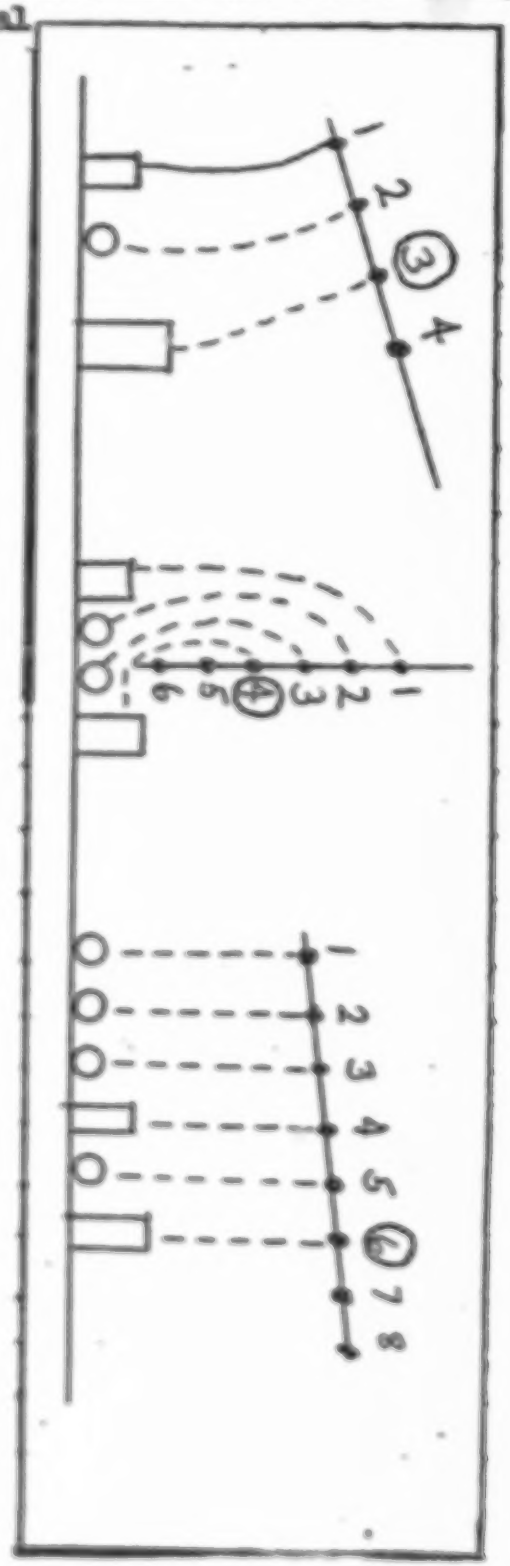
Use numeral tags to label the points on the line, (3).

Let the children help you complete (4) or re-arrange your number line, (scramble the numerals and have the children put them back in the correct order), (5).

4.

5.

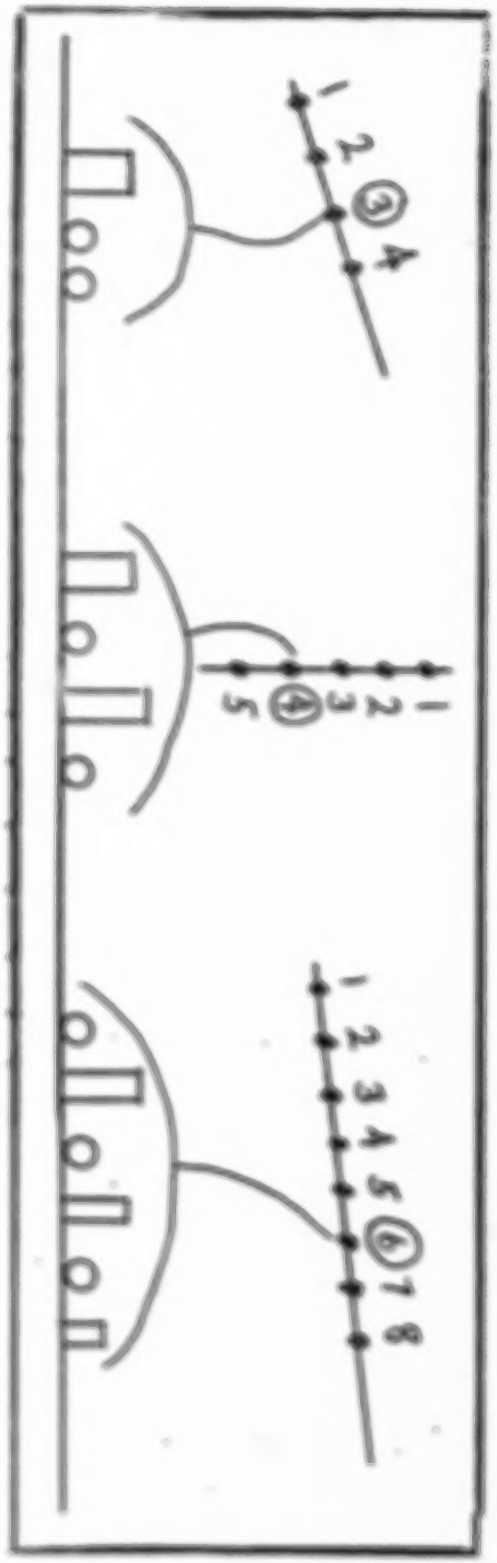
Use of number line for counting objects -- nonverbal



Draw several lines on the chalkboard at various angles numbered from left to right. Under each arrange groups of objects such as rulers, erasers, blocks, circles, etc. 13:3

Draw one line connecting an object with one on the number line above it--that is all you will need to do. Everyone will want to help.

This proceeds at a fast pace. (It might be best to line them up before you begin, since once you have lost your vocie you really ought to let it stay lost.)



Now erase all connecting lines. Make the kinds of connections seen at the left.

The activity is very different here. In the preceding activity one-to-one matching is done, connecting objects to numerals on a number line.

This activity is more sophisticated. Children must count a group of objects and select the number that is the answer to "how many?" 13:3

Please don't explain these ideas in words. Communication would be broken and misunderstanding develop.

As a final variation, leave all the grouping lines and the connecting line but remove all the objects and place them on a table nearby.

Look for help. As volunteers replace the objects there will be no concern about having them in the same order or groups as before. This is counting and we can count any collection to suggest the idea of the number 5.

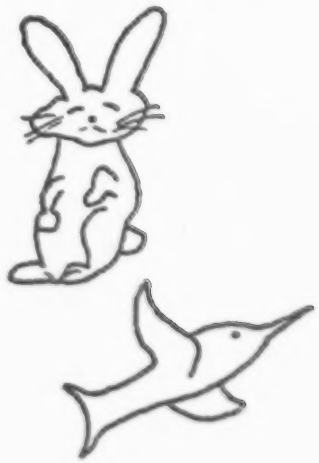
After numerals have been introduced from 1 to 10 a good game for recall and review is creating the numeral sets and numeral words on tagboard and placing into individual pockets. The pockets are placed in order (1-2-3-4) down the cardboard or chipboard booklet.

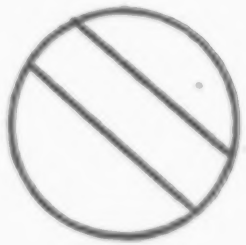
They will take all of the little cards out at one time (1-10) and place numeral 1 - set 1 - word one, first --- 2 second etc.

This will be good for the numeral set concept and also reading each of the numeral's names.

Make clowns out of tagboard and have numerals on each clown. (1-10) Each clown has a hat which belongs to him. Match the numeral hat to the numeral clown by placing hat on clown's head.

Ordinal numbers first, second and third





The five year old is egocentric. He thinks of things in terms of "me first." He learns that other positions have special names. 3:40

Place three objects on the flannel board. Establish a beginning point. Kindergarteners are not left to right oriented so he may begin at the right. Point to the first object. Now ask another child to point to the second object. A third child may point to the third object.

<u>Variation</u>. Say: I am thinking of a bird. What is its position?

Play "switcho." Ask a child to switch the first and third objects. Switch the second and first objects. 4:40

Have three children stand before the group. Have the children listen carefully and follow the directions. The first child must hop on one foot, the second child clap hands two times, etc. Vary the directions.

Ordinals 1-3

This Little Clown

The first little clown is fat and gay 4:41
The second little clown does tricks all day;
The third little clown is strong and tall;
I like the clowns, I like them all.

(See Appendix pp. xii-xiii)

Little Kittens

Three little kittens; see them play. 4:42
The first is brown, the second gray.
The third one has soft paws and a wee little nose;
He hears a noise, and away he goes.

(See Appendix p. ii)

Three Little Oak Leaves

Three little oak leaves, red, brown and gold, 15:119
Were happy when the wind turned cold.
The first one said, "I'll be a coat for an elf;
He'll be able to warm himself."
The second one said, "I'll be a home for a bug,
So he will be cozy and snug."
The third one said, "To a tiny seed I'll bring
A coat to keep it warm till spring."
Three little oak leaves, red, brown and gold,
Were happy when the wind turned cold.

Ordinals 1-5

Five Little Puppies 4:44

Five little puppies were playing in the sun;
The first one saw a rabbit, and he began to run;
The second saw a butterfly, and he began to race;
The third one saw a pussycat, and he began to chase;
The fourth one tried to catch his tail, and he
went round and round;
The fifth one was so quiet, he never made a sound

 (See Appendix P.xiv)

This Little Chick

4:44

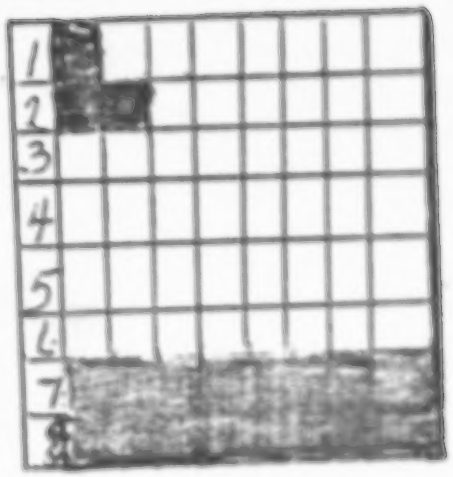
The first little chick ate corn today;
The second little chick ate worms, and they say:
The third little chick ate yellow meal;
The fourth little chick ate potato peel;
The fifth little chick, like a fluffy ball,
Ate a teeny, tiny bit of all!
Corn today! Worms they say! Yellow meal!
Potato peel!

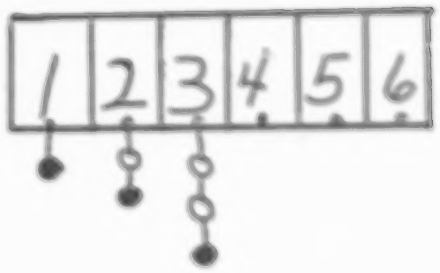
(See Appendix p. xv)

Five Little Mice

15:89

Five little mice on the pantry floor;
The first little mouse peeked behind the door;
The second little mouse nibbled at the cake;
The third little mouse not a sound did make;
The fourth little mouse took a bite of cheese;
The fifth little mouse heard a kitten sneeze;
"AH-CHOO!" sneezed the kitten and "Squeek!"
they cried,
As they found the hole and ran inside.





To help some children gain a better understanding of the numeral eight, provide graph paper with 1 inch squares. This paper should be numbered down from one to eight. Have the children color the correct number of squares that each numeral designates. By having the children color each set of squares in a different color it will help them see that each set is "one more" and how many are in a specific set. This can be done with several numerals.

The immature child may find this activity frustrating. He can be given square blocks of wood to line up in this same fashion.

As each numeral is studied, a variety of visual aids will provide interest and excitement. A number line can be constructed from heavy cardboard with a hole below each numeral. Suspend painted beads fastened with bent wire from an old coat hanger. Use contrasting colors to show the "one more" pattern. 4:27

The children enjoy manipulating a wooden number line with cup hooks to which may be attached 5" wooden numerals. As the class becomes more familiar with the sequence and appearance of each numeral, they will profit from a game in which the teacher mixes up the number line while they are out of the room. This gives many opportunities for language development as they direct the correct placement. "The 3 has to be on the left of the 4;" "The 9 is backwards, because the stick must be on right," etc.

The teacher must be completely unable to hear incomplete directions such as "over there," or sentences which give less than a complete thought.

ORDER AND RELATIONS

Kindergarten

Kindergarten

Vocabulary development.

Size relationships:

largest, smallest, nearest farthest, larger than, smaller than, taller, shorter, under, over, on top, here, there, together, apart, equal, not equal, the same size as, today is, tomorrow will be, yesterday was, the same number in a group.

Although most children enter kindergarten today with a broader background of experience than in former years, it cannot be taken for granted that all concepts they bring to school are really clear to them. The teacher, by listening to a child talk and asking him questions, can evaluate the true understanding that he has in relation to his environment. Some children come with a void of math understanding. Verbalization and manipulation of objects ready such a child for basic math concepts needed at this level.

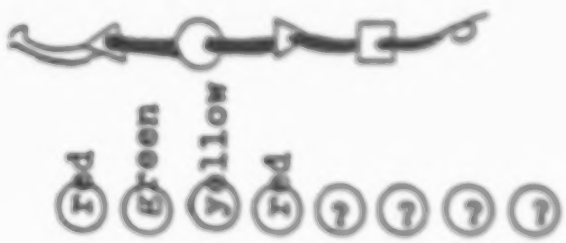
Relative sizes through comparison.

size of shoes of classmates, size of children, size of children's desk and teacher's desk, etc.

Lead the children to understand that we must have two objects in order to compare and use the "larger" and "smaller" when speaking of two objects: "largest" and "smallest" when comparing more than two objects.

Patterns and sequence.

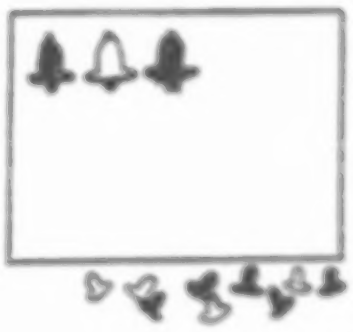
An understanding of patterns and orderly arrangement is basic to an understanding of number sequence. Many opportunities present themselves for recognition and repetition of pattern in the classroom: geometric



1. girl, boy, girl, boy.
2. girl, girl, girl, boy,
   girl, girl, girl, boy.

shapes, children in a line, flannelboard objects, stripes on a boy's shirt or a girl's dress.

Pupils may develop patterns using different colored beads on a string. ("Put one red bead on the string, then one yellow bead, then one red bead. What color do you think will come next?")

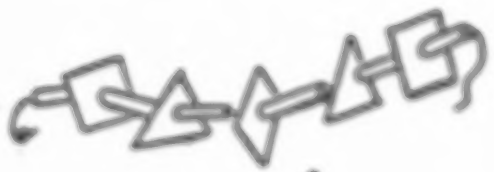


Use colored felt cutouts of bells, apples, and houses. Using the flannelboard, start a pattern. Let volunteers choose and put up the next one. Have pupils tell why they chose the one they did. Other objects that can be arranged might include spoons and cups, cans and lids, bottle caps and jar lids, square and triangular shapes (pre-cut forms and gummed stickers). Encourage the children to feel the edges of the shapes (trace around them with their fingers).

Play "Guess My Pattern." Line up red shirt, blue shirt, red shirt, and see if they can tell what the pattern is and which child should go next. This can be done with long hair, short hair, long hair, or sometimes blue

shoes, red shoes, blue shoes, depending upon how the children are dressed.

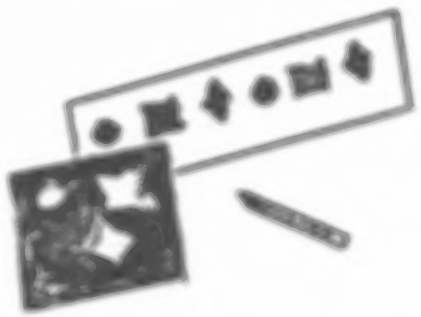
Pattern



Have pupils string pre-cut paper shapes with short spacers cut from paper soda straws. Macaroni may be painted with tempera and strung for Indian necklaces. 3:18

Pupils can cut and paste their own patterns on stripe to form Indian head bands and belts.

Block prints may be made, using potato blocks that are textured by scraping with silverware. For "gadget prints," use bottle caps, kitchen tools (egg whips, potato mashers, etc.), dipped in tempera. Clay may be used for hand-formed blocks.



For a stencil pattern, the teacher may cut simple stencils from tagboard and have the children use them as guides for crayon coloring.

Lead children to say what the meanings of pattern and repeated are, in their own ways of thinking.

After many, many experiences with concrete objects in pattern-making and pattern-repeating, children may be asked to color the remaining members of a set of alternately colored objects.

Equipment for Experimenting with Numbers.

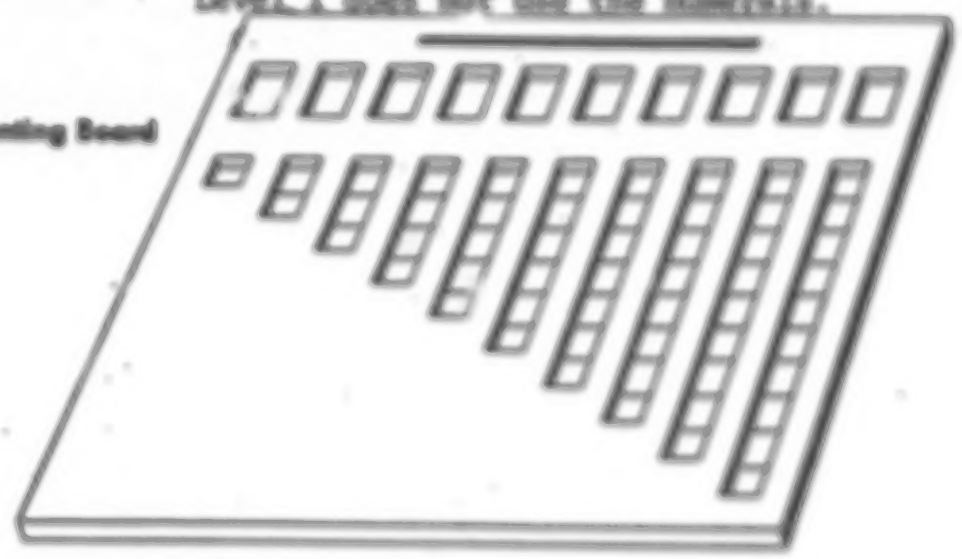
The drawing below and on the next page show all the equipment that is used in Experimenting with Numbers. This equipment may be purchased as a set (Kit 1, Part A), or as individual pieces, from Houghton Mifflin Company.

The Stern Structural Arithmetic Kit has an excellent program for the initial learning without use of number name or counting in developing patterns, estimating comparing, finding errors and making corrections by themselves. The blocks are designed so that with a minimum of "telling" by the teacher, children are able to make discoveries in sound mathematical ideas without number or counting. (Level 2 does use number and counting.)

Number Guide

Level 1 does not use the numerals.

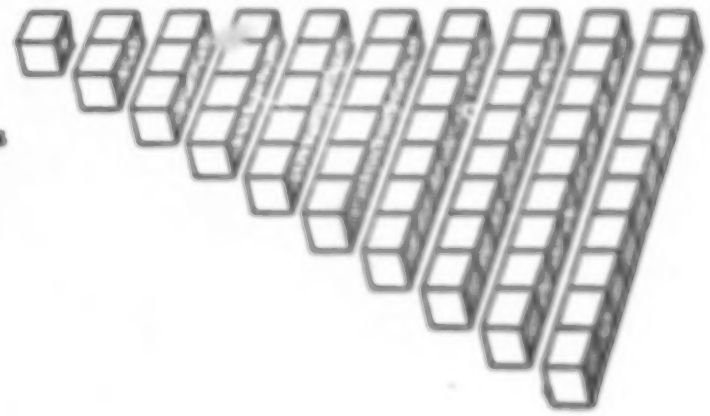
Counting Board

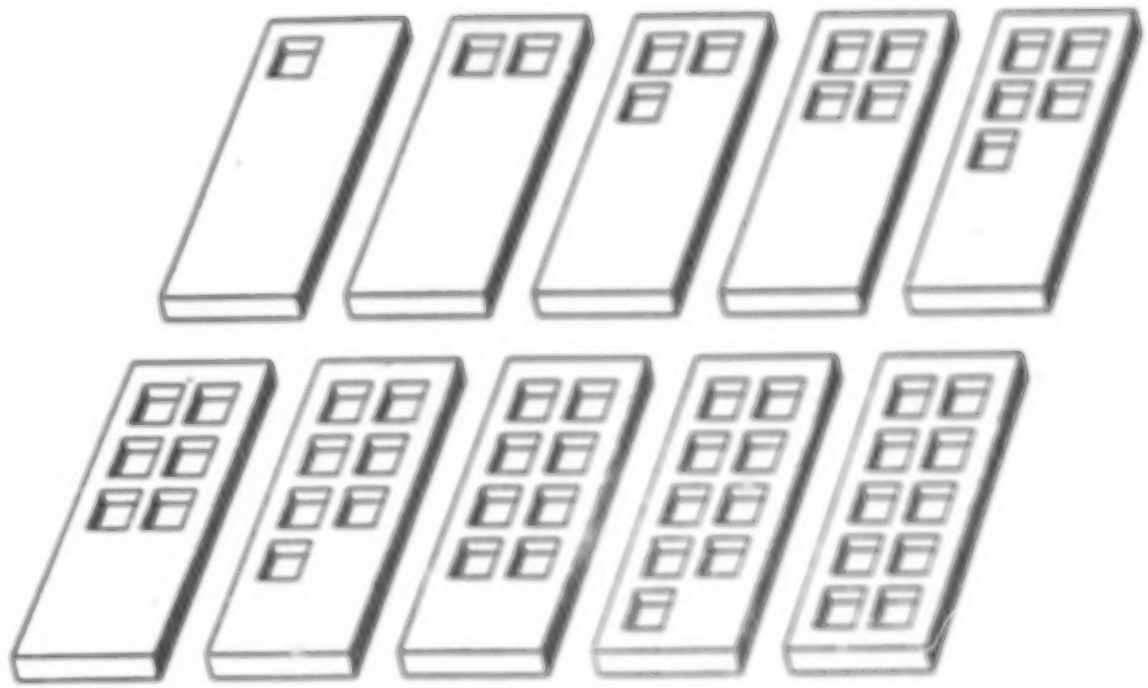


Number Markers (2)

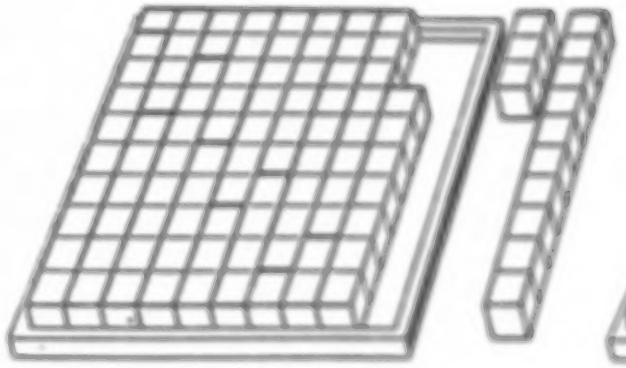
Level 1 does not use the numerals

Unit Blocks

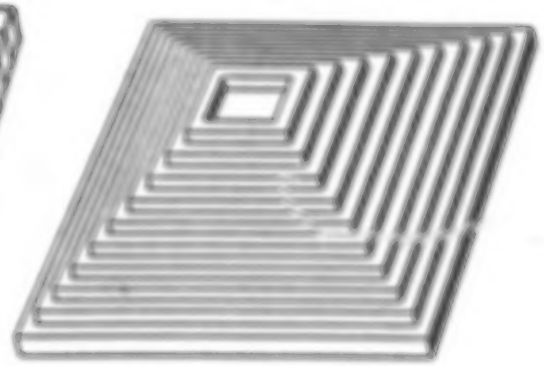




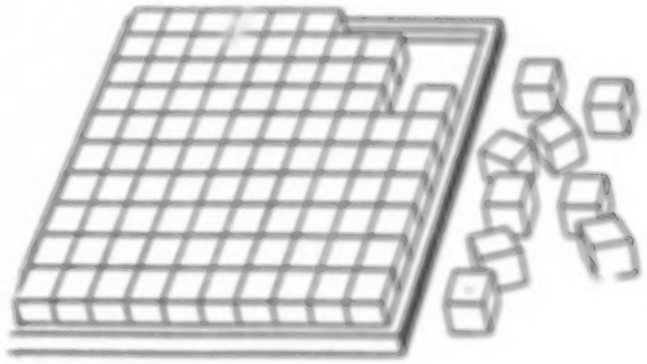
Pattern Boards



Unit Box with Unit Blocks



Number Cases



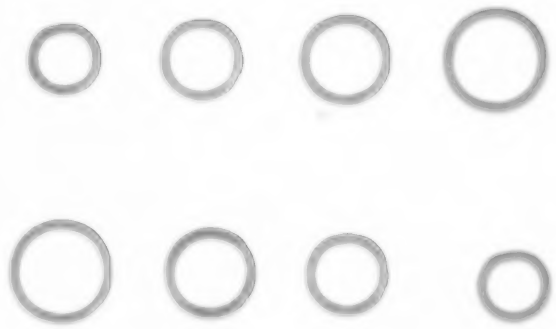
Box of 100 Cubes

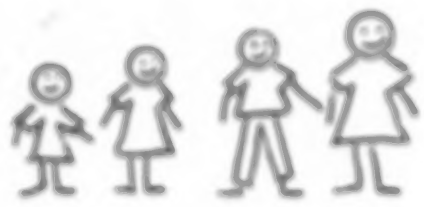
Miscellaneous Markers

"Repeat the pattern."

Number lines can be given to pupils to match samples on chalkboard. Pupils can be given pieces of colored paper and asked to line up in front of the class to form a pattern. 13:10

Seeing Patterns.





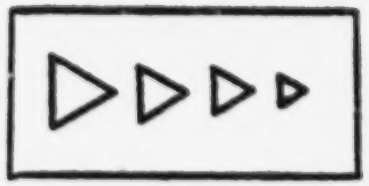
Extending a pattern can help to develop the feeling of sequence and relation and provide excellent groundwork for counting beyond ten.

Place the smallest shape on the flannel board; ask a child to place the next larger shape beside it; a third child may then extend the pattern with a still larger shape. By placing the first shape well to the left, the children will automatically choose a left-to-right sequence. Produce a still larger shape and ask where it should go. Use the term "increasing" to describe this arrangement. 4:20

The shapes may be removed from the flannel board and the pattern can be constructed in "decreasing" order. Some classes like to call these patterns "get bigger" and "get smaller."

Children will find and bring in increasing and decreasing patterns in many concrete objects--rocks, acorns, seashells, jar lids, stuffed toys. They enjoy arranging these on a table top or gluing them to pieces of oak tag. One child reported that their small car and station wagon were parked in the driveway. When he put his two-wheeler beside the little car, he had a "get bigger" pattern. Another class was delighted to find a "get bigger" pattern in the book "Fly Went By."

This is another chance to play a line-up game, choosing children by size in order than they can make either an increasing or a decreasing pattern. Four children of varying sizes can be chosen, while a fifth child "manipulates" them into the proper order.



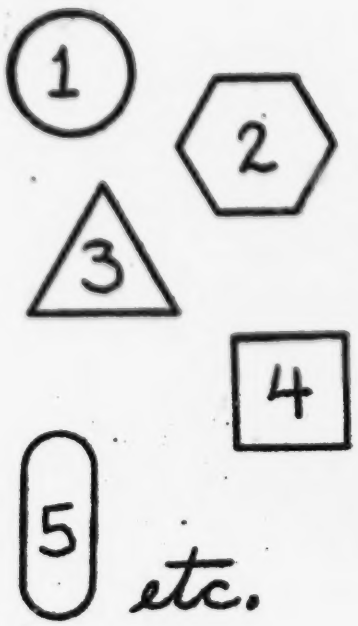
Worksheets which require the children to extend a pattern are very difficult; their hand-eye coordination is not developed enough to allow them to decide the size they wish the shape to be, and then to draw it that size. Rather, they can successfully show their understanding of the concept with cut-and-paste geometric shapes, if verbal evaluation does not seem sufficient.

Many teachers have found it wise to allow some time to elapse between the introduction of "Repeat the Pattern" and "Extend the Pattern," to avoid confusion.

Display any number from 1 to 10 and ask the children to hold up the number which precedes it, then the one which follows it.

Assign a number (1 - 10) to each of ten children and have them form a line in mixed numerical order.

Tell them they are the boxcars on a train which is mixed up and must be reassembled in proper order from one to ten with the engine (No. 1) on the left. Then time them to see how fast they can line up properly. This is a good outdoor activity.



Put numerals on differently shaped cards and have each card a different color.

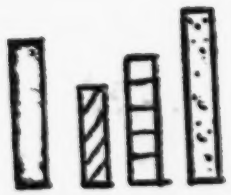
Ask questions such as: "Compare the number on the blue circle with the number on the red square. Which is greater? Which is less? How much less?"

ADDITION & SUBTRACTION

KINDERGARTEN

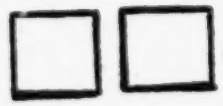
KINDERGARTEN

Adding and subtracting



Which is the largest?

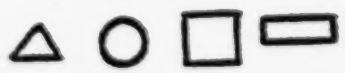
one

 one more

more than...

less than

Complete the pattern.



Which shape is missing?

ADDITION & SUBTRACTION

Early in the year, begin having the children decide on larger, smaller, more than, less than, using different sized colored sticks, pieces of paper, etc.

Use blocks to show one more and one less before using the names of the "numbers."

Supply many experiences for comparison of size and amount before starting formal "arithmetic" presentation. This "pre-number" arithmetic builds vocabulary and concepts involving relationships (greater than, less than, equal to), sequence (patterns of increasing or decreasing sizes of shapes).

Wherever possible, have pupils use "holdable" materials rather than workbook or duplicated materials. Students will be able to grasp ideas better.

Combinations through five.

Some children may be able to grasp the abstract idea of the operations. More, however, need to continue using concrete objects such as the Stern blocks, beads, bottle caps, children, etc. 12

Subtraction as the inverse.

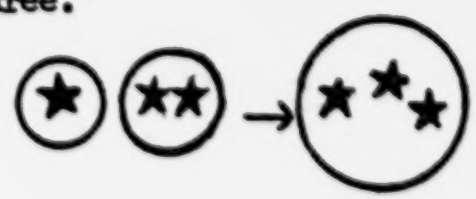
The commutative idea can be demonstrated easily and applied by manipulation of objects, the use of the flannel board, using groups of children, etc.

The nested blocks of the Stern Structural Math is one excellent source for building this concept in a meaningful, useful, concrete, way.

Readiness for addition and subtraction through work with sets of objects.

Do not write number sentences. Let the children put their reactions and discoveries into words. (Writing sentences can come late in the year.) 3:61-79

A set of "two" joined to a set of "one" gives a set of "three."

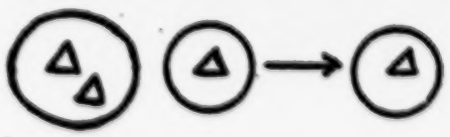


Children should be led to discover each of the members of the other two sets.

Only <u>disjoint</u> sets (no elements in common) will be used on this level. 13:10,11

Later, the name "sum" will be applied to the union of two groups.

A set of "two"...remove a set of "one"... gives a set of "one."



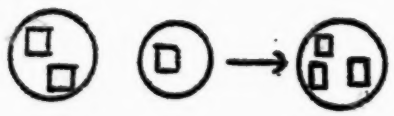
Terms such as "set union," "set separation," "commutativity," etc., need not be used with the children. <u>Ideas</u> are more important than <u>vocabulary</u>.

Use many manipulative devices. Have pupils use domino cards, felt cut outs, fingers, yarn outlines, etc. to tell number stories.

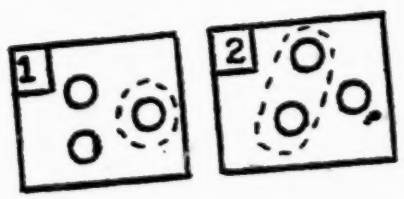
Refer to groups as they naturally occur during the day: number of people in areas; milk cartons carried by 2 or 3 children.

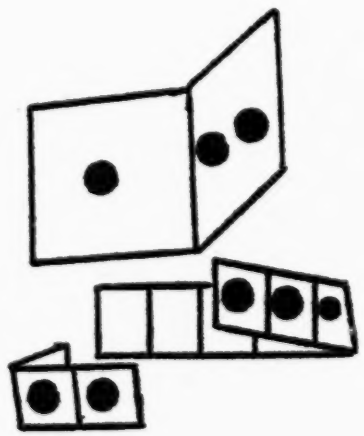
Commutative idea through sets.

A set of "two" joined with a set of"one" gives a set of "three."



A set of "one" joined to a set of "two" gives a set of "three."





Use solid objects, such as apples, oranges, etc. to act out these number stories. 13:16,17

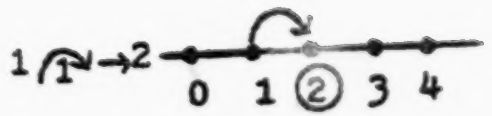
Finger games can be used: ask pupils to "fix your fingers so there are three fingers up...can you do it with one hand? Have one finger up on one hand. How many on the other hand?..." (Change hands or cross arms so that the idea of "order makes no difference" is seen). Have pupils give other "problems."

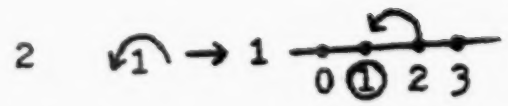
Have pupils circle the correct number of objects drawn in groups on the chalk board. "How many are there? Draw a ring around one... How many are left?"

Fold back one side of a domino card. "How many more will make three?"

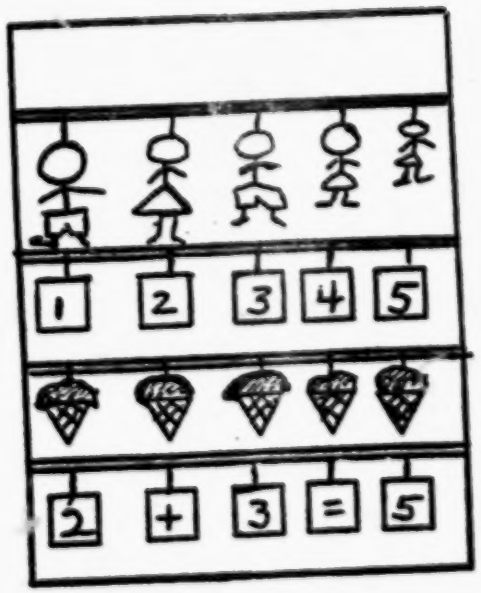
"Counting strips" of different lengths may be made of tagboard, divided into squares (about 3"). 13:14 On one side, make solid black disks.

Number line activities to demonstrate the joining and separating of sets.





Addition and Subtraction



Use floor number line until pupils are acquainted with the use and meaning of this device. Pupils step or jump from one numbered place to another. Include 0-10 spaces. Mystic tape is usable. 12" to 18" intervals may be "comfortable" spaces for steps.

After pupils become accustomed to the floor number line, introduce a similar one on the chalk board. A suggested "code" is shown: 13:12,13

Start	Number of jumps	Landing Place
1	1	2

Little by little, develop different "jumps," and work in opposite directions; show the "undoing" property (inverse), but don't burden the child with vocabulary beyond his reach. Let <u>him</u> explain what happens. Have pupils close eyes and <u>listen</u>.

This board has proven to be a useful manipulative device which allows pupil participation and avoids teacher verbalization.

Place several rows of cup hooks or "L" hooks on a large piece of plywood or peg board. Make pipe cleaner people (with oak tag skirts and pants) which can be placed on the top row of hooks to represent "My Family." Tagboard cards containing numerals can be added on the second row of hooks for counting purposes. Have a set of ice cream cones; find out if there is one cone for each family member.

Addition and subtraction rhymes

Relocate the people so there is a set of grown-ups and a set of children. Mature children will enjoy cards which tell a number story. As the families are placed on the hooks, many children will automatically locate them from left to right, and in an increasing or decreasing pattern. As Daddy leaves to go to work, subtraction concepts are reinforced. Grandma may come to visit--there are many ways to vary the use of the board.

The following rhymes can be spoken and illustrated on the flannel board to help kindergartners with their number concepts.

TEN FLUFFY CHICKENS

Five eggs and five eggs, 15:30
That makes ten;
Sitting on top is the Mother Hen.
Crackle, crackle, crackle;
What do I see?
Ten fluffy chickens
As yellow as can be!

ARITHMETIC PROBLEMS

One and one are two. That I always knew. 15:56
Two and two are four. They could be no more.
Three and three are six.--whether books or bricks.
Four and four are eight. I can keep them
straight.
Five and five are ten. Write them with a pen.

This little honey bear was playing peek-a-boo.
Here is another. Now there are two.
Two little honey bears said, "Let's climb a tree."
Up came another. Now there are three.
Three little honey bears said, "I wish there were some more!
Along came another. Now there are four.
Four little honey bears said, "Let's find a bee-hive."
Here comes another. Now there are five.
Five little honey bears climbed up that tree.
Two fell down. Now there are three.
Three little honey bears said, "Let's climb some more."
Back came another one. Now there are four.
Four little honey bears said, "Let's go to the zoo."
Two of them went. That left just two.
Two little honey bears said, "We've had our fun."
They both went home. Now there are none.

(Winnie-the-Pooh would make a cute bear to illustrate this number poem on the flannel board.)

<u>Bingo</u>

The caller (teacher or child) calls out a number and the children cover that numeral with a blank square of paper.

The first child to get three across or three down wins the game.

(Be sure to put the numerals in different places on each card. Also use the numeral "0" (zero).

4	3	1
2	5	9
7	2	6

1+1	1+2	2+0
3+0	0+1	3+1
0+2	2+1	4+0

Bingo (for the advanced)

The teacher is the caller. If she calls "three" the child must find the combination or combinations which match. (3 + 0 and 2 + 1, etc.) The child then covers these combinations with blank squares of paper.

The first player to have three combinations covered, horizontally or vertically, wins the game.

(Make sure the cards you make fit the ability of your group).

PROBLEM SOLVING

Kindergarten

PROBLEM SOLVING

We can help children in problem solving skills by helping them to become aware that a problem exists. They can learn to recognize a problem in a situation, define it in their own words, then translate it into mathematical terms with the teacher's help.

The teacher must provide an environment which will motivate the children to question, think, discover, and try to solve their own problems.

Many classroom situations (such as easels, blocks, doll, corner, etc.) involve pairing sets to find more than, fewer than, and as many as. These often can be used as problem solving situations. Children will enjoy some "story problems" such as those that follow. Mature children may like to make up story problems of their own for the class.

1. There were 6 cups on the table, and there was a saucer under each cup. How many saucers were on the table? (Six)

2. Five children were going to draw pictures and each child needed a pencil. How many pencils did the pupils need? (Five)

3. There was once a party with three little girls. The set of girls had as many members as (was equivalent to) the set of dolls. How many dolls were there? (Three)

4. There were seven children at a party. Each child had a hat, and there was a feather on each hat. How many feathers were there? (Seven)

5. There were four dogs. Each dog had a cat, each cat had a rat, and each rat had a mouse. How many mice were there? (Four) Were there as many mice as cats? (Yes)

6. On the table is a set of bottles and as many straws. There are eight bottles. How many straws are there? (Eight) Is there a straw for each bottle? (Yes)

7. Six toy soldiers are on the table. Six toy guns are on the floor. Are there as many soldiers as guns? (Yes)

8. How many balloons are in a set of balloons where there are enough for 7 children to each have one? (Seven)

9. Four children can work in the house corner. How many are there now? (Three) How many more can go in? (One)

10. Alice had four paper dolls. Susan had two. Who had more dolls? (Alice) How many dolls were there when they played together? (Six)

Informal problem solving analysis through questions, discussions, and activities

Make the problems mean something to the child. Let them make up their own problems. Make up finger exercises using "no thumbs" to show a different organization. For example, "Hold your hands in your laps with your thumbs down. Make "five." Students hold up four fingers on one hand and one on the other to use an unfamiliar grouping.

Precise language "If-then" logic.
Problem solving analysis.

A blue box, red box
and a green box.
Three pieces of chalk.

Three boxes--a thinking game--this game could be played with any number of boxes, but less than three is limiting and more than three is confusing to Kindergarteners.

Start with one empty box. Put one piece of chalk in the box. Say, "Can you tell me anything about the box and chalk?" "Yes, there is a piece of chalk in the box."

Start again with two boxes and one piece of chalk. Put the chalk in one of the boxes. "Can you tell me anything about the boxes and piece of chalk?" "Yes, there is a piece of chalk in one and nothing in the other."

Now begin with two boxes and two pieces of chalk. Put a piece of chalk in each box. "Can you tell me anything about the boxes and chalk?"

Someone says, "There is a piece of chalk in each box."

"Are you sure?" Discussion should develop.

"Can you tell anything about the two boxes and two pieces of chalk and be sure it is true?"

If there is one chalk in the red box there is one piece in the blue box. If there are no pieces in the red box, there are two pieces in the blue box. There are not more than two pieces in each box. There is at least one piece of chalk in one of the boxes.

You can use this game as many times as the children like. They learn to reason mathematically. Each "If ..., then ..." statement employs mathematical logic or reasoning. The game is no fun unless we are sure that everyone says exactly what he means. This gives you, the teacher, an opportunity to emphasize the need, in certain circumstances, for precise expression.

If you have a mature class you may want to continue with the following:

Now begin again with three pieces of chalk and two boxes. "What can you say now and be sure what you say is true?"

There will be many answers, one of which should not be overlooked. If it is not given, ask, "Can you be sure there will be at least two pieces in one of the boxes?" The answer is, "Yes," but the reasoning may not be obvious to the child.

Now use three boxes and one piece of chalk. There cannot be two pieces in any box.

Use two pieces of chalk. There is at least one empty box, then there are two empty boxes.

Use three pieces of chalk. If there are not two pieces in at least one box then there is one piece in each box. If there are only two pieces in the blue box, then the green and red boxes cannot both be empty and they cannot both have pieces of chalk.

Then use four pieces of chalk with the three boxes. "Can you be sure there is at least one piece in every box?" (No). "Can you be sure there are at least two pieces in one box?" (Yes) There are many responses possible, i.e., "If there are two pieces in each of the boxes then the other box is empty."

Finally, use five pieces of chalk with the three boxes. If one box is empty, then there are at least three pieces of chalk in one of the other boxes. "Can we be sure there are not at least two pieces in each box?" (Yes, we would need six for that to be true.)

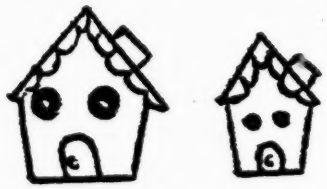
"If there are two pieces in one box, can we be sure there are at least two in one of the other boxes?" (Yes)

MEASUREMENT

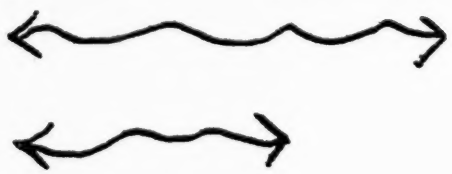
KINDERGARTEN

KINDERGARTEN

Comparison of objects with each other.



Larger than - Smaller than



Longer than - Shorter than

MEASUREMENT

The purpose of having children participate in measurement activities is to establish for them the idea of:

1. Selecting a unit, and
2. Observing and recording an approximate comparison.

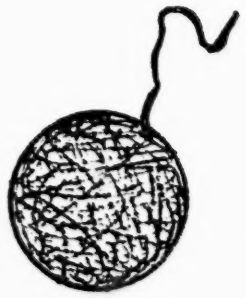
Make daily use of the natural classroom situations which arise--"Is Mary's painting smaller or larger than Tom's?" 6:27-31

"Which is longer, the jump rope or the cord on the record player?" 5:1-6

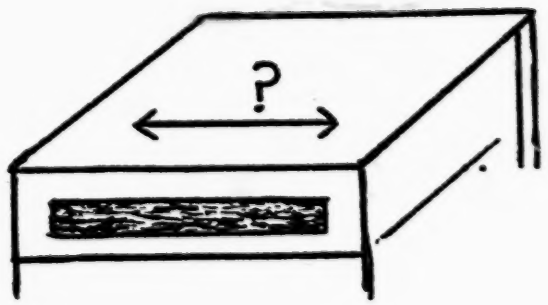
"Who threw the ball the farthest?"
"Give me a sheet of paper the same size as Janet's."

Place three or four colored felt strips horizontally on the flannel board. They should have a common beginning position. Ask which is longest. Repeat with short and shortest, etc. Have a child put them in order from shortest to longest. Test for length by moving one edge against another. 10:423

A guessing game



Use of nonstandard



Which is Longest?

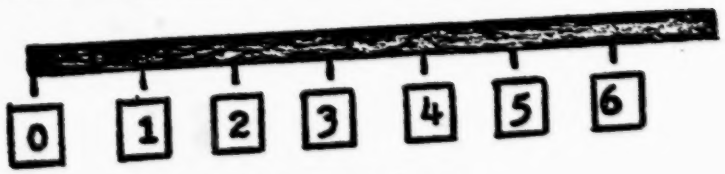
10:407-438

Put three or four masking tape lines in different areas of the classroom. Pose the problem of how you can use a string to compare their lengths. Lay a piece of string beside one line segment and then, holding it at the end points of the line segment, carry it carefully over to another strip of masking tape (line segment). Discuss their relative sizes. Have the children invent their own units and use them. (Jump rope, paper strip, toy broom handle, etc.)

Use someone's feet for nonstandard units and 7:136-9
have them pace off the classroom. Discuss the fact that in order for them to find out how many "shoe lengths" wide or long the room is (stepped off toe-to-heel-to-toe) they must decide whose shoes will be used. They can match shoe lengths and children with "approximately" the same length shoes can check the results. 8:437-46

What part of our body could we use to measure the desk? If the palm of the hand is suggested whose hand shall hand shall we use? Is the thumb to be excluded? Is the hand to be squeezed together? What if the measurement doesn't come out even? Whose opinion is to be accepted? The children will have to make a decision on a standard of measurement at this point.

Standard units on a number line



Vocabulary of comparison

Money

Put down a masking tape number line on the floor and play number games by hopping the line using "equally spaced hops." In this instance each hop is a unit of measure. Compare line segments on this number line using a string.

Let's consider the vocabulary concerned with measurement. The correct and frequent use of terms throughout the school day is important. Ask the children to tell their own number stories about objects.

"Who can tell me a story about these coins?" 2:89-104

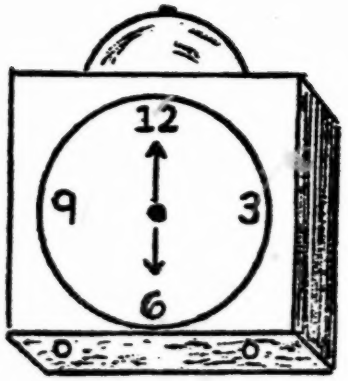
"A nickel is worth more than two pennies."
"A nickel will buy as much as 5 pennies."

Avoid saying a nickel is "equal to" five pennies. Say it is "worth the same" or "equal in value."

Display a child's toy, such as a magic slate, that costs a quarter. Using real coins or play money have the children choose pennies, nickles, dimes, and/or quarters that add up to the price of the toy.

Do the same with a ten cent candy bar, a five cent pack of gum, etc.

Time

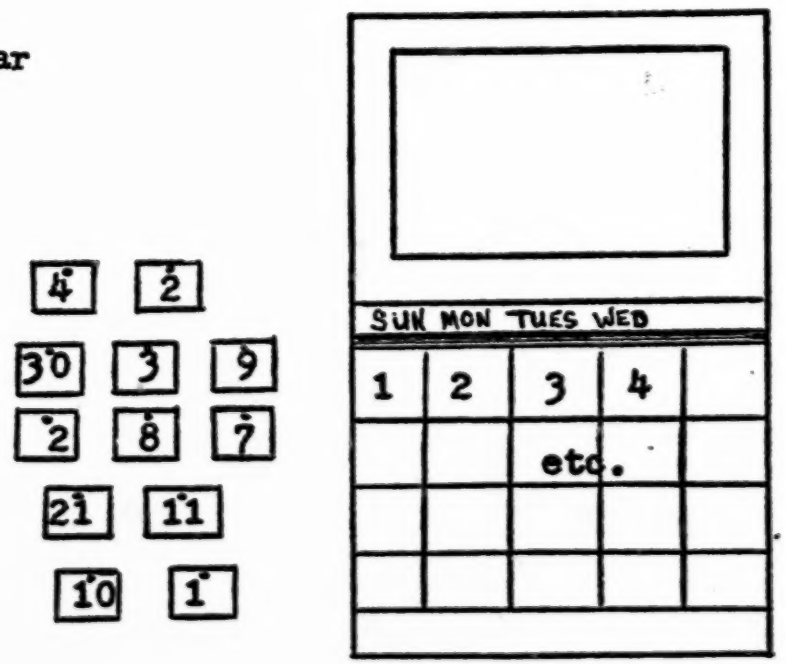


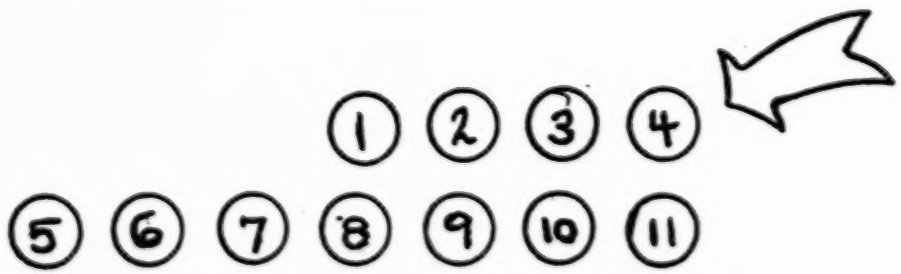
Compare a large manipulative clock to the one on the wall. "It is time for lunch." "We are ready for recess later than we planned to be."

"...Incessantly upon the shelf
It chatters, chatters to itself.
Without a word for anyone,
It never walks, will only run.
And if you ask the simplest thing,
It points instead of answering..."

Phyllis McGinley

Calendar





Keep a large calendar for each month. Display <u>all</u> of the numerals for the whole month on the calendar in black. Then hang identical numerals in green (or some other color) next to the calendar in any order. Each morning have one child choose a green numeral for that day and hang it over the black numeral. They must choose and match numbers to succeed.

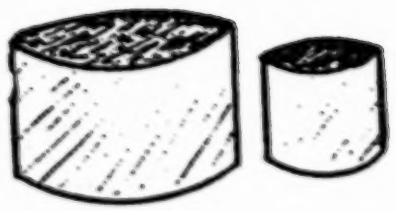
"The name of the number is ten, or eleven, etc."

"Mary's birthday is next week on the 12th. How many more days till then?"

Early in the year there will be more chance for the less mature child to experience success if the moveable numeral cards are arranged by weeks. Children can "take the top numeral on the <u>left</u> and put it on the <u>right</u>, next to 4."

Later they will enjoy choosing from the scrambled numerals "the one that comes next after 4." Near the end of the year most children will be able to handle the numerals in many ways, i.e., "April fourteenth comes after April thirteenth."

Volume



Shape does not determine volume

Provide cans of different sizes and sand to do "pouring experiments." Have the children guess which of the cans will hold more sand. Then have someone pour the sand to check.

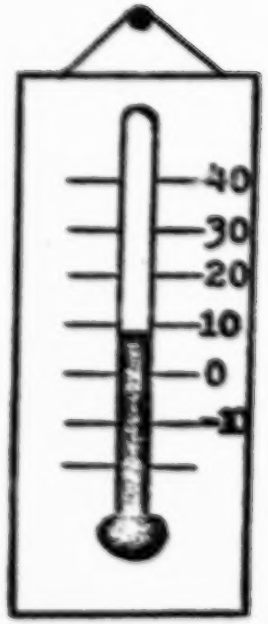
During an art lesson have the children form two balls of clay of about equal size. Then roll one ball into a "snake." Is there as much clay in the "snake" as there was in the ball? Yes, we haven't removed any. Do they look as if they are the same? No, they fool you.

Provide a pitcher of colored water (food coloring), measuring cups, and various size jars or cans. Guess which holds more water. Put them in order from least capacity to most (or most to least). Pour water to check accuracy of guesses.

Set up these different sorts of measuring equipment on or near the sink for an independent activity.

To help children understand the importance of measurement the class could make a simple play dough for attractive Easter eggs.

We need:
- one cup of salt
- one cup of flour
- one half cup of hot water
- a few drops of food coloring



This can be kneaded by hand into a dough with an unusual and interesting texture. But--what happens if we use a whole cup of water? Or less than a half cup of water? Allow the children to discover the answers by experimenting

Have a large paper manipulative thermometer with an elastic to reproduce the temperature reading on a real model. The red liquid goes up when it gets warmer and down when it gets colder.

Place the real thermometer in a pan of ice so children can see a fast drop. Then put it near the heater or in hot water to watch the mercury rise.

"This morning our thermometer reads 68° and now it reads 72°. Is it warmer or cooler now?"

Choose a "weather man" to "read" the outdoor thermometer every morning and afternoon.

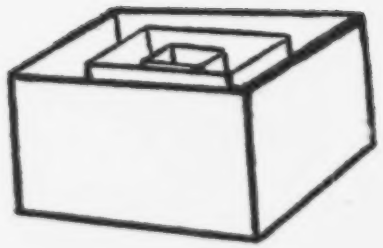
Weight

This small brick is much heavier than our big red ball. Lead the children to verbalize, in their own words, that size does not always determine the weight of different objects.

Arrange several objects on a table (toy truck, boxes, doll, book, etc. Include at least two things that weigh the same.) Have the children estimate if one object is heavier or lighter than another.

Then use a simple balance scale to compare weights. Show that objects that have the same weight will balance. If a teeter-totter is available take the children out and compare their weights. Ask them how a teeter-totter is like a simple scale.

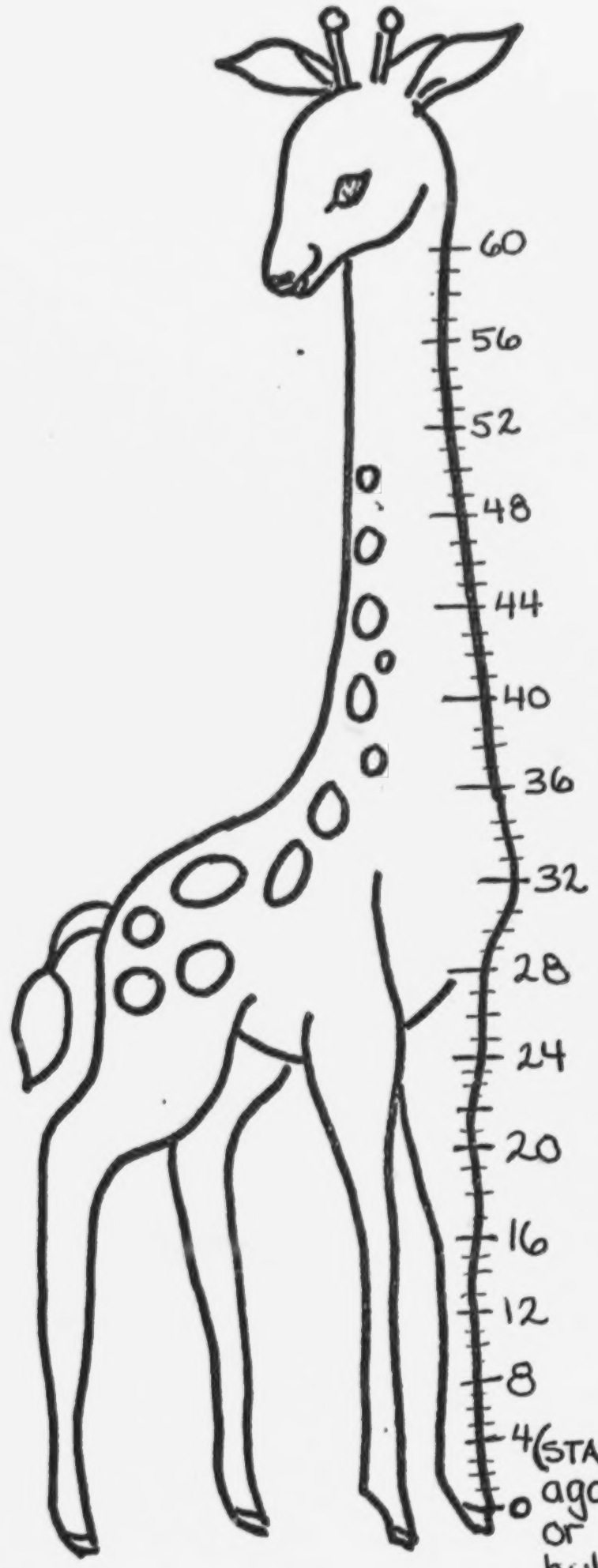
Size



Use nested boxes or cans the children can disassemble them as a game. Large cardboard boxes from the grocery store can be used for this activity.

Height

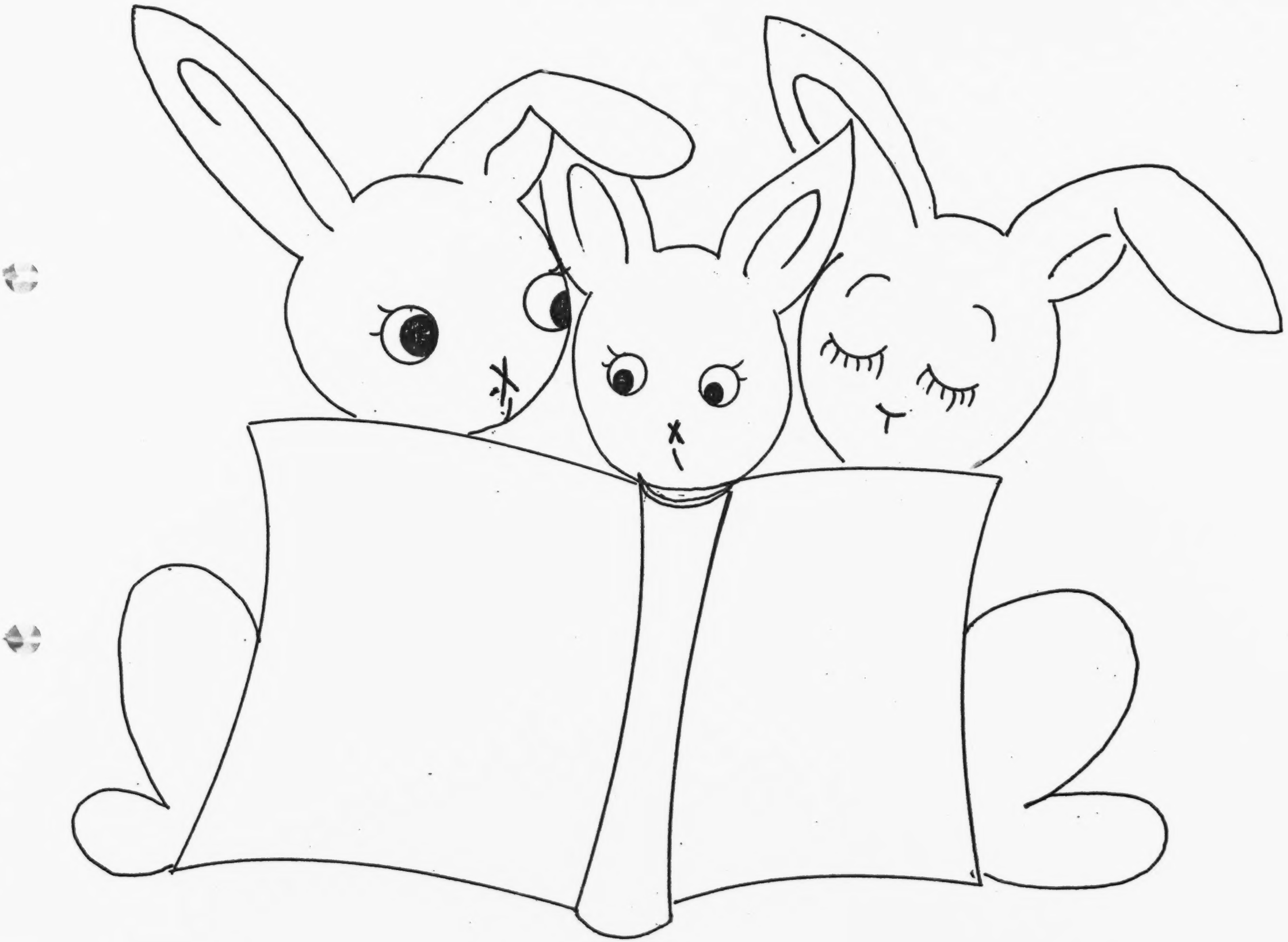
Tack large wrapping paper to a long bulletin board. Have the children stand in a line from shortest to tallest and mark each child's height and name on the paper. Compare heights and then put the chart away. In a few months hang the paper again and recheck each child's height to make new comparisons.

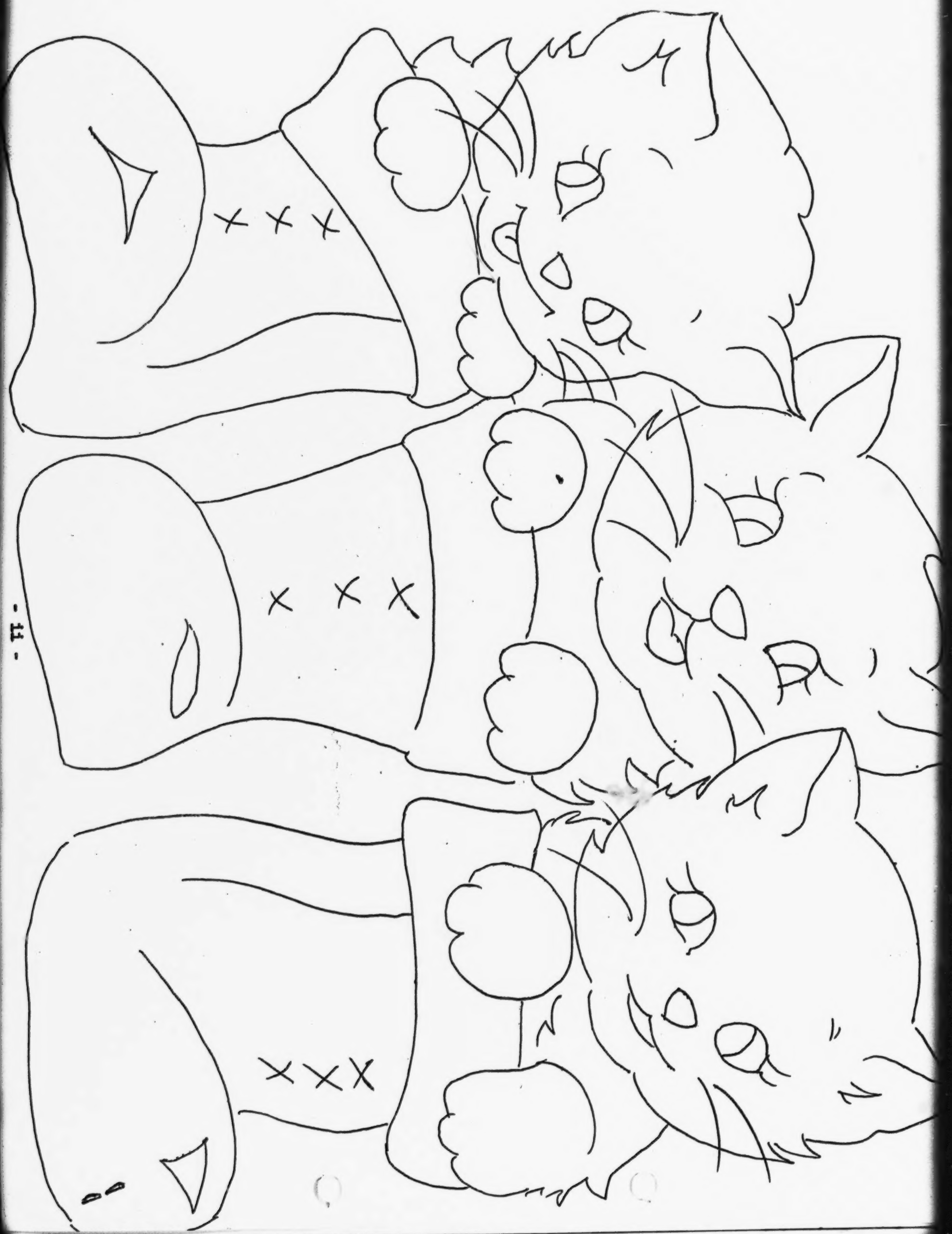


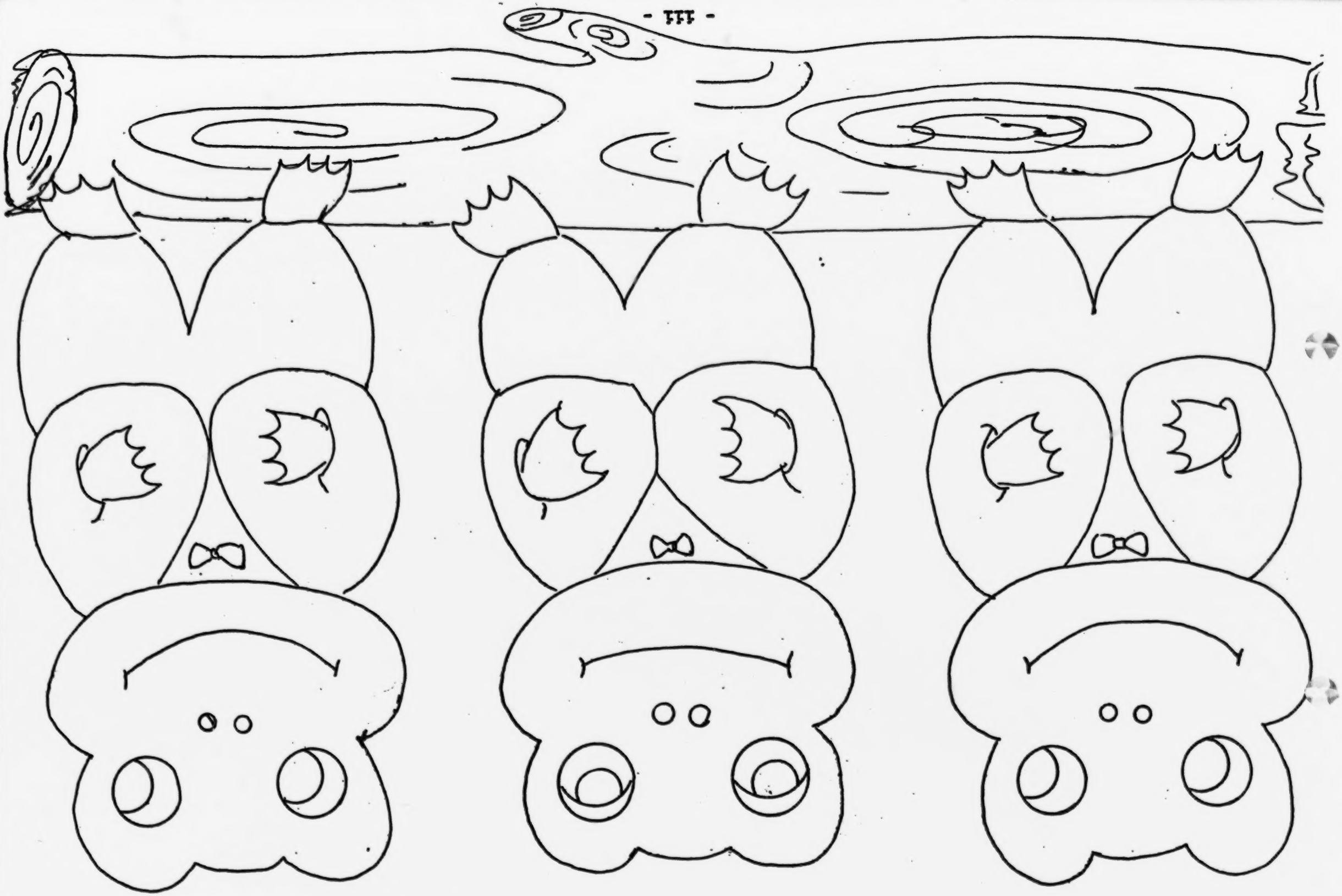
HOW TALL are YOU?

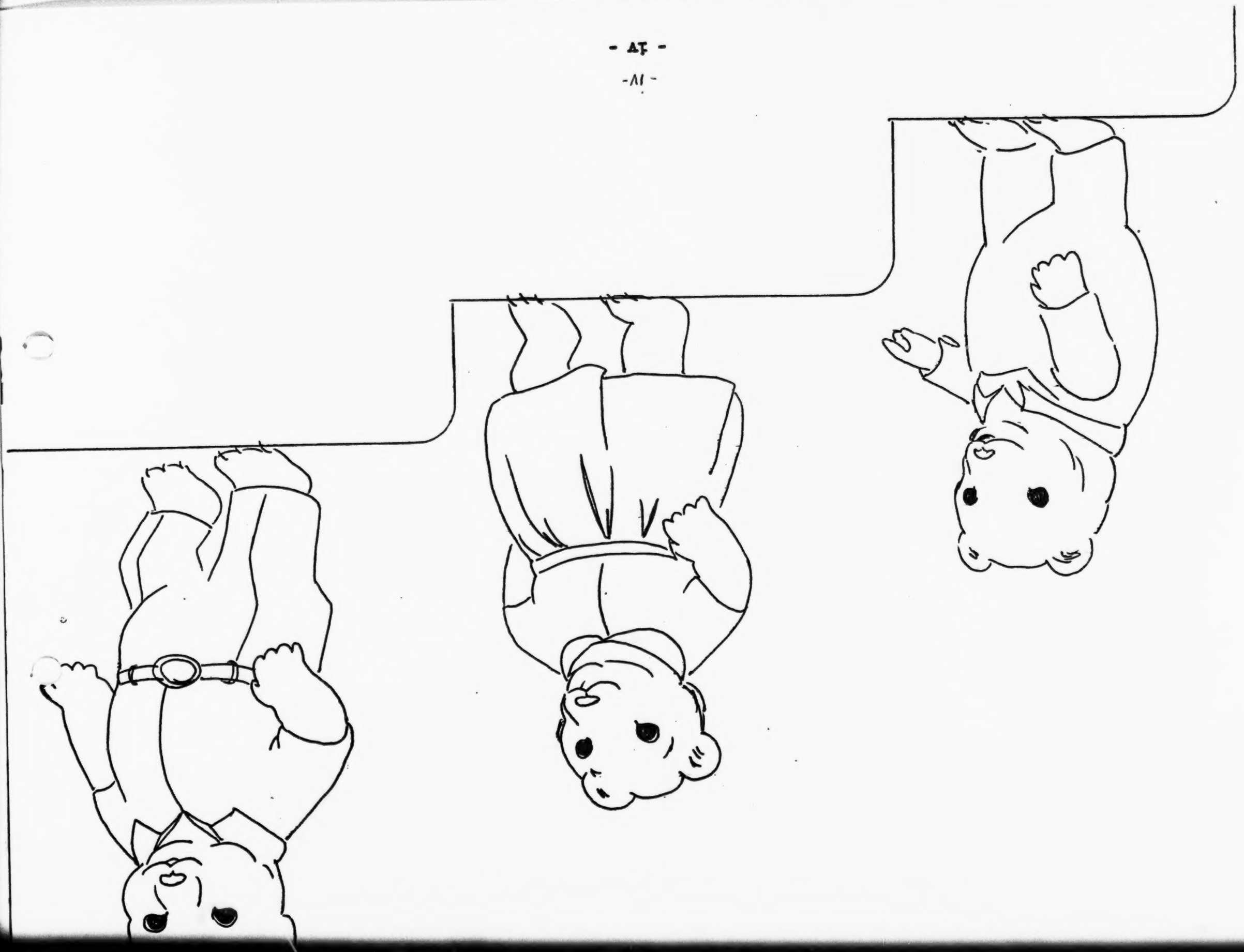
(STAND THE GIRAFFE against the wall or pin to the bulletin board)

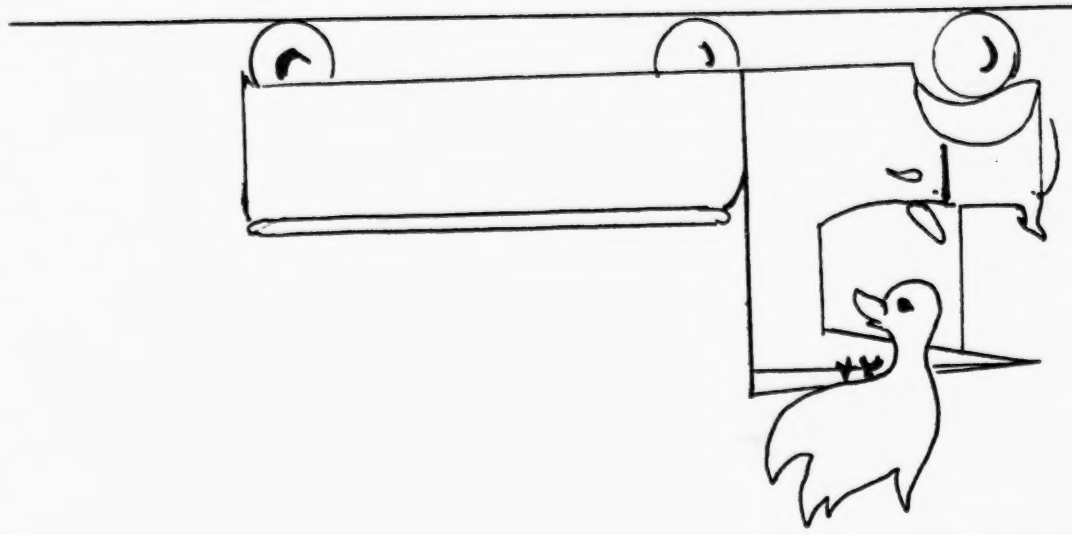
APPENDIX

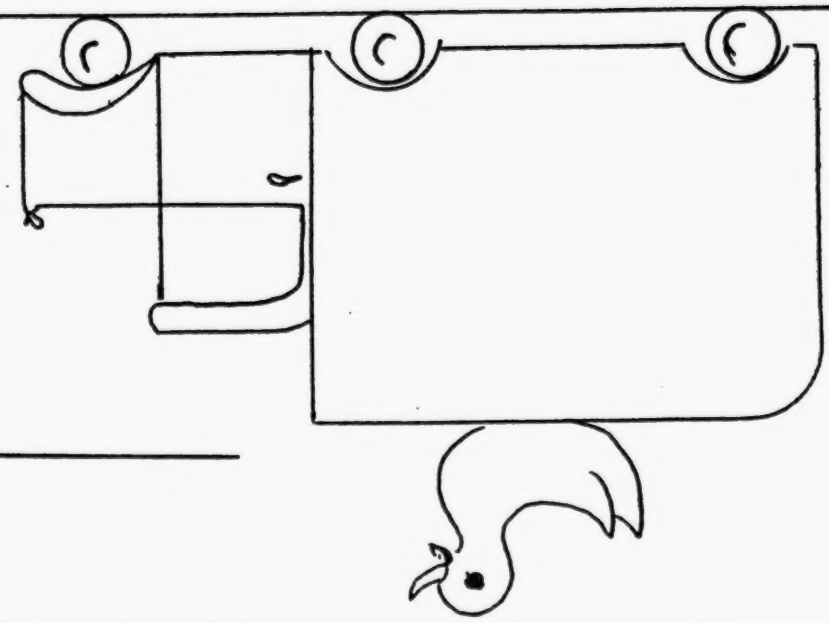


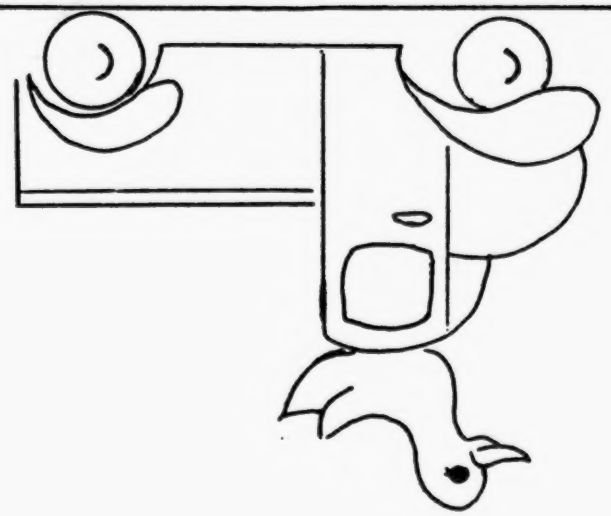


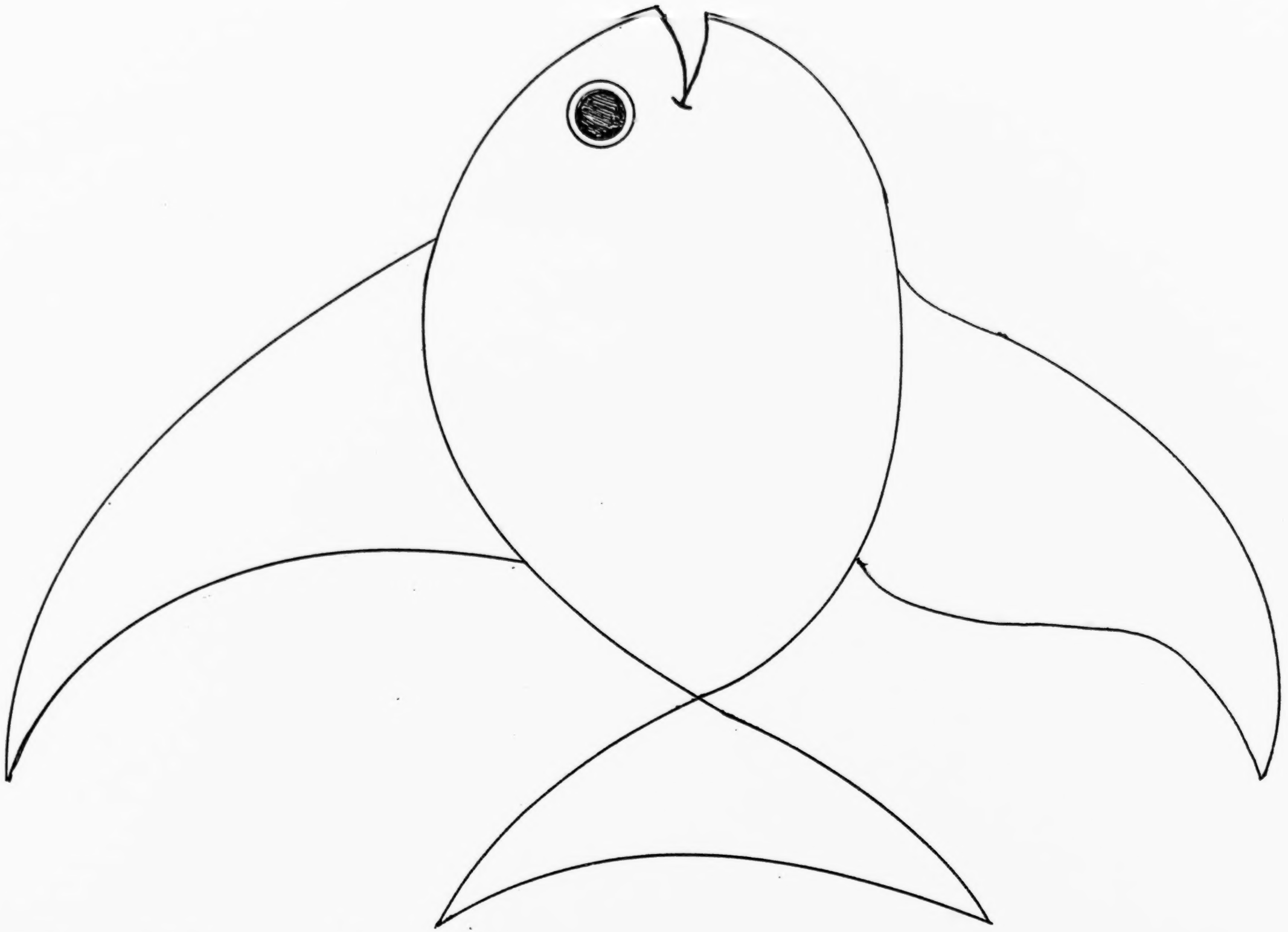


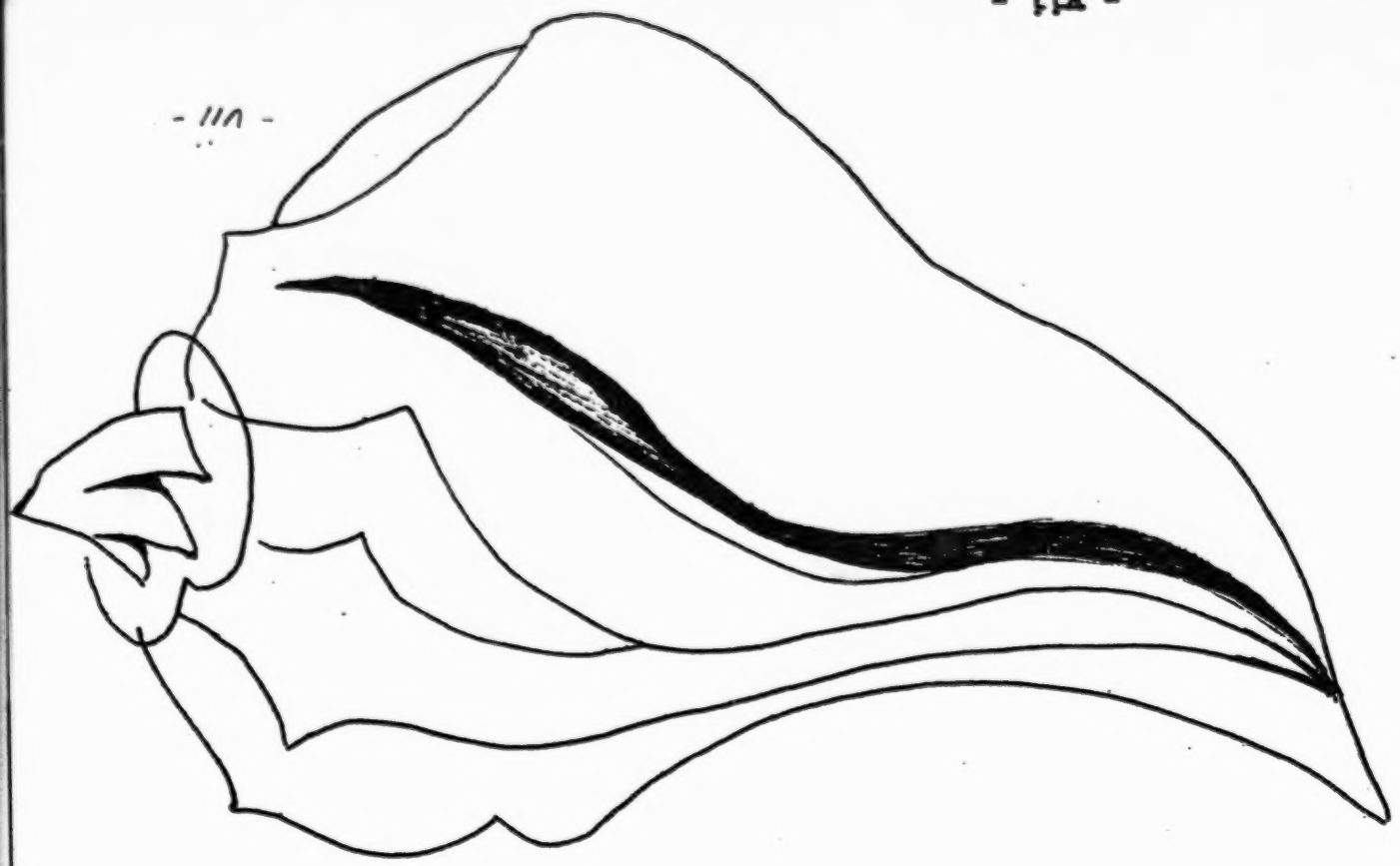


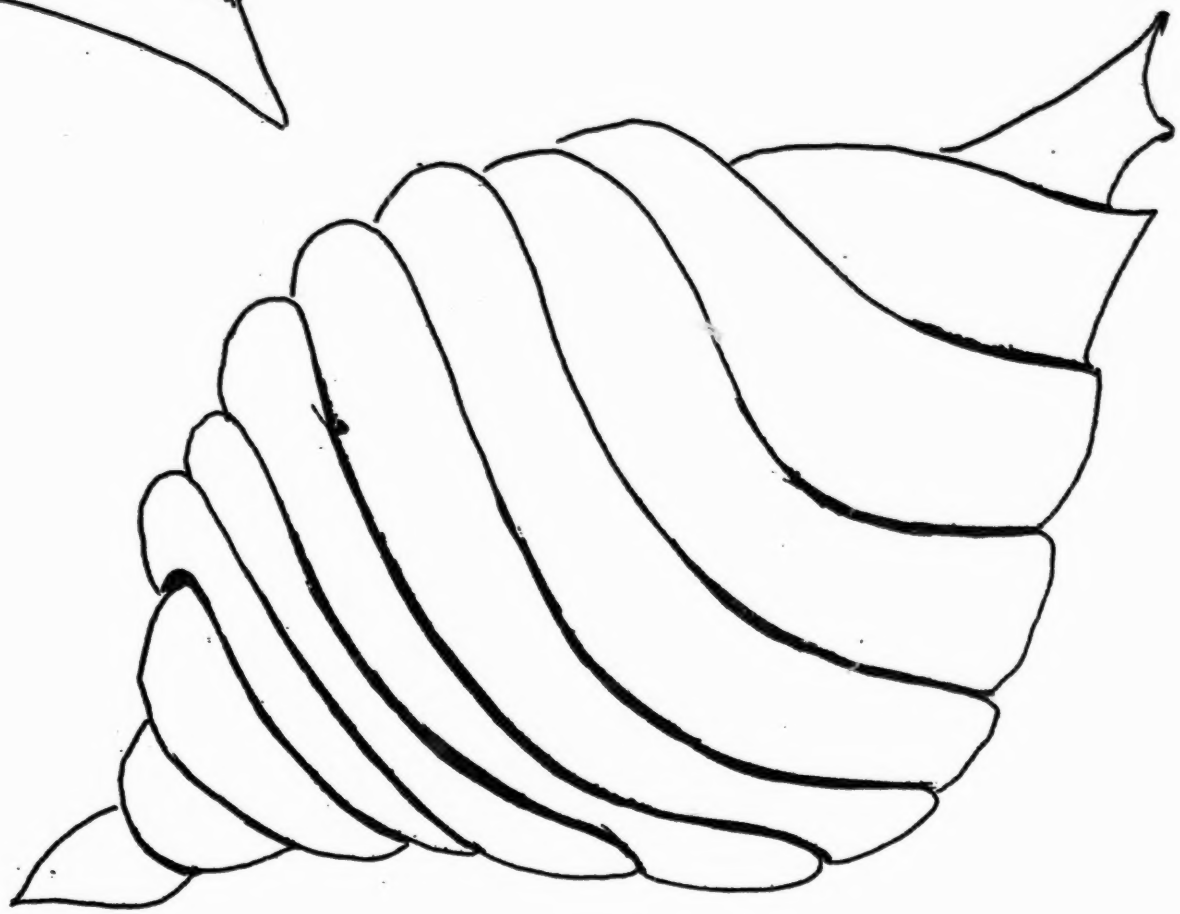


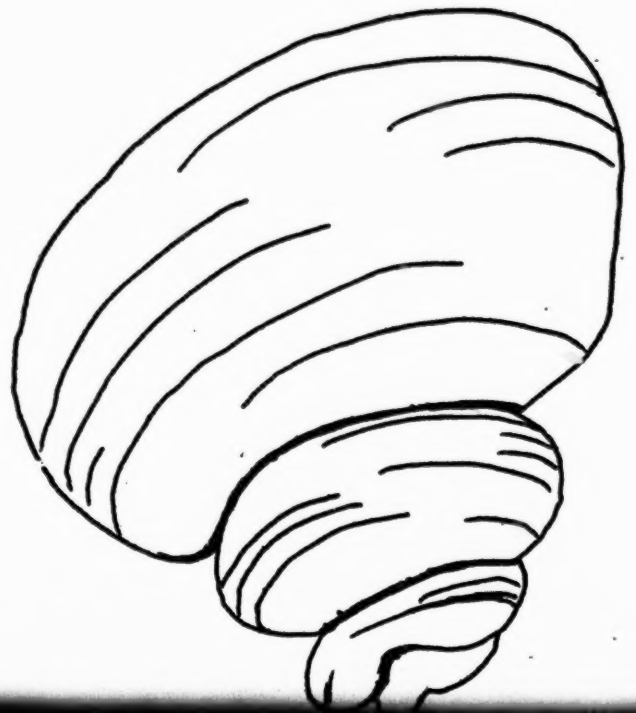


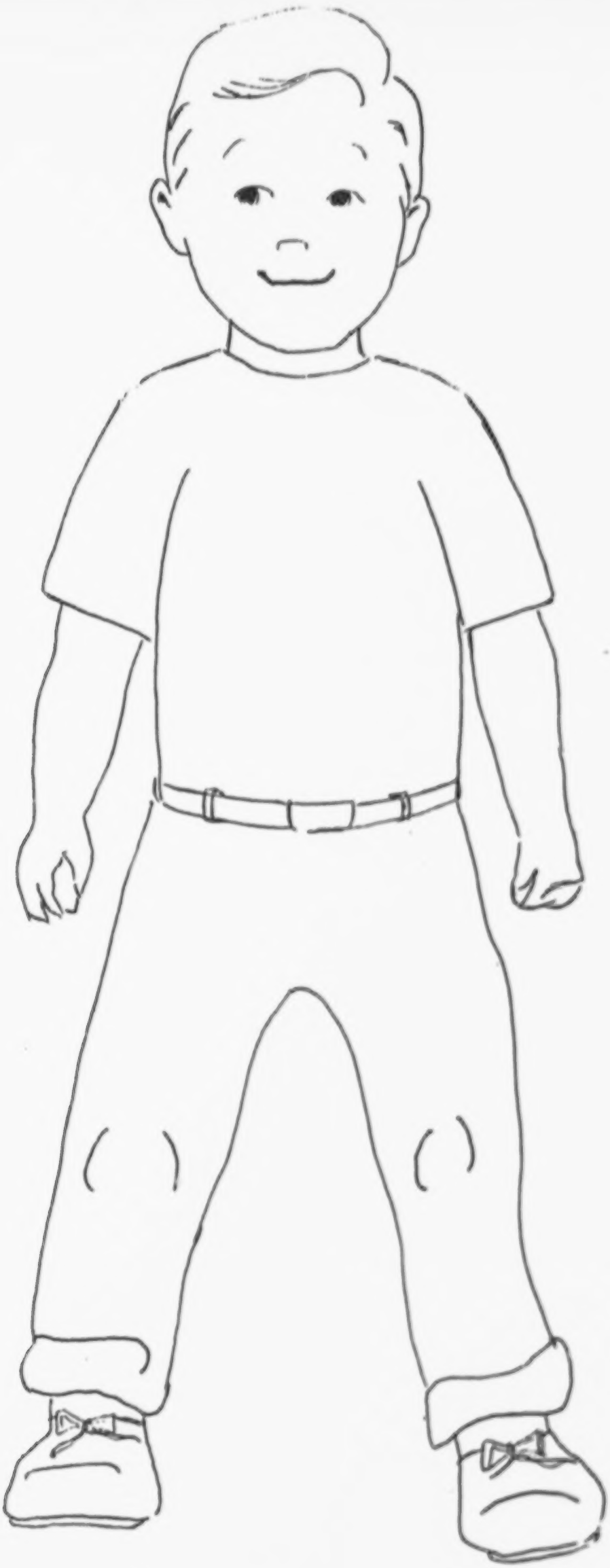


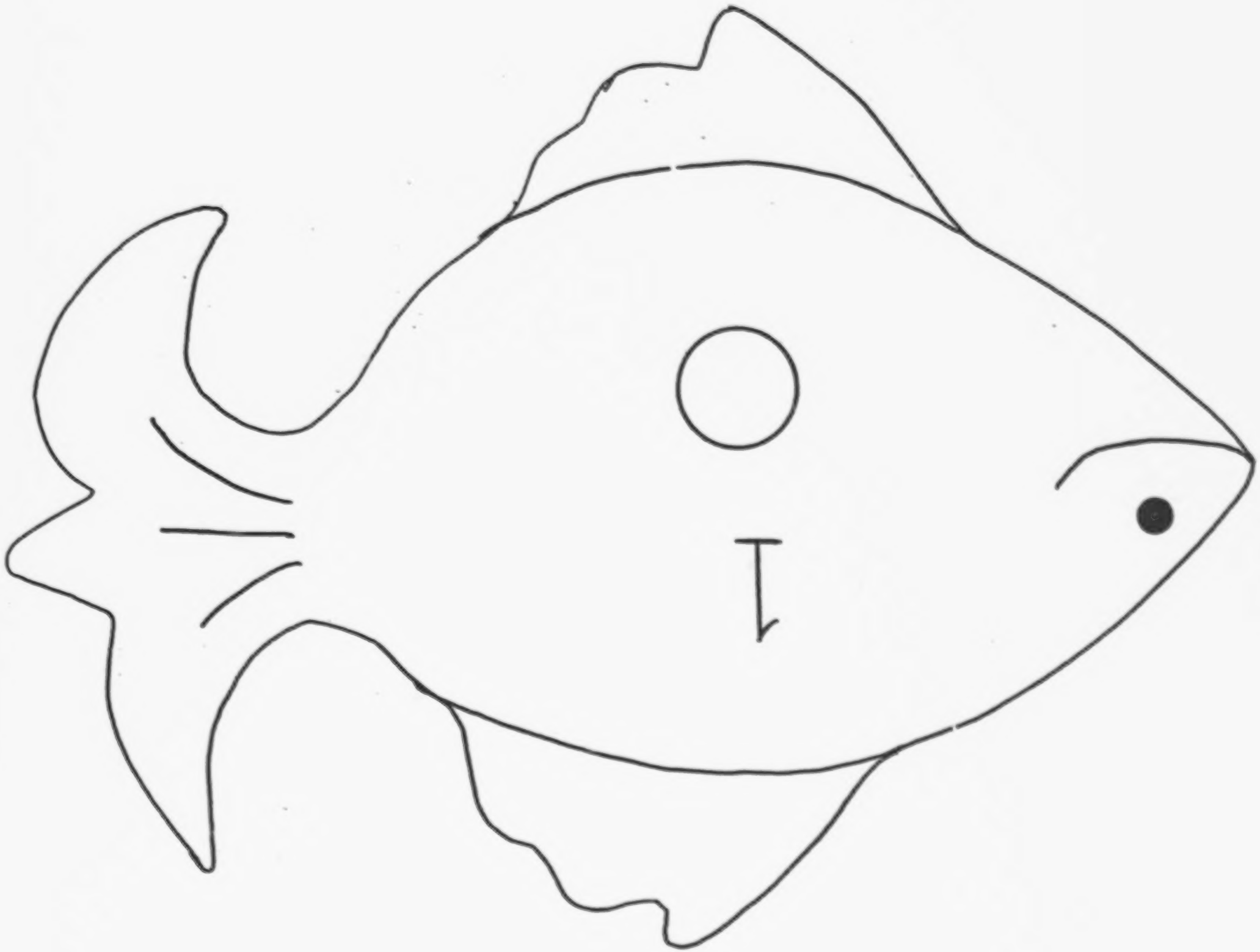


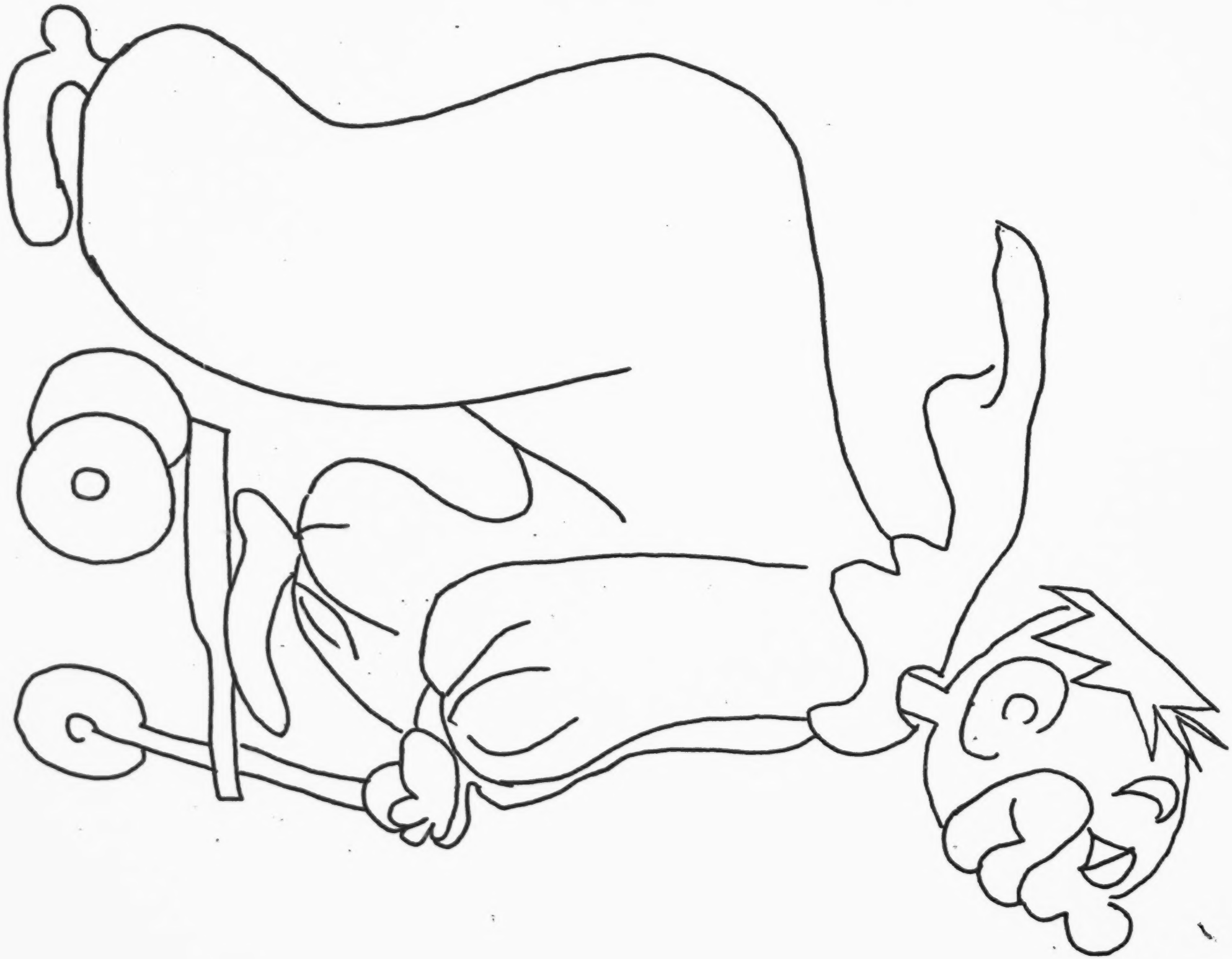


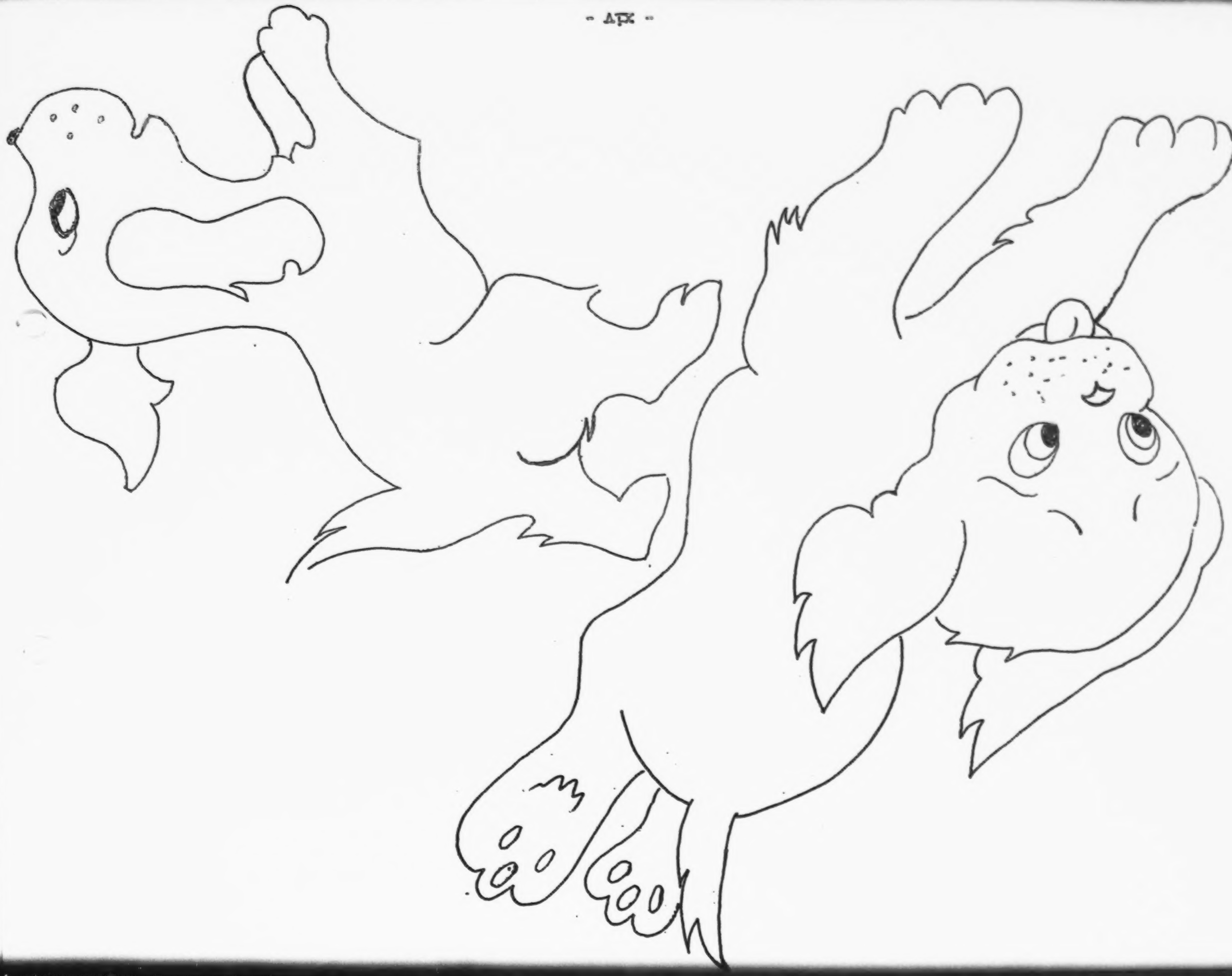


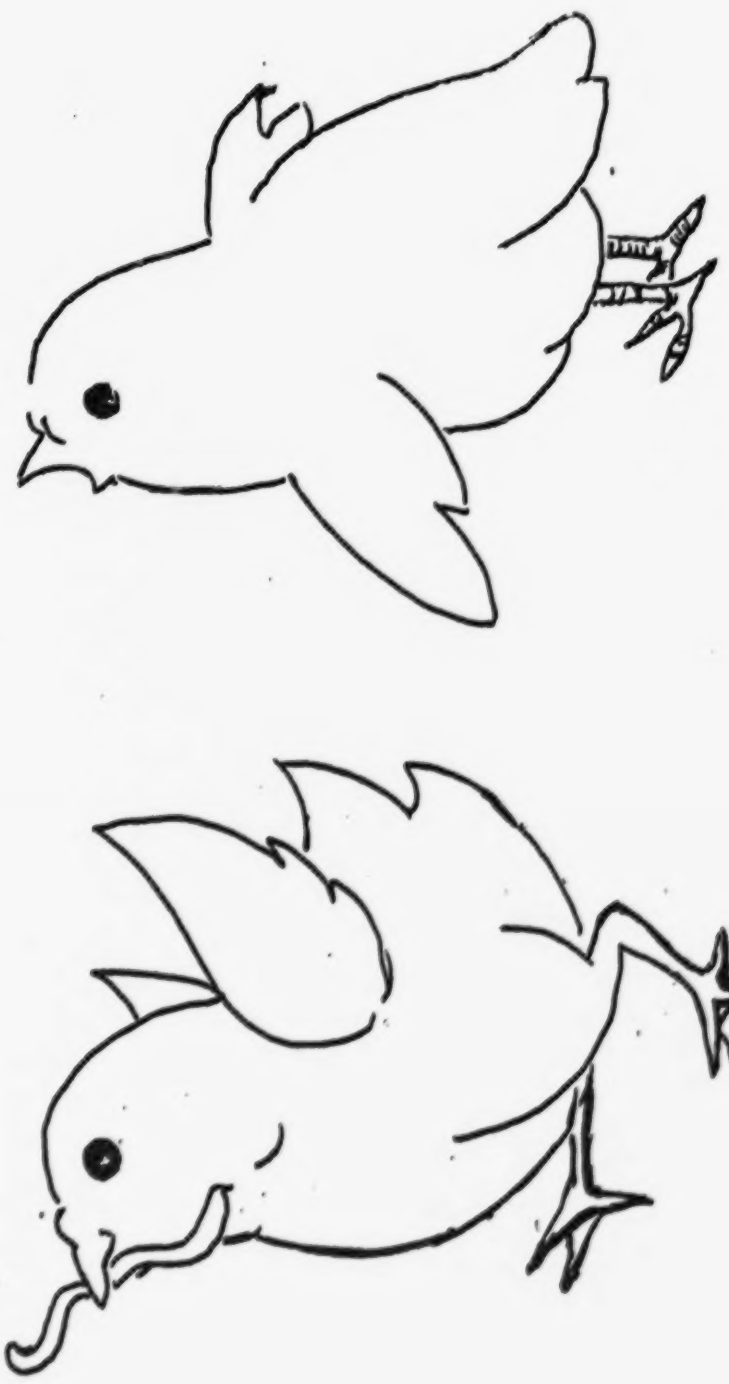


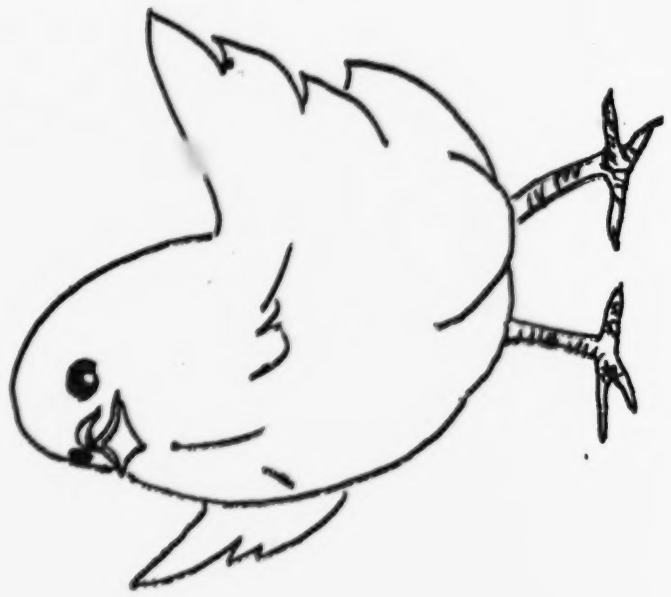


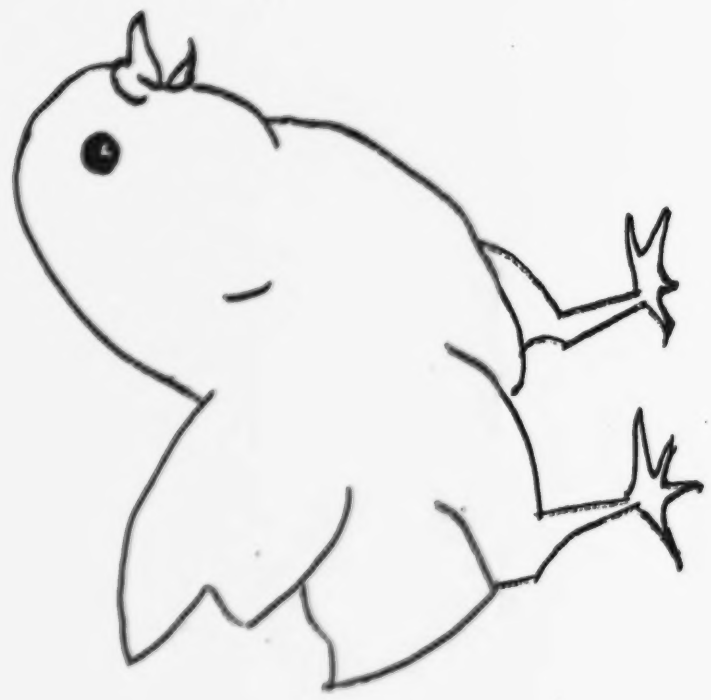












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O.C.S.E.I.P. SYLLABUS

Grade 1

SE006 069

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
OFFICE OF EDUCATION

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PREFACE

The Orange County Science Education Improvement Program (O.C.S.E.I.P.) is sponsored by the National Science Foundation and hosted by U.C. Irvine. It is a cooperative venture undertaken by the University of California, Irvine, California State College at Fullerton, the Orange County Schools Office and local school districts throughout Orange County. This syllabus was written by O.C.S.E.I.P. to help teachers teach the best aspects of recent mathematics programs. It is not meant to be another textbook for a new program. Instead, it is meant to be a sharing and synthesis of effective teaching methods. The outline of topics is a minimum coverage which is common to all schools in Orange County. Topics adequately covered in the majority of texts in use are given a minumum treatment in the syllabus.

The first draft of this syllabus was written during an 8 week session at University of California, Irvine during the summer of 1966 by:

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The first draft was evaluated and revised by the following members of a University of California, Irvine Extension class during the school year 1966-67:

Sylvia Horne - Master Teacher
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We wish to thank all the participants in this program for their hard work and fine cooperation.

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NUMBERS AND NUMERATION

GRADE 1

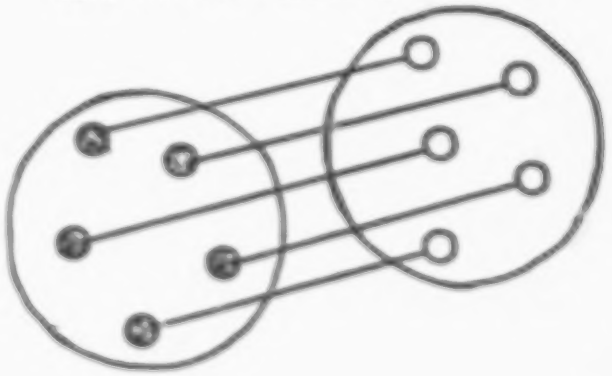
GRADE 1

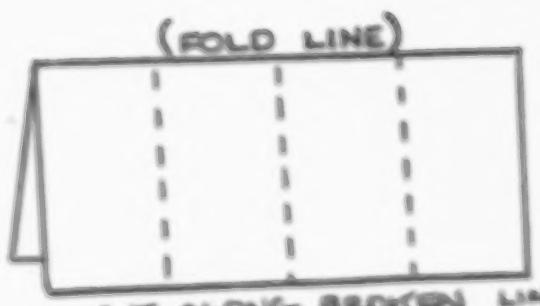
NUMBERS AND NUMERATION

In general, the steps in mathematical learning are:

4. abstract-numerals only.
3. semi-abstract pictures and numerals
2. semi-concrete pictures
1. concrete manipulation of real objects

One by one counting by one to one correspondence.





Some children will need to take longer in reaching the abstract step than other children. Those who are capable of "going on" should not be delayed by the "slower achieving." Grouping by ability groups in a classroom or individual teaching is strongly urged.

Before children can learn to add, they must be able to count one object at a time and arrive at the right number for the set. This is called one to one correspondence. One to one correspondence may also be used to determine whether sets are equal or not and if there is enough of an item--for example, "If everyone has one cookie, will 30 cookies be enough for us?"

Children need experience in counting many kinds of objects to be adept at this.

Flip pictures are easy to construct and fun for children to use. Develop flip pictures by folding a large piece of construction paper (9 by 12) in half, lengthwise. Lift up the top section and cut six sections, cutting up to the fold. These sections form the "flip picture." On the flip-up section, write a numeral such as 1, 2, 3, etc.

Children then are directed to lift up the flip picture and draw the set that corresponds to the numeral.

One to one correspondence - a reinforcement activity.

This activity will show children that a numeral may represent a child. When the children must return items to school (such as Federal Survey cards) make a problem solving situation out of it and solve it with one to one correspondence. Show the children the stack of cards that have been returned and ask how we may find out how many have been brought back. Count them. Then ask how we may find out how many have not been returned. Explore with the children all their suggestions until they can see that they are wrong or can be proven correct. Use a number line to solve the problem. Put one card on the chalk rail by the number line for each numeral starting at one. Then let the children explore different ways of finding out how many have not been brought back. Tell the children how many are in the class and mark this by the number line. Dramatize the problem until someone sees that he must count the numbers that do not have a card by them up to and including the numeral marked as the total number of children in the class. On the next day, the process can be repeated, using the new numbers as more children will have returned their cards. Usually by the second or third day only a few will not have been returned and the names of those who have not returned them may be placed on the board. Then count the returned cards and the names on the board to prove that they total the number in the class. This may be done for field trip permission slips, PTA memberships or Emergency Data Cards. This activity also promotes problem solving techniques.

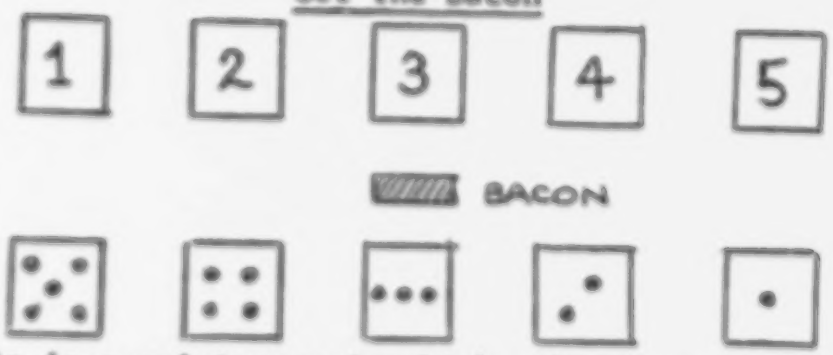
In the beginning of the year in first grade, some children have difficulty in visualizing the concept

of one to one correspondence. This example gives visual evidence. It also provides experience in counting and recording. The child is to match each object with a numeral and record the total on the tag.

**Reference. Math Workshop for Children. Level A. Encyclopedia Brittanica Press, 1964.

One to one correspondence and Recognition of Number

Get the Bacon



This is a quiet game in which the class is divided into two equal rows. If there are 32 in the class the teacher prepares two sets of 16 cards. One set bears the numerals from 1 through 16 and the other shows dots in sets of 1 through 16. One row of players is then numbered from 1 to 16 and the other row holds the dotted cards from 16 to 1.

The teacher silently writes a numeral or draws a set of dots representing a number on the board. (The teacher may use a set of number cards instead.)

If, for instance, "3" is written or displayed, the two players holding corresponding cards race to get the bacon. The winner takes it back to his side, receives a point for his team, and the game continues with a new number.

The bacon may be an eraser or handkerchief.

Numbers and Numeration

Sets of Whole Numbers 0-150
(reading and writing)

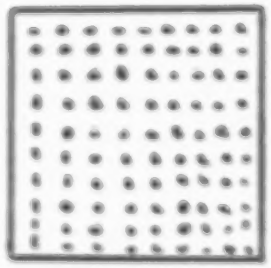
This is a game to further recognition of numerals. This game is best performed outside. Each child is given a card with a numeral on it, using numerals 1-30 (depending on the size of the class). The children form a circle. One child is chosen to be the telephone operator. The operator calls out a number. The child with that number runs around the circle with the operator in pursuit. The child to get back to the center first is the new operator. As more numerals are learned, larger numerals can be written on the cards given each child (Example: 70-100).

Telephone Game

The operator calls number 3 and the operator chooses the person with number 3 around the circle.

The teacher, working with a small group at the flannel board, places animals, triangles, carrots, etc. in numbered groups. The children then write the numeral represented on their individual clip-boards. The teacher leaves each number group up for an appointed time before removing. The children receive a point for a correctly-formed numeral and a point for accuracy. Cards with dots may be used later for the larger numerals.

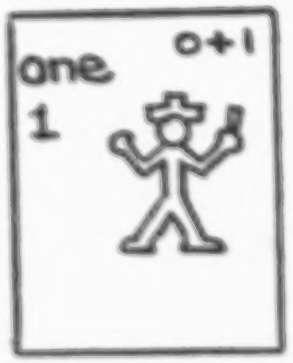
Sets of Whole Numbers.



Supply the children with peg boards and varied colored pegs or golf tees. Acoustical ceiling tiles with uniform holes are excellent. Ask the children to duplicate various sets of pegs that represent numbers. Any order is acceptable. Point out that 6 pegs is still 6 whether shown in one, two, or three rows.

The peg board is also useful to show the "one more" concept.

Numbers and Numeration



On a large sheet of tag board, draw a countingman showing the number one, with the word and numeral for one. When the concept has been introduced, add 0 + 1. The children then find pictures of sets of one in magazines from home which are pasted on the chart.

As each new number is introduced, present the corresponding chart.

Charts can be placed on chart rack or bulletin boards, but preferably where they are easy for children to look at.

Zero--an Empty Set

Have several boxes containing sets of blocks (pencils, beads, etc.) on the table. Let a child choose a box and either find or write the numeral that tells how many are in that set. Do this with several sets. Then show a box with nothing in it.

Discuss the name for this empty box or set. Zero is the numeral for an empty set.

How many boys have we in our room? (a set of 15)
How many girls? (a set of 17)
How many pink rabbits? (a set of 0) We have an empty set, or no pink rabbits.

Odd and even counting by groups of ten, five and two in increasing or decreasing order.

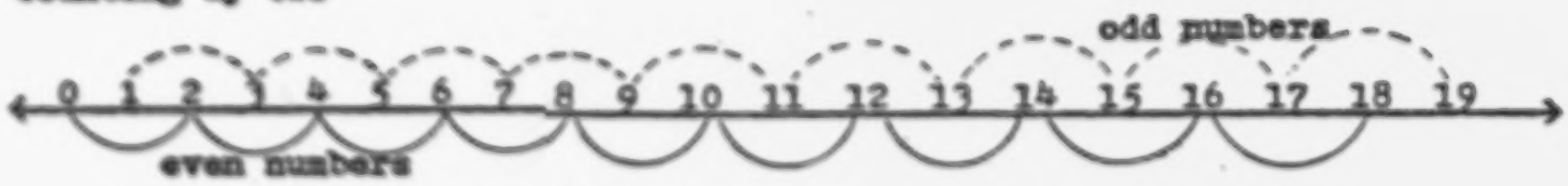
Numbers ending in 1,3,5,7,9 are odd. Numbers ending in 2,4,6,8,0 are even.

Illustrate odd and even counting by paper frogs who are a "two hop" frog, a "5 hop" frog, and a "10 hop" frog.

Counting by five

Have these frogs hop back and forth on a large blackboard number-line counting as they go. The children should have the opportunity to manipulate these frogs during the lesson.

Counting by two



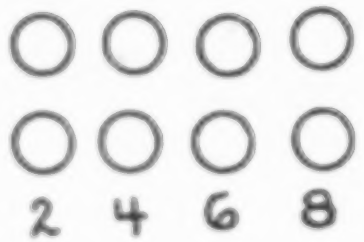
Individual number lines at each student's desk also offer meaningful manipulation of ideas.

Odd and even counting by 2's in increasing and decreasing order.

Draw lattice on the chalkboard. Tell the children that the numerals in the top row represent even numbers and the bottom row odd numbers. Point out the repeated patterns of numerals in the ones-place: 0, 2, 4, 6, 8, 0, 2, 4, 6, 8 or 1, 3, 5, 7, 9, 1, 3, 5, 7, 9, 1. Then write the numeral 76 on the chalkboard and ask, "Is 76 an odd or even number?" Repeat the activity by writing numerals such as 95, 37, 42, and 60 on the board.

A frog making two hops at a time serves to further illustrate the concept behind this lattice: 0-2-4 as well as 1-3-5, etc.

Put felt discs on the flannel board in sets of two. Have the children count by ones to determine the number of disks in all the sets. As each set



(Counting by 2's starting with odd numbers.)

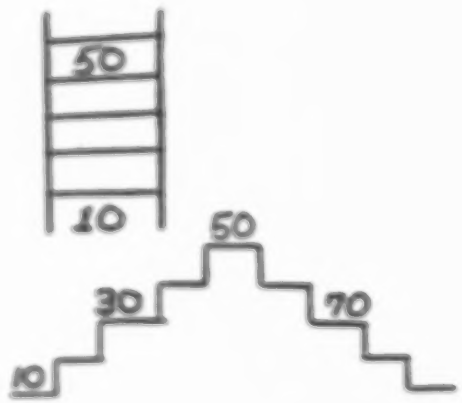
of two is counted, have them place a felt numeral beneath the set to indicate how many disks have been counted up to this point. Then have them examine the number sequence which appears. Help them note that each term is two more than the term preceding it. You can vary the activity by beginning with 8 or with 16.

Remove 2 at a time and have the children give the number of disks that remain each time.

Repeat above procedures starting with odd numbers--1, 3, 5, through 11. Remove disks two at a time to show decreasing pattern.

Provide bundles of sticks, each containing ten sticks, and have the children practice counting by tens to determine how many sticks are in various sets.

Hundred-Chart made from 24"x 36" tag.



Mark off ten rows of ten boxes each and with the numerals 1 through 100 in the boxes. Discuss the number sequence which appears in the first column: 1, 11, 21, 31, through 91. Help the children see that 10 is added each time. Ask them to read the same column from bottom to top: 91, 81, back to 1, and have them discover that 10 is subtracted each time.

Allow the children who need more experience in using number sequence to work in a small group at the chalkboard. Draw ladders and steps. Have the children count by tens to find the missing terms and have them write the corresponding numerals on the steps of a ladder. They can read the number sequences in increasing and decreasing order.

Numbers and Numeration

Number sequences, counting by tens in both increasing and decreasing order.

On the board have the children draw ten rows with ten circles in a row. Put a line at the end of each row. Have the children count the circles one by one recording the number counted at the end of each row.

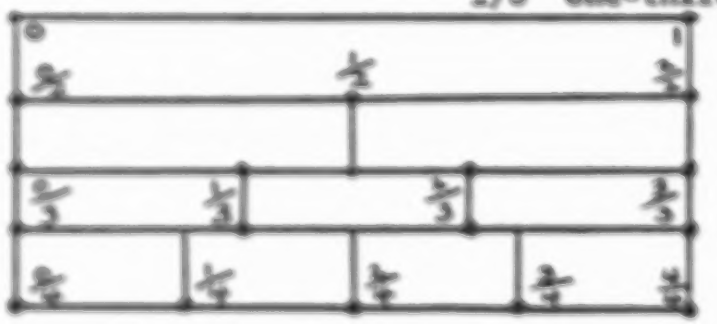
They will soon discover with your help that since there are ten circles in each row it is easier to find the total number of circles by counting by tens. Have them read the number sequence which appears in a column to the right of the circles: 10, 20, 30 through 100. Point out that this is an increasing sequence; ten has been added each time.

Again have the children tell the total number of circles-erase the last row and ask how many circles remain. Continue to erase one row at a time, with the children giving the number of the remaining circles. After all the circles have been erased, ask the children to read the number sequence from bottom to top: 100, 90, 80 back to 0. Point out that this is a decreasing sequence; ten has been subtracted each time.

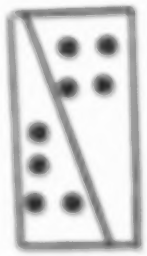
Fractional Numbers.

Recognizing fractions as equal parts of a whole.

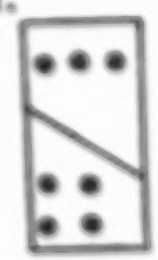
1 whole
1/2 one-half
1/4 one-fourth
1/3 one-third



This concept is one that can be used in many different situations. In art activities the children are many times asked to fold paper in halves and fourths. In science the children are asked to fill a container half full. During physical education the class is divided into two or four equal teams. These instances are excellent opportunities to point out equality of parts in halves, thirds, fourths, etc. The exposure and brief explanation in varied situations will make the concept of fractions meaningful.



"Yes" would be printed on the back of this card.



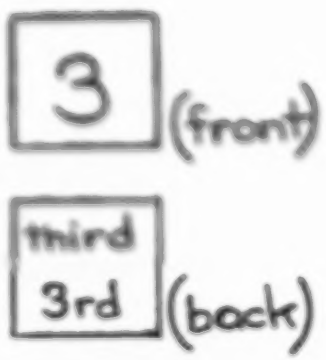
"No" would be printed on the back of this card.

It is important that children fully understand the division of one object into equal parts before they are asked to divide sets. The number line is helpful in the division of sets.

A set of cards can be used to tell if the children understand one half. Make a set of 10 cards (3 by 5 oak tag) with sets of dots separated into two groups. These can be presented to the class in the following manner. "This game is called yes or no: If the dots are divided evenly into two sets then you'll say yes. If the sets are not evenly divided, you will say no. After you decide to say yes or no, you turn the card over and find out if you gave the correct answer. The correct answer is on the back of each card."*

*Reference: Morton, Robert. Modern Arithmetic Through Discovery, Book 1 Morristown, New Jersey: Silver Burdett Company, 1966.

Ordinal Numbers.



Ask ten children to line up to board an imaginary bus. Allow them to discuss who in this set of passengers is first. Call him Number 1. He may hold a large card with the numeral "one." Who is second in line? Call him number 2, etc. What else can we call Number 3? (Third) Have the word "third" and "3rd" on the flip side of the number 3 card. Play this questioning game back and forth till each child in line has flipped his card more than once and everyone has had the opportunity to verbalize and see demonstrated the words "first, second, third, fourth, fifth, sixth, seventh, eighth, ninth, and tenth.

Numbers and Numeration

Ordinal Numbers.

Develop understanding of the use of numerals to denote position within a group.

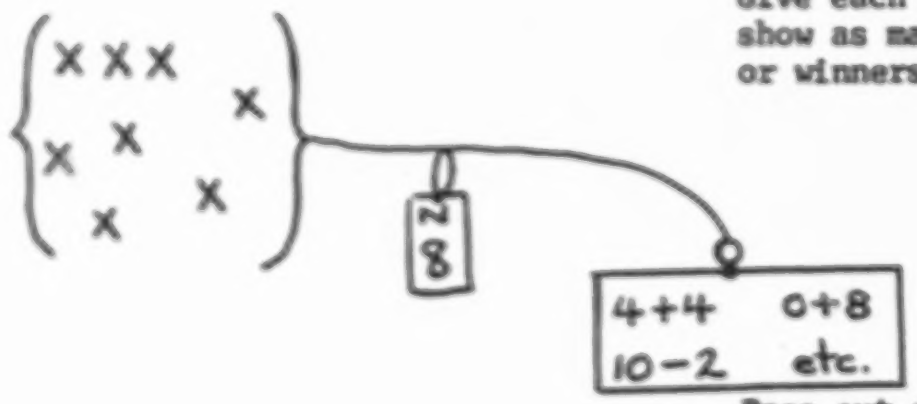
As a quick drill to reinforce ordinal numbers, call out a direction to a child but instead of using his name refer to him as "second person in fifth row." This idea can be used for fun: "The third person must stand on one foot" or to name the child who comes to the flannel board, goes to the office, etc.

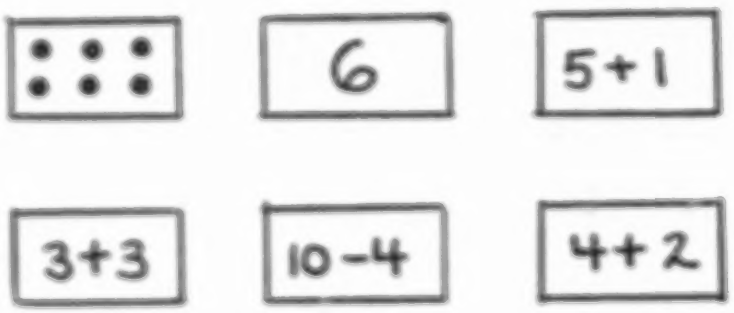
A Number has Many Names

Children will do more creative thinking once this concept is fully understood. Ask the children to guess the number of ways there are to name 8.

Give each child a sheet of paper and have him show as many names as he can for a number. Winner or winners choose the next number to be illustrated.*

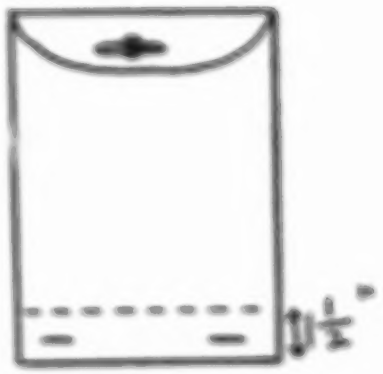
*Reference: Math Workshop for Children, Level A, Encyclopedia Britannica Press, Chicago, Illinois, 1964.





Pass out set cards once through eight and numeral cards one through eight and different numeral cards 1 + 1 through 7 + 1. Tell the children to examine the card they have, then close their eyes and listen. Tap a wood block a certain number of times. Children having the set cards that correspond then come forward. They stand in a line to illustrate the many names for that number.

Innovative drill

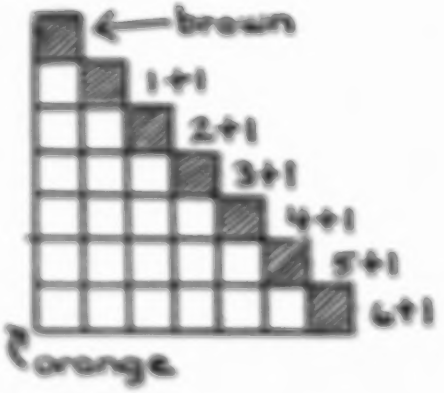


6 x 9 envelope. Fold up lower edge 1½" and staple.

Materials: use manila envelopes, size 6 by 9 and turn up the bottom edge 1½ inches and staple to form a holder for numerals. Inside place two sets of numerals 0-10. For example: two 2's; two 3's; etc.

Suggested uses for everybody show. Children can keep these in their desk to be ready for informal drill practice during the day.

a. Teacher writes a problem on board. Children select answer, puts in holder, hides, reveals when teacher says "Everybody Show."
b. Teacher can give <u>oral</u> addition or subtraction problem. Children selects answers, etc.
c. Teacher can give oral problem solving equations.
d. Teacher says, "Find another numeral for (3 + 1) three and one more. Children select numeral from packet and place holder until the command "Everybody Show."



This activity will help the children visualize that each counting number is one more than the one preceding it. Use 3/4" manila graph paper. Direct the children to use two colored crayons--brown and orange. Color the top left square brown. Color the one directly under it orange and the one beside the orange square, brown. Continue in this manner until the paper is used. Let the children count the colored squares and write the numerals for each line such as 1 + 1, 2 + 1, 3 + 1, etc. The children may cut along the brown squares to make steps.

Numbers and Numeration
Expanded Notation and Place Value

11 is 1 group of 10
and one more.

17 means __tens __ones.
23 means __tens __ones.

17 = 10 + 7
23 = 20 + 3

The operations that children perform with two-digit numerals are better understood through the use of expanded notation in which numerals are broken up into particular component parts before performing the operation. In this way numerals are explained and understood in terms of place-value. A thorough understanding of this concept provides a basis of work with addition combinations whose sums are greater than ten. 9:93 4:159

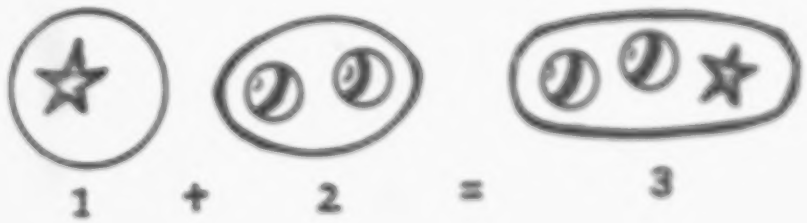
Have children group ice cream sticks into bundles of 100's, 10's, and 1's. Illustrate a numeral on the board such as "42". Ask someone to put this many bundles of sticks in place value cups. Ask someone else to write out what has been done thusly: 42 = 40 + 2 and 42 = 4 tens and 2 ones.

ADDITION AND SUBTRACTION

Grade 1

Grade 1

Addition through set union



"One plus six equals seven."

$1 + 6 = 7$

$$\begin{array}{r} 1 \\ +6 \\ \hline 7 \end{array}$$

Addition is an operation performed on the cardinal numbers of disjoint sets, producing a single cardinal number - the sum.

Set union is joining together the elements of two or more sets to give a third set.

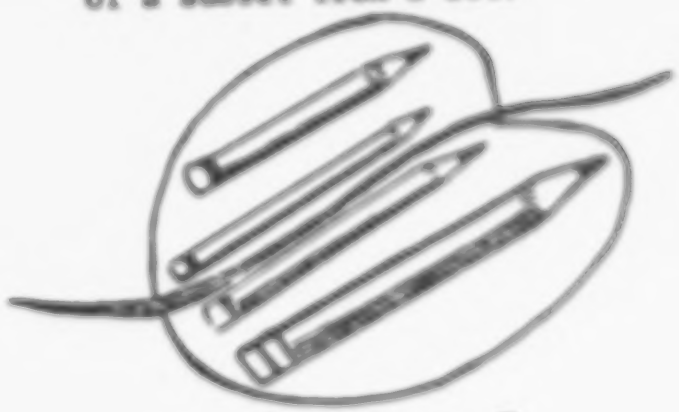
The cardinal number tells how many members are in each set or group. (Only numbers can be added. Members of sets can be joined.)

The math sentence, initiated in the primary grades, is expanded and reinforced in every other grade. In mathematics, ideas are expressed by mathematical symbols which are arranged in meaningful patterns called mathematical sentences. The algorism is the end result for quick computation.

Dramatize number combinations by using children. Select three children to stand in a straight line in front of the class. Tell the other children to close their eyes and not to "peek". Whisper to two of the children to remain standing. Tell the other child to sit on the floor. Have the class open their eyes. Explain that the class must "read" from left to right and tell what addition combination the scene makes them think about. Then have the children read from right to left and tell what addition combination the scene makes them think about. Then have the children read from right to left and tell what related subtraction equation the scene makes them think about. 4:87

Choose other children to represent other combinations of three. Continue the game as long as interest is shown.

Subtraction is the removal of a subset from a set.



$4 - 2 = 2$

$$\begin{array}{r} 4 \\ -2 \\ \hline 2 \end{array}$$

Addition through union of disjoint sets; subtraction through set separation.

Subtraction is the removal of a subset from a set. Along with the idea of the removal or separation of sets should come the subtraction equation (mathematical sentences).

The teacher should first start with the joining of one set to another set. With the idea of set union taught, she may use this union of sets to illustrate the removal of a subset from a set.

The words "remove" or "take away" should be used when doing the physical operation of removing subsets from sets. "Minus" or "subtract" should be used when working with a subtraction equation.

Place two objects on the flannel board. Bring them close together and explain how a new set of 2 has been formed by joining a set of one with another set of one. Explain terms, such as equals, equation, number sentence, plus, add, and join, as necessary. A yarn or string divider may be used 4:75-77

2 + 4 = □

1 + 1 = 2

to show separation and union. Remove it to show union, or place it between members to show separation.

"What's My Rule?"

2	5	1	4	0
3	6			

2	4	5		9
0	2		1	

A game to promote discovery can be played at the chalkboard, or can be played on dittoed sheets.

The teacher puts a numeral in one box of a grid, and explains that he is going to "do something to this number to get another number. If anyone thinks he knows what the rule is, he cannot tell the rule but may suggest the answer for another number."

Inverse relationship of addition and subtraction.

3+⬡=5

5−⬡=3

Draw a large birthday cake on the chalkboard. Draw three candles on the cake. Tell the children this cake is for a little girl who is five years old. Ask the class, "Are there enough candles on the cake?" Under the cake put the equation 3 + □ = 5. Have one child use a different-colored chalk to draw as many more candles as are needed to make a set of five. Have another child tell what was done and complete the placeholder equation.

Then tell the children the little girl blew out her candles to make a wish, but she did not blow hard enough and three candles remained lit. Explain that we (the class) must decide how many candles the little girl blew out. Under the equation, write the placeholder equation 5 - □ = 3. Have a child erase the number of candles he thinks were blown out and ask if he is correct. Then complete the placeholder.

Continue this procedure until you have used all combinations with sums of five or six.

Subtraction as the inverse operation of addition.

$(4 + 3) - 3 = 4$

The -3 undoes the +3.

As early as the time when children are able to add combinations whose sum is as much as 4, they should be led to understand the inverse operation which is subtraction.

Manipulation of counters (bottle tops, beads, pegs, etc.) provides actual experience and should be used when each new combination is introduced.

$2 + \square = 4$
$\square + 1 = 3$
$3 + \square = 5$
$4 + \square = 6$

$3 - 1 = \square$
$4 - \square = 2$
$6 - \square = 4$
$5 - \square = 3$

Have on the chalkboard two columns of placeholder equations. 4:101

Call on a child to complete the first addition equation. Have a second child draw a line from this completed equation to the subtraction equation he thinks belongs with the addition equation.

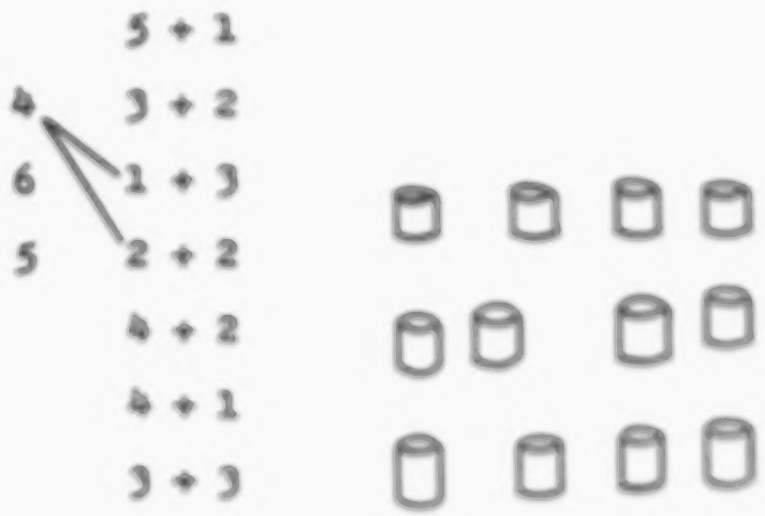
Have a third child complete the subtraction equation. At this point discuss the numerals used to complete the equations. Continue to work the equations the same way discussing numerals used.

Subtracting a number from itself.

The teacher should encourage the child to discover what happens when you subtract a number from itself, as in $4 - 4 = \square$.

Place four beads on a wire. Ask how many beads must be removed so that no beads will be left on the wire. Illustrate the equation on the chalkboard. $4 - 4 = \square$

Addition and subtraction with mathematical sentences.



Let pupils find some "other names" for numbers at the chalkboard.

Use solid objects, grouped in various combinations.

Pupils may arrange the objects in as many ways as they can. "Write a number sentence that describes what you see."

Without speaking, write an open number sentence on the chalkboard.

Hand the chalk to someone to finish the sentence. Smile at him for any correct answer. Silently encourage many responses.

(6 = 2 + 4 , or 2 + 2 , or 1 + 1 + 1 + 1 , or 3 + 1 , or any other name for 4.)

$6 = 2 + \square$

The addition and subtraction algorism (vertical form).

2	7
+ 5	+ 2
84	84
+ 12	+ 12

Column Addition.

Expanded notation for problems such as these should be used until the teacher is sure that students know they are adding ones and tens. Be sure the children start with the 1's column.

Addition is an operation on two numbers (binary). When we have three addends, we must decide which two numbers to add first; then we add this sum and the other number. Therefore, to name the sum of 2 + 1 + 4, we may think of grouping the

addends as 2 + (1 + 4). Then we think 2 + 5. We see that the sum is the same either way.

A. 2 B. 3
1 △ 2
+4 +1 △

In Example A and B, two of the addends are grouped and the result of this grouping is put in the triangle. This enables the child to keep in mind the processes of grouping as they are being completed.

"Grandma's House"

The children pretend to be going to Grandma's house. The only way to get there is across a stream using stepping stones. If the answer to each equation (written on the stepping stones) is known, then the child can reach Grandma's house. If the answers are not known, he falls into the stream.

This game works well on the chalkboard. Draw a stick boy, a stick girl, a stream, and Grandma's house. When you feel the answers have been mastered, change the equations.

Addition and Subtraction combinations
0 - 9.

"Going Walking"

Mark off nine 9-inch squares on the floor. Write the numerals 0 through 9 to label the squares. Ask the class to sit in a circle around the strip of nine blocks. Select one child to be "it". Tell the rest of the children

to close their eyes during the game. Have "it" start at 0 and walk forward, stepping on each of the numbered lines, in turn, and tell the rest of the class where he is. He may say, "I've walked forward five steps, I am standing on 5."

Then let him walk forward again and tell the class how many more steps he has moved. "I've walked forward two more steps."

"It" will call upon someone to tell where he is now. The child who answers correctly then becomes "it," and the first child sits down. The game then continues.

The class should consider two related experiences. Have one child move five steps to 5 and four more to 9. Have another child first move four steps to 4 and then five more steps to 9. Ask the children to state the related addition combinations (five plus four equals nine and four plus five equals nine).

Combinations through 10.

Addition and Subtraction combinations
1 - 9.

Bead sticks are a very useful tool to teach combinations. Use a metal hanger to make the bead stick. Cut the curved part of the hanger with metal cutters. Put ten colored beads on the stick - 2 green, 2 red, 2 blue, 2 orange, and 2 purple beads. Then turn up the ends of the hanger.

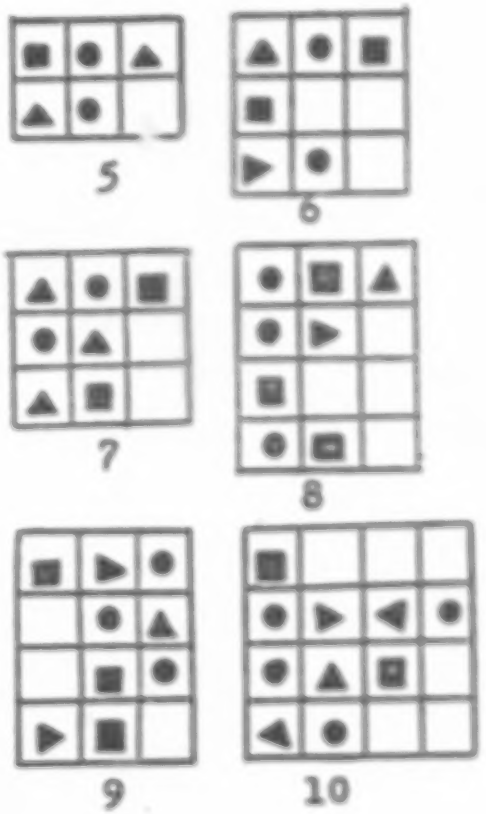
The children can visualize combinations as well as counting by twos and fives.

"I Am Thinking"

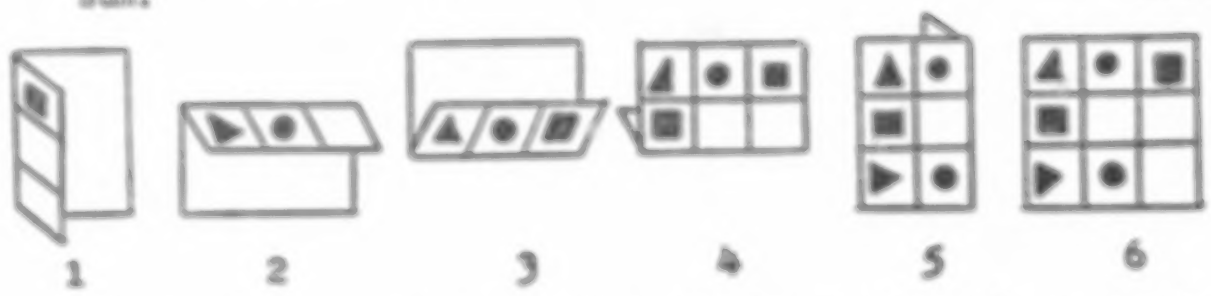
Play a guessing game with the children using addition combinations whose sums are ten or less.

The teacher or a child may hold a flash card so the class cannot see the numbers and say "I am looking at two numerals that make 8. One of the numerals is 5. What is the other numeral?" The first child to give the correct answer gets the card and then can be "it".

Another variation of this game is to give a flash card to the child that gives the correct answer and continue until all the cards are given out. The child who has the most cards wins the game.



Number pattern cards may help pupils practice their addition facts. They may be folded to show any one addend. Pupils supply the missing addend for the given sum. 17:49 20:17



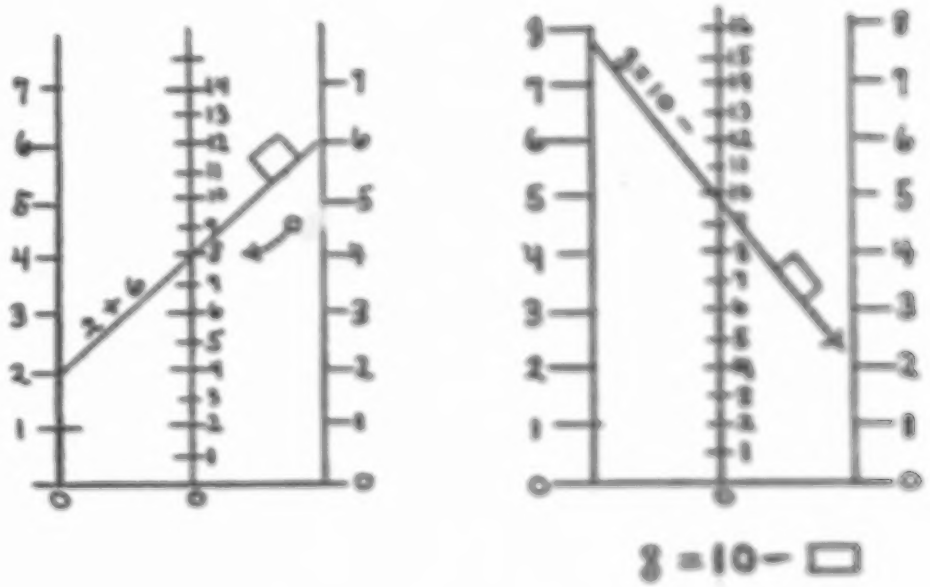
The teacher may ask the class, "How many shapes do you see? How many do I see?" (Number memory, and addition and subtraction can work in.)

Reinforcing addition and subtraction.

The "number ladder" may be useful in furthering the concepts of addition and subtraction. The two outside lines are of equal unit spacing, while the central line is scaled of ½ size units.

Pupils draw in the straight lines from addends on the two sides and find the sum in the middle. Subtraction may also be shown.

The "Number Ladder"



Practice of addition facts.

Addition facts: "What's your number's name"

8 + 2	6 + 4	6 + 8			
3 + 3	8 + 1	4 + 7	10 − 2	10 − 4	5 − 1
9 + 2	6 + 1	9 + 5	6 − 2	3 − 1	7 − 5
7 + 5	6 + 6	4 + 3	7 − 1	8 − 1	9 − 2
			6 − 1	7 − 2	9 − 1

"Cork Drop" is a "bombsight" game to practice adding.

Players have up to five tries to drop numbered corks into a coffee can. As soon as a player has 2 corks in the can, he adds the numbers. Player with the highest total wins. 1:8

Here is a flash-card game, played by two pupils. Each pupil takes six of the twelve cards. One player tells "another name" for his card (for example, if the player has "8 + 2," he would say, "You could call it '9 + 1'." The other player guesses what he thinks the first player's card says. If wrong, he hands one of his cards to the first player and guesses again. If correct, he takes that card from the first player. The second player then adds this card to the bottom of his stack and gives "another name" for one of his cards. The game continues until one player has all the cards.

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

"Fact chart"(For display)

Subtraction cards may be substituted. A "fact chart" may be posted to refresh memories--and stop arguments over answer.

Reinforcing combinations.

7	15	5
4	6	1
18	16	9

"Facts Bingo"

Give each child a card on which different sums and differences are written in squares. Combination cards are shown to the class and children place lima beans (or any other cover) on the correct answer if it is on their on their cards. Continue until someone "Bingo's".

All responses are checked by going through the cards again orally with children giving orally the correct answers.

"Math Bee"

Relay: Divide class into two teams with a score keeper if odd number of children. Teacher calls combination to first child and if he answers correctly, he scores a point for his team. As soon as his turn is over, he goes to end

of the line and the leader of the opposite team has a turn. A penalty can be imposed on team which "prompts" current leader. The combination can become steadily more difficult as year continues.

Number Facts.

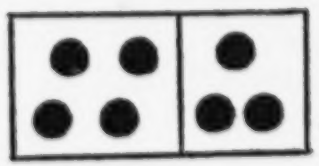
"I Am Thinking of Another Name"

Tell the children that you are thinking of another number name for the number six. The class should try and guess what this name is - such as 7 - 1, 3 + 3, 4 + 2, 6 + 0, etc.

Have the child who guesses correctly whisper to you the numeral he would like the class to guess next.

The order property; commutative property of addition.

3 + 4 = 4 + 3



The order property is the commutative property. There is no reason to subject first grade children to the word commutative. It is the application of the property that is important. When children understand the function of the order property, they can see that all the addition table need not be memorized.

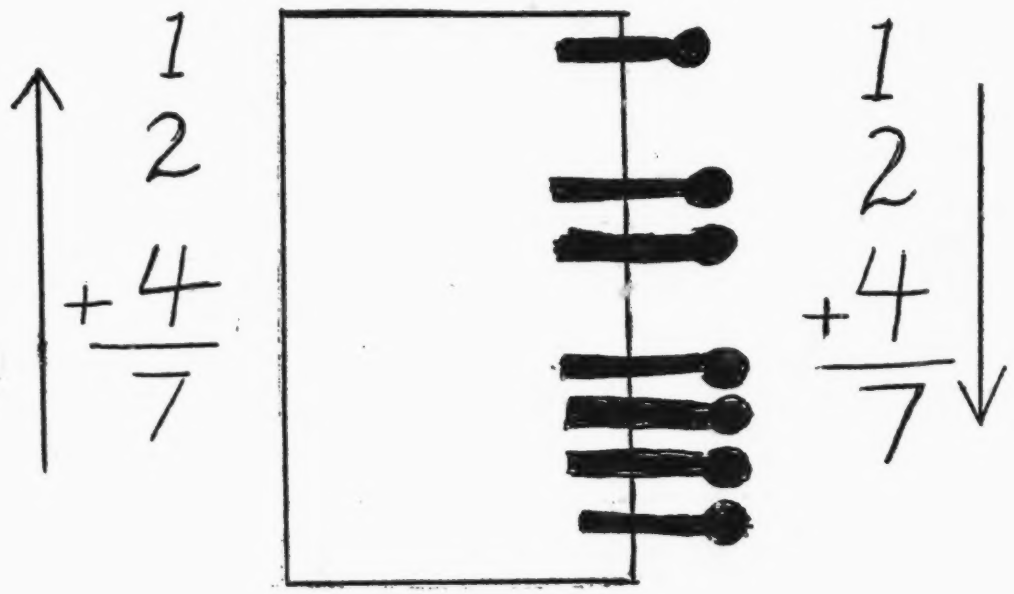
Lead them to conclude that the order or grouping does not affect the number. You may call this the "In-any-order" rule.

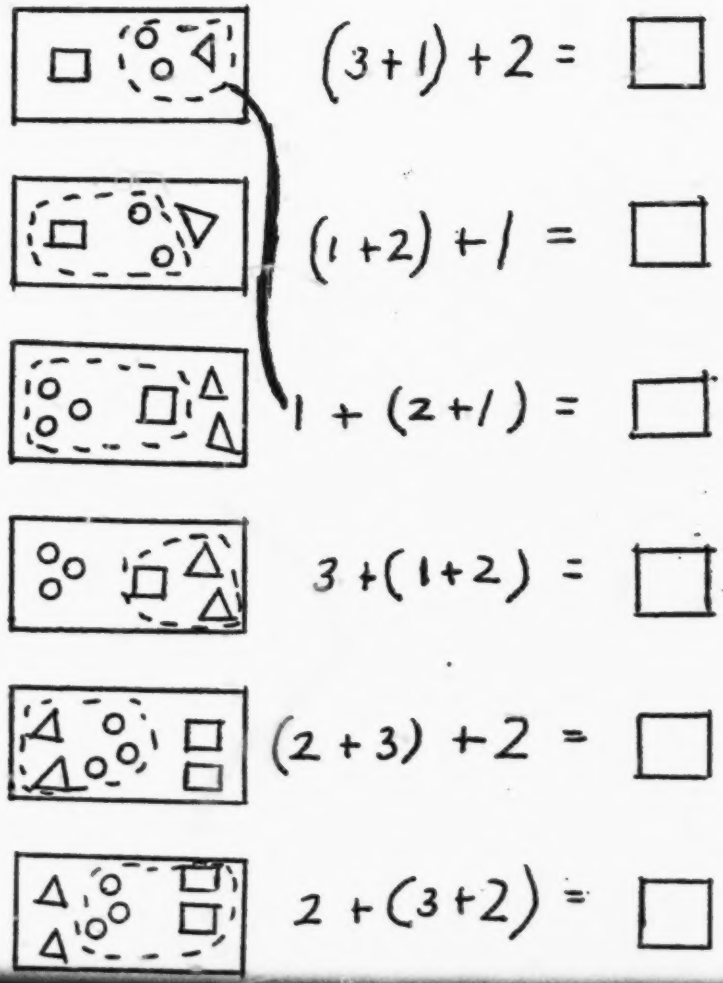
The number line also may be used. Pupils should find as many different orders as they can for adding a group of numbers and step off the "adding" on the number line to see if they all end up at the same finishing point.

Circle number puzzles give lots of drill, using the facts for any addend.

Solid objects may be used to introduce these puzzles, laying the groups out in yarn circles on the flannel board.

The Associative Property - grouping.





In order to help the children discover the properties of commutativity and associativity, give them an opportunity to "scramble" some clothespins.

Use a piece of chipboard or cardboard, 8½ by 12", and supply one green clothespin, two yellow clothespins, and four red clothespins. After the child (or teacher) has placed the clothespins along the edge of the chipboard, encourage the child to "read" from the top to bottom and then place the related addition combination on the chalkboard.

After you read from top to bottom, then have the child read bottom to top and again place the equation on the chalkboard. Children will then discover that in both equations the sum is the same.

Ask the children "What will happen if we change the order of the pins?" Have the children "scramble" the pins, observe the results and give the number combinations involved in each situation.

Guide them to conclude that it is possible to change the order of the addends and arrange them any way and still react the same sum.

It is possible to "scramble" the addends in column addition problems.

Duplicate the following activity for the less able learners who need more experiences working with the grouping property.

The pupils are to match each picture with a sentence and name the sum.

Combining Addition Properties of Commutativity and Associativity.

$2+1+4=\square$

$2+\square+4=7$

$2+1+\square=7$

$\square+1+4=7$

Prepare number cards showing placeholder equations involving three addends. Have the placeholder appear in various positions. Expose the cards, viewing one at a time, and have the children read each completed equation. Some may not be mature enough to work easily with these equations while others will find these a real challenge.

As the child is introduced to column addition, the teacher should guide him to apply his understanding of the commutative and associative properties and discover that he can start at the top or the bottom and still arrive at the same answer.

It is important for the teacher to realize that getting the same answer when adding from the top or the bottom involves both communativity and associativity.

For Instance:

$$\begin{array}{r} 3 \\ 4 \\ +1 \\ \hline \end{array} \qquad \text{OR} \qquad \begin{array}{r} 3 \\ 4 \\ 1 \\ \hline \end{array}$$

$$(3+4)+1 = 1+(4+3) = 1+(3+4)$$ } commutative property

$= (1+4)+3$ Associative Property

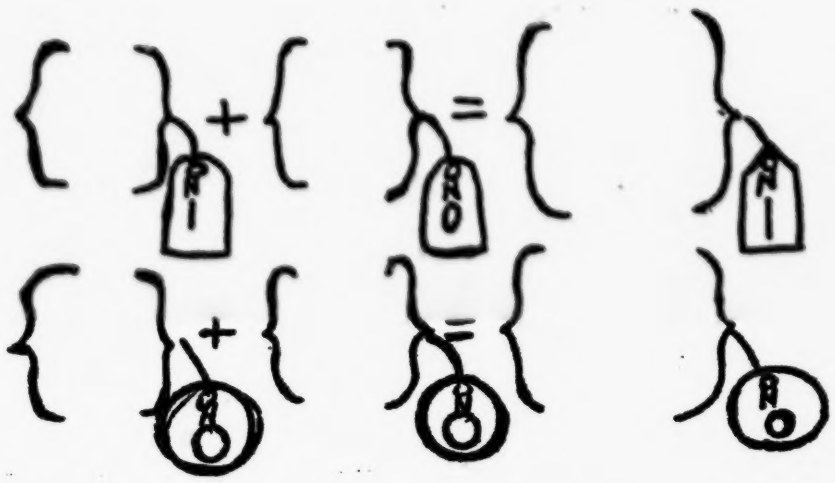
By combining the properties of commutativity and associativity, the child will discover that he may order and group the addends any way he pleases.

The teacher may wish to call this process "scrambling" for the child does actually scramble the addends. As the children progress in arithmetic, he will again and again be exposed to these properties.

Zero as the identity element in addition and as the subtrahend in subtraction.

The children have been exposed to zero when working with sets of whole numbers. They are aware that zero is the numeral for an empty set. Now they will discover that when an empty set is joined to another set, the new set is the same as the other set. Zero is called the "identity element" of addition. 5 + 0 = 5 4:103

In subtraction, the children will discover that when an empty set is subtracted from a set, the set remains the same. 5 - 0 = 5

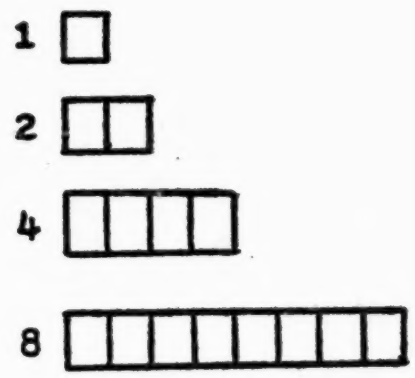


Have the children draw as many things inside each bracket as the tag tells them. This exercise will help them visualize that zero being added to a number does not change the number. The children will see that when you add zero to zero the answer will be zero.

Wertz, Robert et. al. Math Workshop for Children A. Encyclopedia Brittanical Press, Inc., 1962.

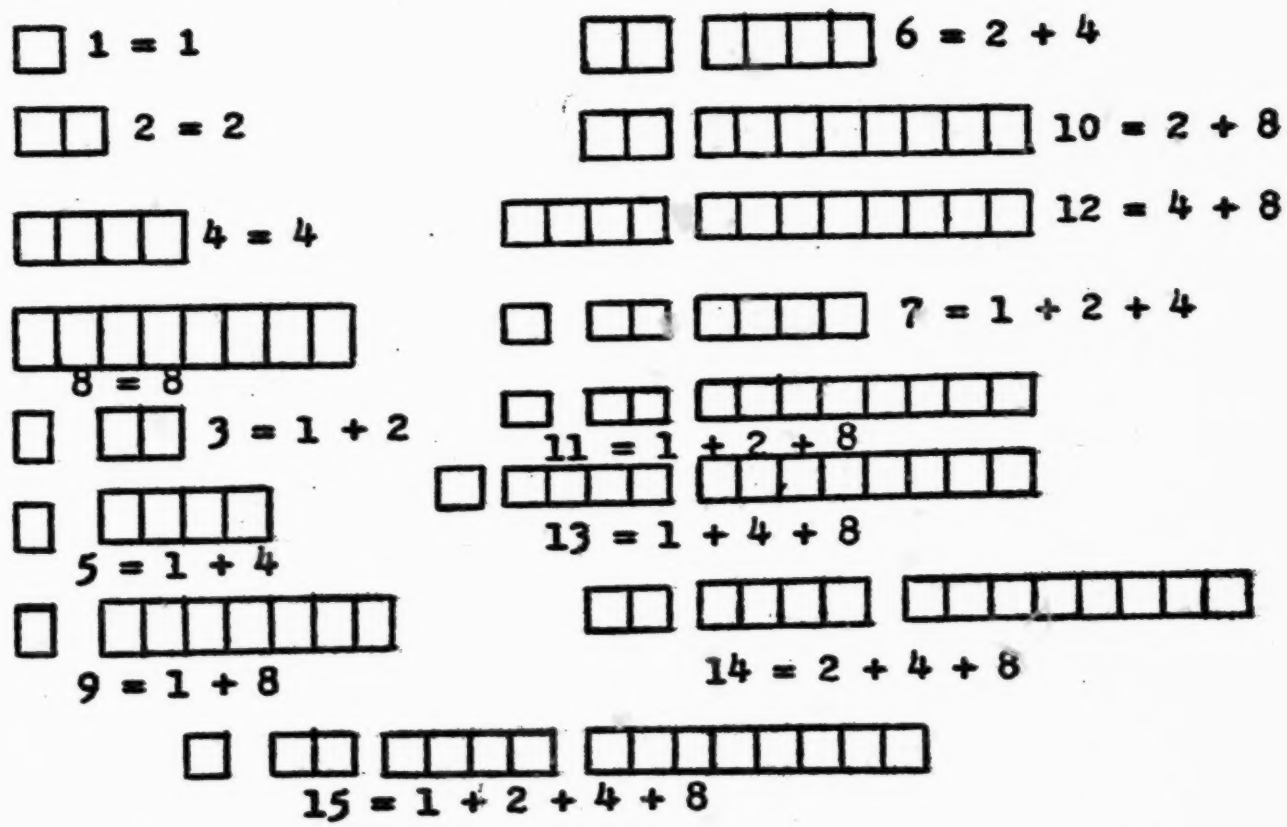
Addition facts to 15.

"The game of 8's, 4's, 2's, and 1's.



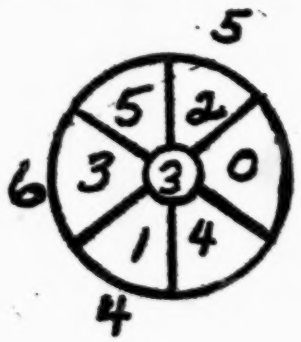
Cut and rule four strips of cardboard into squares, making for 1, 2, 4, and 8. 20:20-21

How many numbers can we show, using one, two, three, or all of these strips?



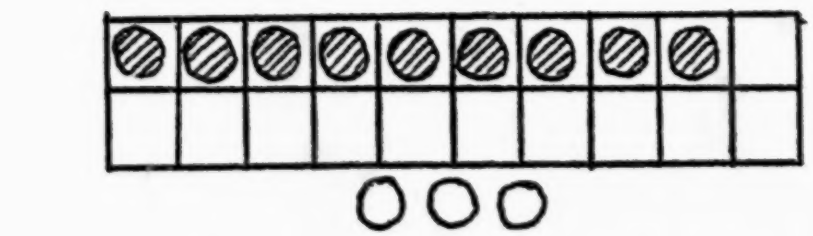
"Does this list show representations for all numbers 1-15?"

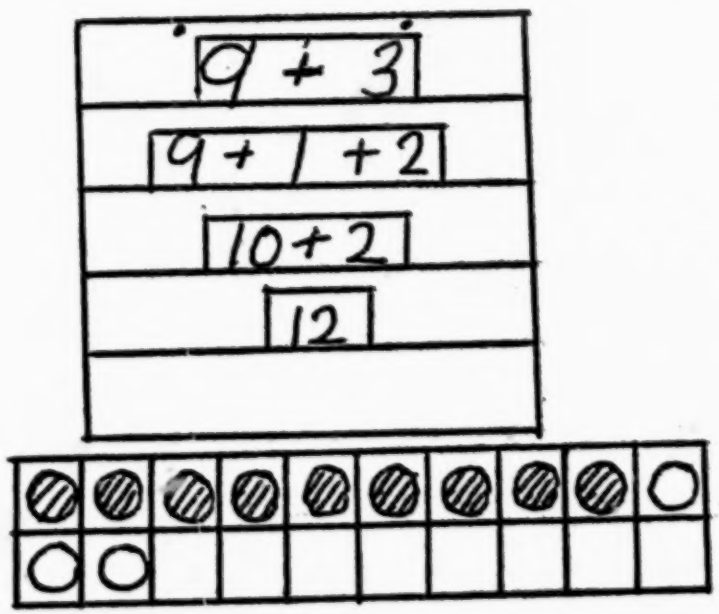
Combinations one through eighteen.



Number wheels made from tagboard can be used to reinforce addition facts. Divide each circle into sections and write one numeral in the center of each wheel. Place other numerals around the edge in the empty sections. Pin the wheel to a flannel board. Have the children place flannel numerals around the outside of the wheel.

Combinations 11 - 18.





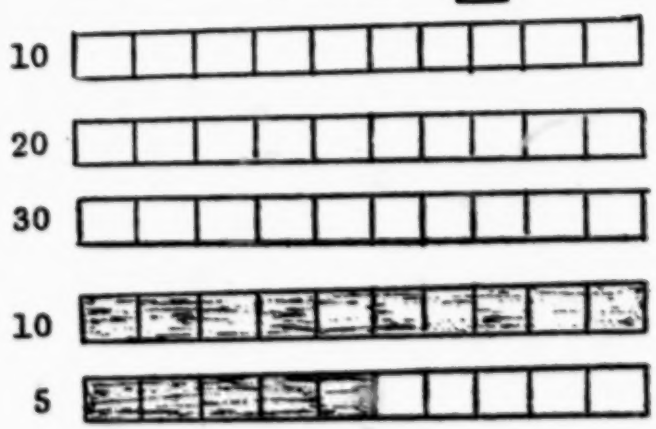
"Operation Big Ten"

Place a piece of lattice work with two rows of ten openings on the flannel board. Place nine counters in the first row and three under the frame. Say that the goal of the game is to make a set of ten as the first step in joining the two sets. Call on a child to perform "Operation Big Ten." As he begins to put the counter with the nine, tell the children that you would like to write a math phrase about the picture, 9 + 1 + 2. Put the picture in the pocket chart. Have the child continue and put the corresponding phrase in the pocket chart.

After the class has worked "Operation Big Ten" together several times, give each child a lattic work. (These can be easily made using 12 x 18 construction paper and a felt pen.)

Addition and Subtraction beyond 18 to 100 using expanded notation and vertical notation without regrouping.

$30 + 15 = \square$



30 + 15 =
(30 + 10 + 5) =
40 + 5 = 45

5 tens 2 ones
+ 4 tens 3 ones
9 tens 5 ones or 95

Number Line to illustrate addition and subtraction.

Place the equation $20 + 15 = \square$ on the chalkboard. Have children place three bundles of sticks on one side of their desks. Ask how many sticks there are. Have them place one more bundle and five loose sticks on the other side of their desks. Tell the children to put all their ones together. They will see that there are only five ones. Have them put their tens together. Now have them tell how many ones and how many tens they have. Have them join the ones to the tens and give the result. Complete the place holder equation. 4:241-244

30 + 15 =
30 + 10 + 5 =
(30 + 10) + 5 =
40 + 5 = 45

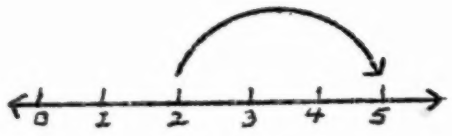
Write addition problems using the words "ones" and "tens".

The use of a number line often helps children become more aware of the orderly arrangement of the whole numbers. Such a line can be drawn on the chalkboard or printed on the floor or playground. The children can make their own number line using adding machine tape or strips of heavy brown paper. Help them to mark off lengths of the same measure and to label each mark, beginning with 0 and extending through 49. Display the number line in a prominent place.

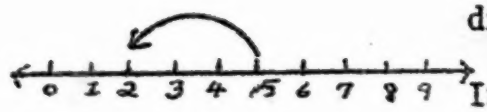
Call at sixteen and count aloud by ones to twenty-four

as he points out successive marks on the number line.

The number line may be used to illustrate the addition and subtraction of numbers. Give pupils experiences with floor number line, and desk copies on which they may mark. Pupils should have many chances to "do and say" what they discern.

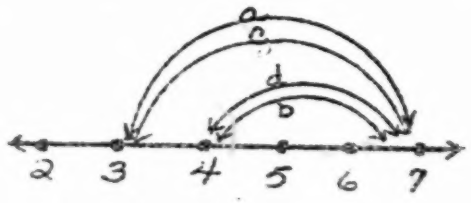


2 + 3 = 5



5 - 3 = 2

Inverse relationship between addition and subtraction can be shown effectively on a number line.



Notation such as this may be used: 5 - 3 = 2 2 + 3 = 5. Show how "subtraction undoes what addition does." Build the idea of "inverse by giving examples such as "Unbuttoning is the opposite of buttoning your coat," "What is the opposite of closing the door?" "What is the opposite of addition?" Use horizontal sentences as well as vertical

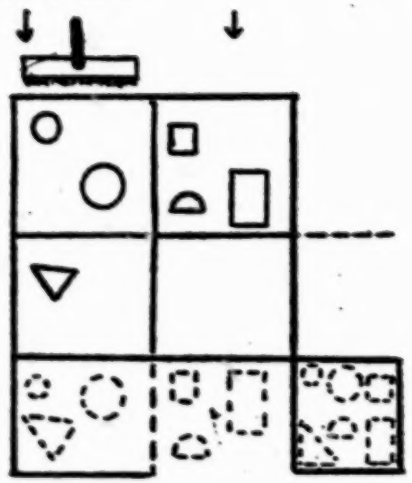
(a) 3 + 4 = 7
(b) 4 + 3 = 7
(c) 7 - 4 = 3
(d) 7 - 3 = 4

Addition table for computation and discovery of number patterns.

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

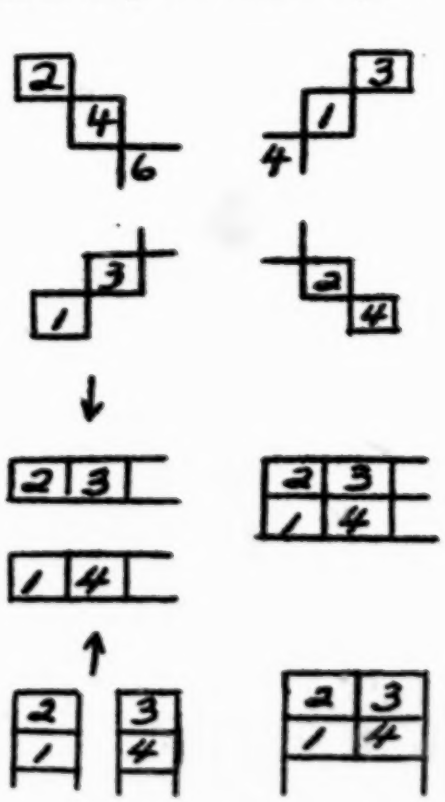
The addition table or grid is fun for fast learning children. It may be used as a supplementary aid to discover number patterns. The primary purpose of the grid is to see number patterns rather than doing the adding to obtain the sums.

Illustrating addition through cross-number puzzles.



To show the grouping involved in addition, a floor-model of the cross-number puzzle can be used, with objects in it being swept together with a push broom. Have pupils actually move the objects, first by sweeping them down in both rows, noting how many there are in each row. Then sweep across into the lower right-hand box to find the number of objects altogether. 17:24-25

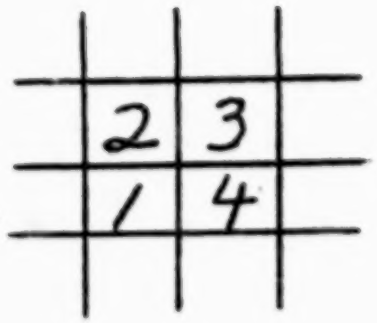
Cross Number Puzzles for computation and discovery of number patterns.



There is no need to become a slave to vertical and horizontal forms. How about a diagonal-down approach? And a diagonal-up approach.

Here they are united into a more compact form.

← Pushed together these become . . .



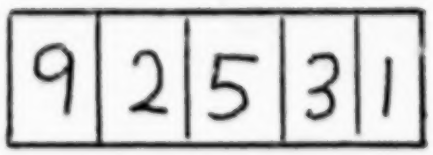
And finally with a few lines added, the equations are compressed into a nice, neat structure. Completed by the children, it looks like the illustration.

Inequalities--
~ relationship
~ comparison.

5 > 3

Display two sets on the flannel board. Have a child identify the number of the set and place the numeral under the set. Then explain to the children that we have a special way to show that five is greater than three. Place the > symbol between 5 and 3. Read the statement (5 > 3): "five is greater than three." Compare other numbers not greater than nine. Explain to the children that the open part of the sumbol is next to the numeral for the greater number.

The relation "less than" can be introduced the same way. Make sure the children understand which symbol is which before going to more complex math sentences.



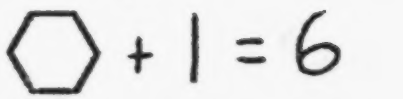
Show the children a card with five numerals in random order. Ask a child to unscramble the numerals and rewrite them on the chalkboard in order of least to greatest. The first time do this with the class, ask which numeral represents the smallest number and proceed to place in order. After they are in order, place symbol < between the numerals, then help children to read them: one is less than two, two is less than three, etc.

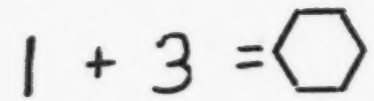
Do this in opposite order sometimes or leave in random order and place symbols between them: 9 > 2 < 5 etc. Have the children read the symbols: "nine is greater than two, two is less than five," etc.

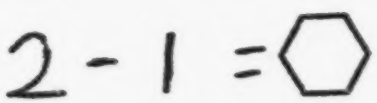
Open and Closed mathematical sentences in addition and subtraction.

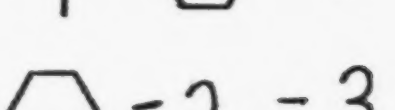
Open sentences are used to present addition and subtraction. Children should read them as sentences before attempting to solve them, reading "what" or "something" for the placeholder. When the correct numeral is in the placeholder, the sentence is closed. This is difficult during the first semester especially for average and slower learners.

Open









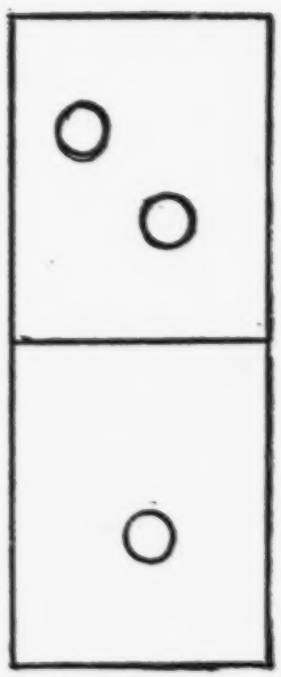
Closed

2 + 3 = 5

5 - 2 = 3

Use the tape recorder to make a tape that directs the children to do each math sentence on a ditto sheet. This ditto sheet should be constructed to emphasize the difficult sentences such as □ - 2 = 3 and □ + 1 = 4. Then give each child in your class a ditto and something to count such as beads on a wire or bottle caps. Next make the tape while the class does the ditto (making the tape while the children are doing the ditto helps the teacher pace the tape correctly).

After the tape has been made, it can be played again for drill for those that require it. For example: if the math sentence is □ - 2 = 5, then the tape could say: "This is a lunch box story. You ate 2 cookies and then you were not hungry any longer, so you took five cookies home. Take 2 counters for the 2 you ate and then take 5 more for the 5 cookies you took home. Now find out how many counters you have. Two that you ate and 5 that you took home - How many cookies did Mother give you? (pause) Yes, 7. What goes in the placeholder (or square)? (pause) Yes, 7.



Draw two dots on one side of the paper and draw one dot on the other side. Fold the card in half so that only two dots show. Show the card to the class. Have one child tell how many dots he sees, then unfold the card and have the child tell how many dots "joined" the set. Next ask a child to write the related equation on the board. 2 + 1 = □

By reversing the procedure, you can present the related subtraction equation. (mathematical sentence)

Open and closed mathematical sentences in addition and subtraction using two addends.

Present story problems to the children by telling a story that can be written in an equation. The teacher may say, "Bobby, put one ornament on the Christmas tree and Susan, put four ornaments on the tree. Ask someone in the class to come and write the math sentence or equation on the board. The child may need the story told again, he then writes the equation 1 + 4 = ☐ on the board. Then ask another child to come and place the sum in the equation.

Then tell the children "there were five balls on the tree but the cat, Blackie, broke four of the balls, how many balls are left on the tree? Then select a child to write the equation on the board. 5 - 4 = ☐

Children at this level enjoy activities that appeal to their sense of humor.

Develop math sentences around riddle games.

To provide sentences that give the children experience with the placeholder in different positions in subtraction equations, present the following riddles:

Say to the class, "If I subtract one from this number, I get two. What is my number?"

Call on a child to write the equation (☐ - 1 = 2) on the board. Ask the class if the equation is correct.

Continue with this procedure using the following riddles.

If I subtract two from three, what number do I have?

If you subtract one from
me, you will have one.
What number am I.

One part of a set of
three is a set of two.
I am the other part. I
am a set of how many
objects.

Open and closed mathematical sentences in addition and subtraction through 3 addends.

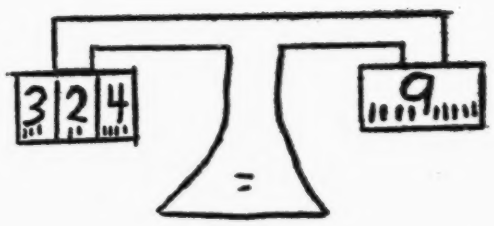
Use a balance scale to illustrate this concept. Commercial ones are available. This can also be illustrated on a bulletin board as per illustration. Marks on cards must balance or be equal on each side of the scale.

Open

$3 + \_ + 4 = 9$

Closed

$3 + 2 + 4 = 9$



PROBLEM SOLVING

GRADE 1

Introducing Story Problems

Problem solving seeks to apply the abstraction of mathematics to the world events. The story problem has been used at the end of units as a test or exercise of the pupils' ability to apply an acquired knowledge of arithmetic. Often the story problem has only been a reading test. If a child could read, the questions are trivial and constitute <u>examples</u> and not <u>problems</u>.

Example: Mr. Jones bought 8 fence posts. He paid $10 a piece. What was his total expense? (If you can read, this constitutes an example of the fact that 8 x 10 = 80.)

Problem: Mr. Jones bought 8 fence posts. He set them out in a straight line, 10 feet apart. What is the distance from the center of one end post? (If you can read, this is a problem that requires careful consideration of the situation presented, with perhaps a sketch to show seven spaces.)

10 feet

Have students dramatize the story of the shepherd who counted his sheep by matching each sheep with a pebble as they came into the corral. Count the students this way as they leave the room for recess and come back after recess. Ask the class "How do you know that everyone returned?" The children have found that sets have the same number of members without counting in the usual sense. 4:3

Make party plans a part of your arithmetic class. Put all number problems related to the party on small pieces of paper. Pick out various cards and work out the problems.

Dramatizing problem situations occurring in classroom and stating problems orally

For example, if three children make place mats for the group, how many will each have to make? How many dimes will be needed to buy milk?

The teacher helps the pupils solve a problem by having them dramatize or make drawings, to illustrate it. Then the pupils write the problem as an equation with a placeholder in it.

When the pupils are able to solve problems by themselves, they may do so.

Example: Ted had six marbles. When he came back from the store with more marbles he had 10 altogether. How many did he get at the store?

$$6 + \boxed{N} = 10$$

Make charts with pictures of objects you might buy in a store and show their prices. Have the class take turns in dramatizing stories of buying and selling.

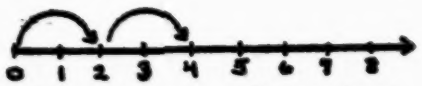
A store may be set up with goods brought from home. This may be a toy store, grocery store, etc. Choose a child to serve as storekeeper. Give the children sets of coins to buy given items. Have the storekeeper give the correct amount of change. (Children could make the money to be used.)

See the game, "Going Walking" in the Addition and Subtraction section.

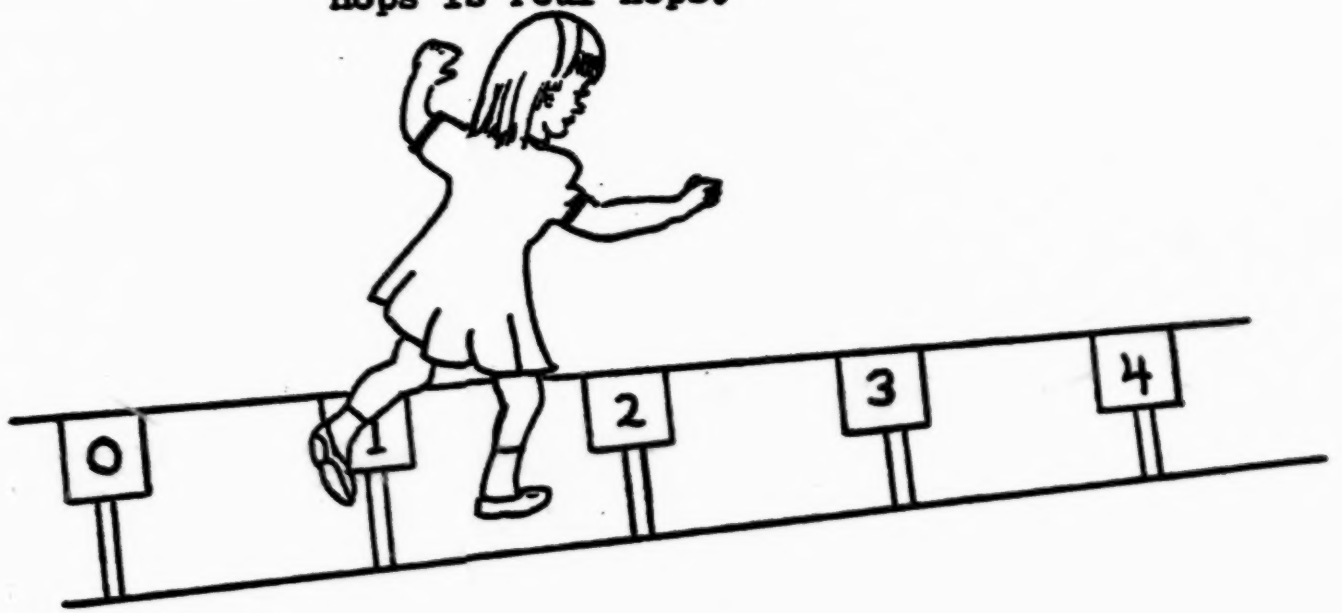
Number line in problem solving

Use the number line in problem solving. The number line should be situated near a wall so children would not look at it upside down.

Have several pupils jump along the line. Suggest that they jump like a frog, hop like a bird, or a rabbit. Ask the class if they know the number of jumps anyone has taken.



Two hops plus two hops is four hops.



Choose children to represent points or "stations" on the number line starting with one. Ask, "How many children are there? How can you tell without counting?"

Give children tags with tally marks. Have them match themselves to the number line.

To solve these story problems the teacher can have the children refer to the class number line or their individual number lines at their desks.

The teacher may say, "There are 28 children in our first grade but one day there were 25 children in the room. How many pupils were absent?" The children can refer to the number line to reach the correct answer. Pupils locate the number 28 and then counted spaces back to 25 to discover that 3 were absent.

The number line can also be used to solve the following problems.

1. There are 17 books on the table. When Mary puts two more books on the table how many do we then have on the table?

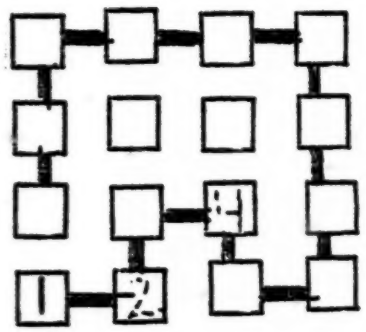
2. Mark had 11 pencils. He lost 3 of them. How many does he have left?

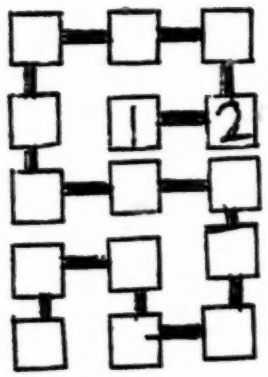
3. John has 14 marbles. Bill gave him 4. How many marbles does John have now?

4. Lucy had nine crayons yesterday. Today she has 5. How many crayons did she lose?

The above problems are discussed or "acted out." They are not written nor are they meant to be read by the children.

Here is an opportunity for the children to see sequence in a different form. They must solve the problem of number order following a path. The children are instructed to follow the path writing the next larger numeral. Start at "1."





Interpreting problems from pictures

First graders can be helped to learn that many combinations have the same sum by the use of large posters.

For example, pictures of mother animals and their young.

On the mother animal write the sum of all the animals. On each of the baby animals the group can write a combination that equals that sum.

Children can make these combinations with the help of the teacher while faster children can make these combinations by themselves.

Translating number problems into math sentences

Make up a problem

Have the pupils work in groups and take turns being "it" as described below. Let one pupil do the timing. Have him experiment with periods of one minute to three minutes to determine how much time should be allowed. (It will be helpful if you can provide a three minute egg timer, watch with second hand, or stop watch.) The pupil who is "it" writes on the board an equation with n to hold a place for the answer. Then he points to someone in the group and asks him to make up a verbal problem to fit the equation. If the chosen pupil can state a problem and give the answer before the time is up, he becomes "it." If he cannot think of a problem, the pupil who is "it" asks someone else to make a problem. Everyone in the group should verify the answers. Supervise the activity to make sure that the pupils do not go beyond the range of processes studied thus far. Permit discussion of all disputed problems.

Charades

1. one tells story with a problem in it
2. one demonstrates with physical objects
3. a team solves the problem

Translating number problems into mathematical sentences (oral and written)

$\square + 3 = 9$

Setting up addition equations

Make cards with numerals on them. Have students pick out numerals and operational symbols, to make problems. Have another student solve the problem.

The teacher should give the pupils an opportunity to compose their own problems using the basic operational combinations being taught so they can learn the combinations.

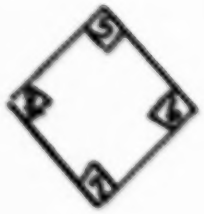
The teacher also assists the pupils in reading and understanding the problems in their text. She helps them to determine the questions asked and to observe the mathematical terms that are used.

Have children dramatize number combinations (addition or subtraction) through 10. Make two sets of cards with numerals 0-9. Pass out the cards, one to a child. One child stands in front of the group and says I am 2. Who can help make me 6? The child with number 4 comes up and he in turn says, "I am 4. Who can help me be 8?"

If the teacher wishes one child can be called to write the story in equation form on the chalkboard.

The pupils may enjoy playing a ball game. Draw a picture of a ball diamond on the board and write a numeral for each base (5,6,7 and 8).

To score, a pupil must give another name for each number that is named on the bases. Write the following number names on the board and have the pupils select an appropriate one for each number name shown on the ball diamond. Divide into two teams and have the pupils keep score.



4 + 1	3 + 3	2 + 6
2 + 3	6 + 0	3 + 2
6 + 2	3 + 5	5 + 2
7 + 1	4 + 4	6 + 1
3 + 4	1 + 7	5 + 0
4 + 2	5 + 1	2 + 4
1 + 6	2 + 5	5 + 3
2 + 4	1 + 5	4 + 3

Matching math sentences with pictures

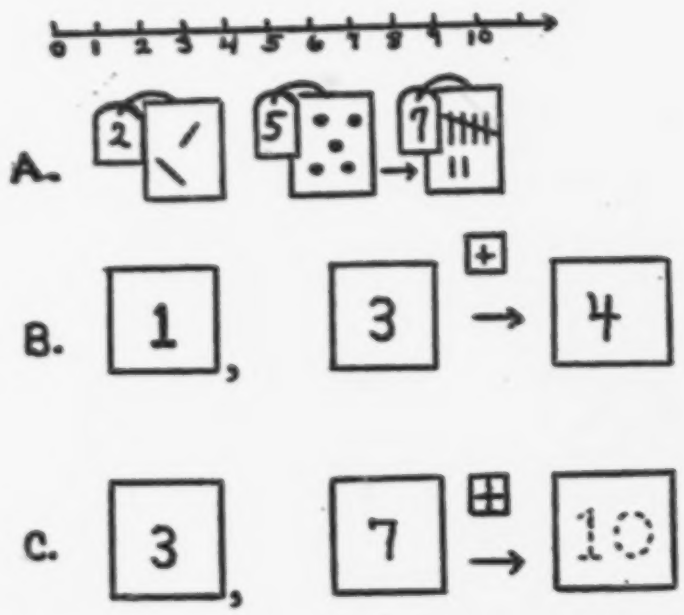
Create problem situations that pupils can dramatize. The pupils may also use objects in the classroom or cutouts at the flannel board to demonstrate the action in a problem. 6:77

Suggested problems:

Three boys were playing marbles. Three boys joined them. Then how many boys were there?

Soon three of the six boys joined another game. How many boys are now playing marbles?

Matching math sentences with pictures



Game

"What's My Rule?"

The secret rule presented here is that the sum of each pair of numbers suggests the third number. 17:58

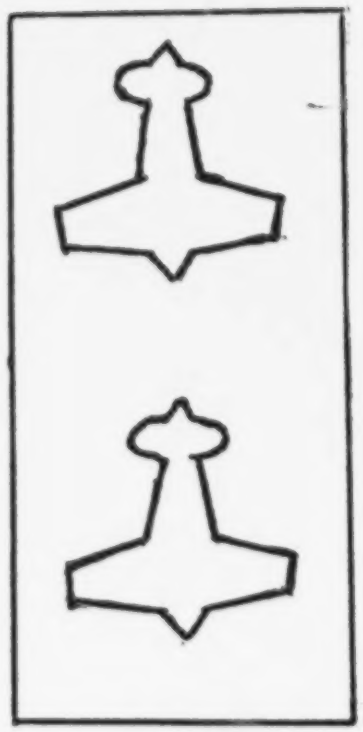
Have the class look at line "A". What is the number of blue bars? What is the number of blue dots? What is the number of blue tally marks?

In line "B" the secret rule "1" and "3" suggests "4." If "2" and "5" suggest "7," and number "1" and "3" suggest "4" on line C, what number do "3" and "7" suggest?

Pupils should see if they can apply the rule to all examples.

E and G have a different rule. What is their rule?

Allow students the opportunity to make up their own rules and try them out on each other.



Anything reasonable should be considered usable. For example, after the game has been played a number of times, the rule might be to combine two numbers so that "1" and "3" would suggest "13" and "3" and "7" would suggest "37". Or perhaps "12" and "49" might suggest "1492" and "28" and "16" might suggest "2168." The rule in each case should be clear to everyone.

Children need not verbalize their understanding of the rules. However, they can express their understanding by offering an example of their own to show that they have found the "secret."

Put a set of two felt cutout airplanes on the flannel board. Have the number of the set identified. Tell the children to close their eyes. Remove one of the airplanes. Direct the children to open their eyes and tell how many airplanes flew away. Have them tell how many airplanes they still see. Have someone tell the set story and give the related number sentence. This procedure can also be used in the introduction of other combinations.

GEOMETRY

Grade 1

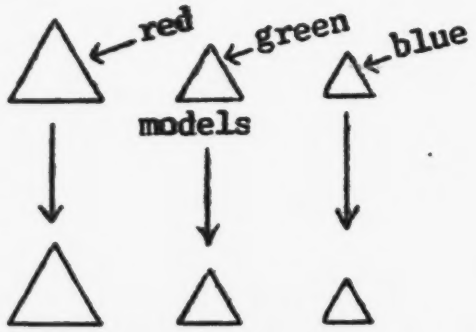
Grade 1

Concepts from Kindergarten

GEOMETRY

Don't assume that pupils have any background in geometry. Due to the lack of a basic text and a variety of content offerings in kindergarten, first grade teachers would be wise to start with concepts introduced at kindergarten level.

Characteristics of circle, square, rectangle and triangle through measuring.



matching white shapes for student to color red, green, and blue.

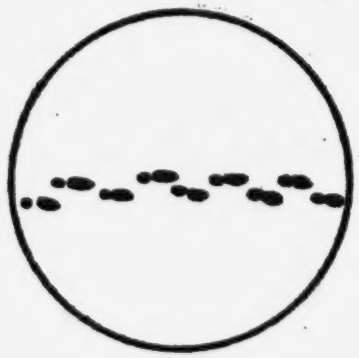
Cut shapes from pieces of colored construction paper. Cut matching shapes from white paper. Ask students to color the white shapes to match the color of given shapes of the same size. Give students at least 3 different sized circles, squares, etc., so they can see the similarities of circles.

18: 17,29, 48,68, 70,80, 94,111, 124,128

11: 8,13,51, 80.98-99, 152

Ask, "How many sides has a rectangle? What can we say about the lengths of the sides of a rectangle?" (Opposite sides have same length). Measure using crayon lengths, widths of finger, etc. "How many corners does a rectangle have?"

Is a square a rectangle? (Yes, a square is a special rectangle.) What is special or different about the square?" (four sides are equal in length)



"How many sides has a triangle? Is a triangle a rectangle? Use what is known about the rectangle to test the triangle." (4 corners? 4 sides? opposite sides the same length?)

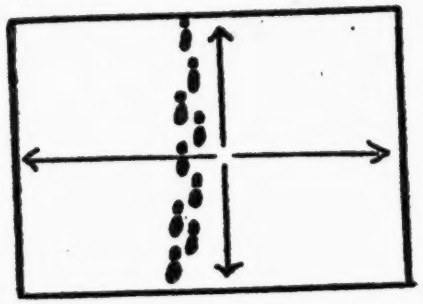
"What do circles look like? Do they have corners? Straight edges? What is the longest walk across the inside of a circle?" (along a diameter, but don't use the term now) Demonstrate the diameter concept by having students join hands and form a circle. Ask a student to show the longest straight path across the inside. Prove by measuring various paths with a string. Have students sit in a rectangular pattern, a square pattern, a triangular pattern. "How many children make up a side of a rectangle if the entire class forms the pattern? (Varies) The square?" (Class members ÷ 4)

When children have learned to recognize geometric shapes, they will enjoy going through old magazines and newspapers looking for pictures illustrating these shapes. They will be surprised to find so many. Have the children cut out pictures of these shapes and paste them on large charts. The individual charts may show objects of one geometric shape or may be a combination of shapes.

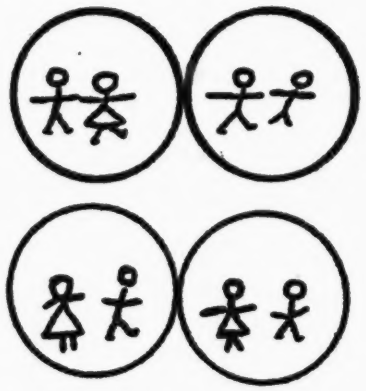
Let the children make geometric shapes from colored paper. Tell the children to create animals from the shapes. The animals may be made from triangles, circles, squares or triangles. Have them describe these animals to the class. What shapes did they use to create the animals?

Spatial relationships between objects (distances)

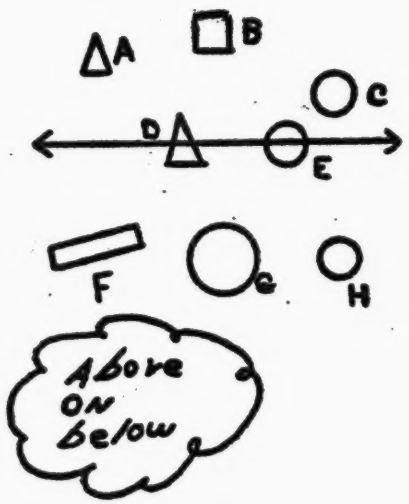
Name two points in the classroom. Ask students to walk from one point to another. Discuss the various paths taken. Ask, "What is the shortest path between the two points?" (line segment)



Use models of rectangles on floor or have students to walk the shortest path from each side to its opposite side. "What two figures are formed if you walk the shortest path between opposite corners?" (triangles) "What can we say about the lengths of the opposite sides of a rectangle?" (equal)

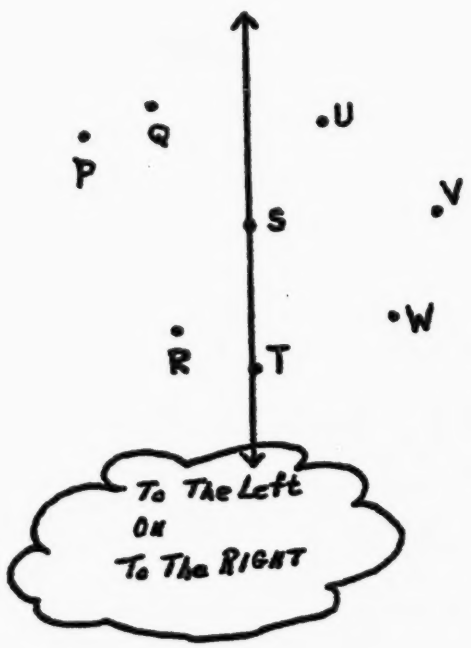


Try the same activity with squares. Ask students to generalize about the lengths of the four sides of any square. Let children decide how many students should sit in each line to form a square. (Separate class members into four sets.)



Develop vocabulary and awareness. Draw line on chalkboard. Draw triangles, squares, circles and rectangles above, on, and below the line. Label each figure with a capital letter. Ask, "Is triangle A above, below, or on the line? How many circles are below the line?" etc.

19:5-8



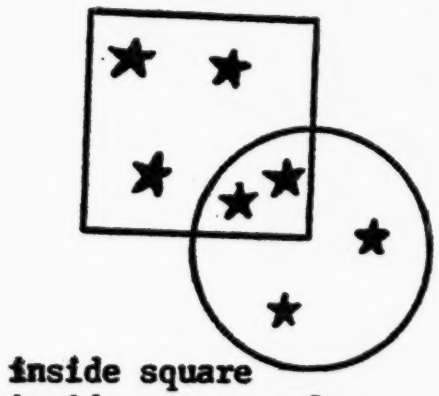
Draw a vertical line on the chalkboard and place points to the left, on, and to the right of the line. Label the points using the capital letters of students' first names. Ask, "Is point P to the left, on, or to the right of the line? Name two points to the right of the line. Is point W on the line?"

On a table, place two objects about 15 inches apart. Have the pupils take turns in arranging a string to show different ways to get from one object to the next. Develop the idea that the shortest distance is shown by holding the string tight. Go back to the drawing of the houses. Develop the idea that the shortest distance or path between houses is measured along a straight chalk mark.

Tell the pupils that such a mark is a picture of a line or segment or path.

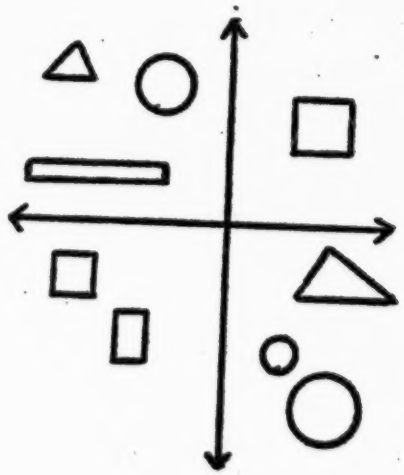
Spatial relationships - interior and exterior points.

Draw pairs of intersecting triangles, squares, circles, and ractangles. Draw stars or points inside the figures. Ask, "How many stars are inside the square? (five). How many stars are inside the square only? (three) A variety of



inside square
inside square only
inside circle
inside circle only
inside square and circle
outside square
outside circle
outside square and circle

shapes and objects could be used. Assign letters to points and ask the children to name the points outside the square, inside the circle, etc.



Draw two intersecting lines as shown in the example. Draw geometric figures, numbers, or letters in the quadrants. Have students ask questions of each other.

1. How many circles are to the left of the line?
2. Name the figures above the line.
3. Name the figures above and to the left of the lines.
4. Point to a circle to the right and below the lines.

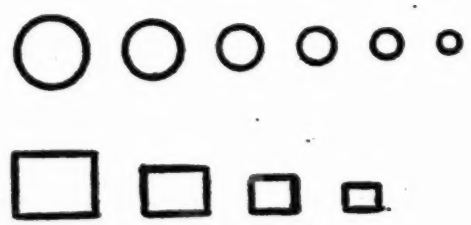
Comparing shapes and sizes of objects.

Show pairs of objects alike in color and general shape but different in size. (books, balls, pencils, containers) "How can we tell the objects of each pair apart?" (size) Show two and then three circles of different size. "How can we

tell the two circles apart? Three circles?" Develop vocabulary of larger than, less than, largest, smallest, and in-between. Show squares, rectangles and triangles in the same way.

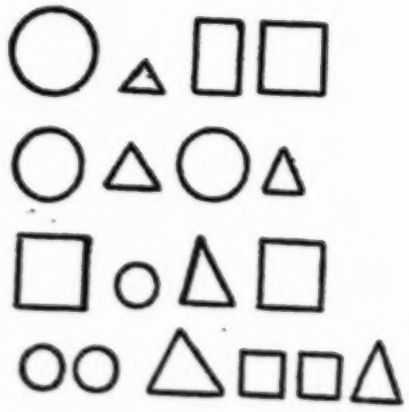
Place the name of a shape (triangle) on the chalkboard. Ask the children "Who wants to draw a triangle on the chalkboard? Another triange? A long thin triangle? A small triangle? A triangle with a very sharp point?"

Continue this activity with other shapes. Then tell the children they are a jury. Their job is to study the shapes - do they all belong? Are all the squares really squares? Are they triangles? Discuss shapes that don"t belong. "Where does it belong?"



Place rows of circles of various sizes and colors along the chalk tray. Have students duplicate the patterns displayed. Encourage discussion and use of vocabulary terms.

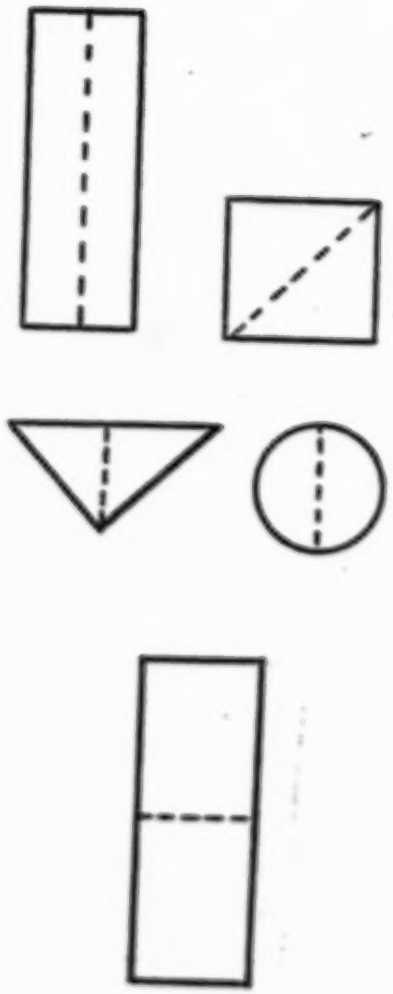
1. Arrange figures from smallest to largest size (left to right).
2. Arrange from largest to smallest size.
3. Vary positions.



Place a pattern of circles, squares, rectangles and triangles on the chalk try. Have individual students match the pattern or allow groups to build a pattern together.

To vary this activity, have a student arrange a model pattern on the chalk tray. Then let class members see it briefly, cover the model and ask the class to reconstruct the pattern by themselves from memory.

Symmetry



Provide students with the following shapes: circle, rectangle, square, and an isosceles (two sides equal) and equilateral (three sides equal) triangle. Important--make all copies of one shape of the same color, i.e., all circles red, all squares blue, etc. Or, make all shapes of white paper. This reduces color confusion and allows student to dwell on the symmetry of the shapes. Students then fold shapes in half. Special help and discussion should develop out of the folding of the triangle shape. Students cut along the folding line and paste the halves on sheets of colored paper.

"Are the two halves alike? Are they exactly alike (congruent)? How can we prove that they are?" (By placing one over the other, by mapping all points of one half on the second half.)

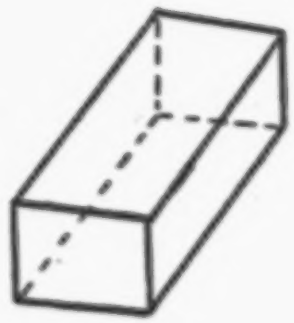
Have students fold 8½" x 11" sheets of typing paper in half. Let them experiment, cutting various shapes, being sure to leave segments along the folding line intact. Paste the designs on colored construction paper.

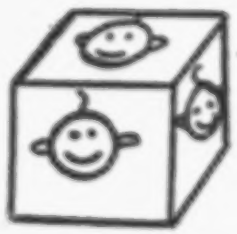
Discuss how familiar shapes such as hearts or circles can be cut out by thinking about the picture of one half of a heart or one half of a circle. Let them experiment to find out how a circle shape must be cut on the fold. Compare shapes by placing them side by side on the chalk tray.

Recognition of three-dimensional objects--sphere and cube.

right circular cylinder

right rectangular prism





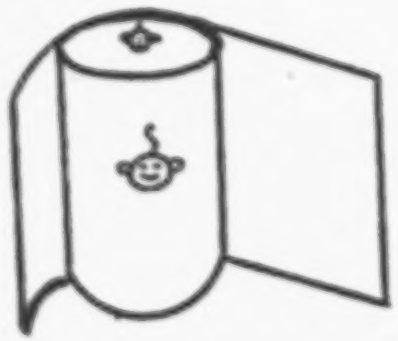
Collect a set of three-dimensional objects such as marbles, balls, boxes, dice, sugar cubes, water tumblers, tin cans, and mailing tubes. 6:117-125

Give the children the objects to feel and inspect. Encourage them to see edges (line segments), faces (surfaces), and corners (right angles).

Lead them to discover specific facts about each object, e.g., hold up a can. "Does it have corners? (No) Does it have edges? (Yes, Have students run fingers aroung the two circular rims.) Does it have faces?" (Yes, three)

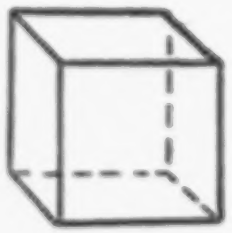
To develop vocabular and concepts sequentially, it is wise to begin with the cube and right rectangular prism (box) before introducing the cylinder. There is sometimes confusion over the edge-face distinction. Draw a funny face on the face of a box and point out that we could

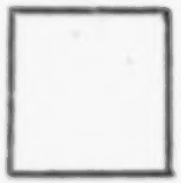
not draw the same picture on an edge.



To show the face of a cylinder, cover the surface of a tim can with a rectangular piece of paper and then remove the can. Open the resulting paper cylinder to show the rectangle shape.

Ask students to hold objects behind their backs while describing the object to the class in terms of edges, faces, shapes, and corners. The class member who guesses the object correctly then becomes the leader.





side view of cube

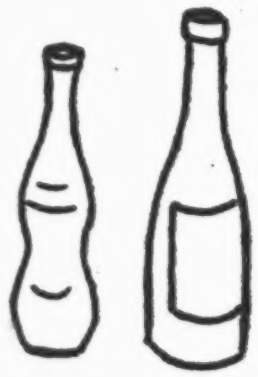
Hold objects so that the children see the two-dimensional shapes found in them, i.e., the circle and rectangle shapes in the cylinder, the square shape in the cube, and the rectangle shapes in the right rectangular prism.

Darken the room and use a light source such as a film projector to project the shadows of three-dimensional objects on the wall or screen. See if students can guess the object by studying its shadow projections, e.g., a cube, a rectangular prism, and a cylinder may all project a square, hence a second projection would be necessary to distinguish between these objects.

MEASUREMENT

GRADE 1

Measurement non-standard units to standard units.



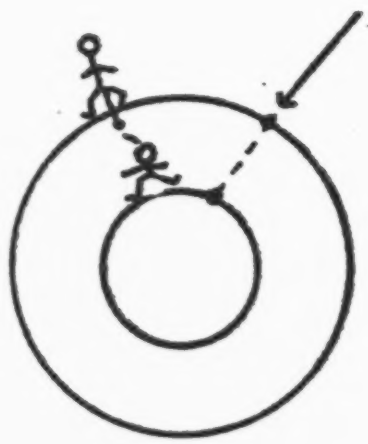
Lead the children to see what is involved in choosing a standard unit of measure, rather than teaching rote use of the ruler or yard-stick. 13:33-41

Ask the children if they are bigger than a soda pop bottle. Have them estimate their height in "bottle" units. Stand one or two children against a vertical piece of adding machine tape. Record and compare their height in bottles to their estimate. Once a comparison is made between one or two children and the bottle, ask the rest of the class to revise their estimates. Then the children, working in teams of 5 or 6, can carry out the experiment themselves.

Make frequent use of terms related to height.

Ann is short, Sally is shorter and Rod is shortest. (Tall, taller, tallest, etc.) 5:50-51

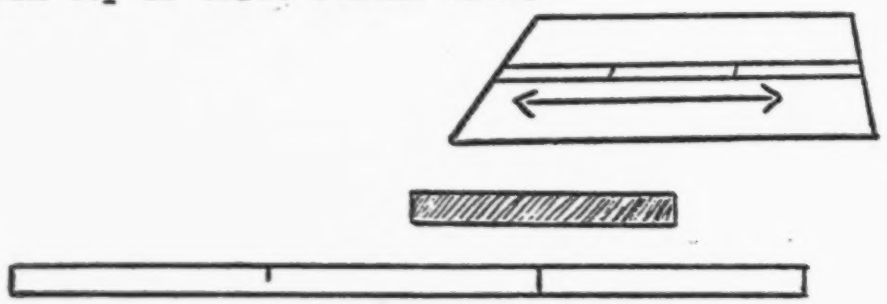
Measurement related to distance around a circle.



Draw two concentric circles in chalk on the playground. Have a pair of children, holding hands, go around both circles while you ask questions about the distance covered by each child. (One child is walking on the inner circles while one is walking on the outer circle.) An attempt should then be made at measurement. The unit of measure could be a child's foot, a strip of tagboard, etc. Let each child in a small group use his own standard so that confusion will arise and the need for a single standard will become obvious.

Measurement using non-standard units to lead to standard units.

Desk top is three sticks wide.



Show several unmarked sticks of various lengths. Have the children, working together in pairs, measure the length of their desk, the chalk tray, the floor, etc. Then compare their results in "stick units." Decide which stick length is the most practical for each situation at hand. 9:449-450
It is important that units of measurement be appropriate. (We don't use inches to find the distance from L.A. to San Francisco or try to weigh gold bars on our bathroom scale.)

Rounding off (approximation)

The children will measure something in stick units which will not fit an even number of times but will have some "left over " They should estimate the amount of the stick unit left over and round it to the nearest whole unit. 6:122, 126-128

Distribute several unmarked foot rulers (wood, tagboard, chipboard). Compare their lengths to the lengths of a yardstick. Decide which, 17:70-71
the yardstick or the foot ruler, you would use to measure the chalk board and which you would use to measure a piece of paper.

Put masking tape strips of different lengths on the floor and walls in various parts of the classroom. Use lengths ranging between 1 ft. and 8 ft. Have the children measure them with unmarked foot rulers, in teams, then record and compare their results.

Ask one child to measure a 10 inch strip using his unmarked foot ruler, round it to the nearest foot, and record it as 1 foot. Ask another child to measure a 3 inch strip with his ruler. Rounded to the nearest foot, it would be called zero feet. Is

Length

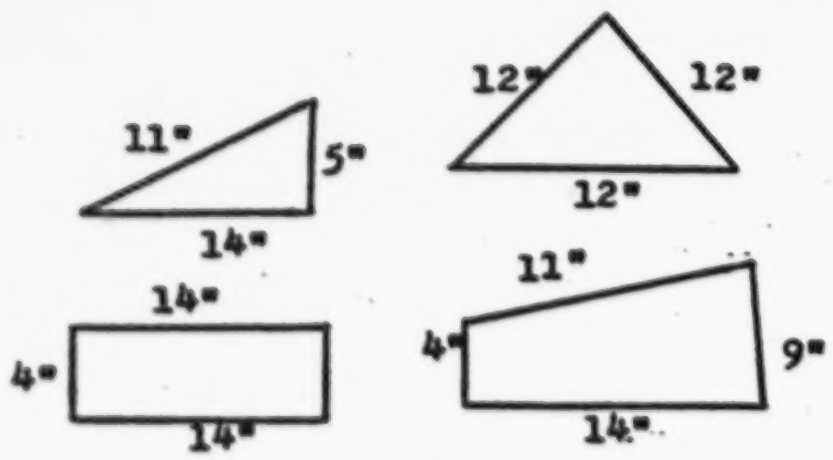
it really zero? This, hopefully, will lead into the idea of using smaller units.

Children can manipulate one-inch-cube blocks to discover how many will fit on an unmarked foot-ruler. We can now say that one foot unit of measure is equivalent to 12 one-inch units of measure. Now the class may be at a point where they can use standard rulers with inch or half-inch markings. 17:97, 128

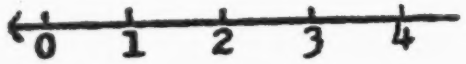
Teachers should remember that when we add we do not add inches, any more than we add apples; all we add are <u>numbers</u>. If we have 4 inches of string and 2 inches of string, we have 6 inches of string altogether--only because:

$$4 + 2 = 6$$

On large tagboard draw triangles and quadrilaterals. Have the children measure the lengths of the line segments forming the sides of the figures. Draw these figures, in much larger form, on the playground with chalk and measure with a yardstick. Select groups to work together measuring objects the teacher has already measured. Have them record the results and check with the teacher to see if they are <u>reasonably correct</u>. Remember that many first grade children lack the dexterity and coordination needed to be precise in their measurements and should not be expected to be accurate. 12:407-438; 6:129-130; 11-60-61,98



The number line as a measuring device.



Children who have not been introduced to the number line in kindergarten should see a floor model first and have the experience of hopping in equal hops. Later there should be a number line on the blackboard (low enough to reach) as well as individual number lines taped on each child's desk.

Number lines can be found several places in the classroom. The clock on the walk, which measures time; the yardstick and the foot ruler which measure distance; the calendar (in number line "chunks") which measures days, weeks, etc.

Put a vertical number line on a wall so that children can measure their height with a partner to assist.

Colored rods as a form of number line.

Example: Cuisenaire Rods

Use of the colored rods is a conceptual approach to math through algebra rather than the traditional approach through counting. The rods are a model of the rational number system. They provide a concrete model of abstract numbers and of relations existing among these numbers. There are 10 colors and each colored rod is referred to by its color. The red rod is written "r," the white "w," etc.

18:24, 52-4, 59, 81-82, 84-85, 87-89, 92-93, 97 99-100

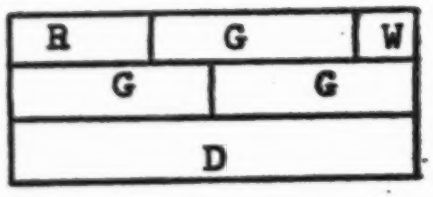
Comparing lengths.

After working with these rods the children come to realize intuitively that there is a standard unit of measurement involved, and that it can be verified. If the lengths of two "trains" (a term used for joining the colored rods together) are the same, we say the trains are equivalent. The sign (=) is used between two expressions representing trains to indicate that the lengths of the train are equal--that is, that each train has the same length.

15:22-23

Equal lengths
(Equivalent trains)

R + G + W = G + G = D



We can write "R + G + W = G + G = D" to indicate that the train made with a red rod, a green rod, and a white rod is equivalent in length to a train made with two light green rods or one dark green rod.

Inequalities with rods

When any rod is compared to a second rod, one of the following situations must occur:

The first train is longer and we write

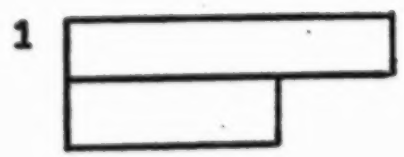
$A > B$

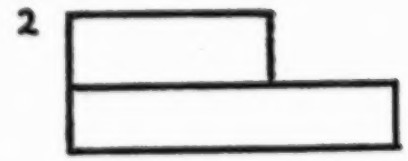
The first train is shorter and we write

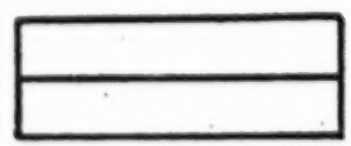
$A < B$

They are equal in length, and we write

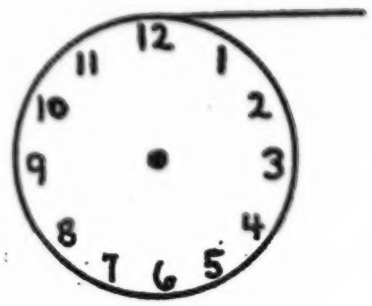
$A = B$







Time (the clock)



Here is an opportunity for the first grader to work with a finite (limited) system, rather than the usual system of whole numbers with its infinite number of elements. How many elements does this clock face have? (12--the numerals for the hours) In the later grades clock arithmetic becomes "modular arithmetic." Ask the children to find a special number line in the classroom. If they have difficulty give them a hint - (it isn't a straight line. It only goes to 12, etc.) A

The hour hand

clock face is a circular number line. If the ends of this line are joined with the points named 0 and 12 overlapping, we have a clock face.

To tell time on this clock face number line we go to the right, just as we add on the straight number line.

Have each child make his own clock from a paper plate and attach <u>just</u> the hour hand. (Add the minute hand later when the need arises and the readiness is present.)

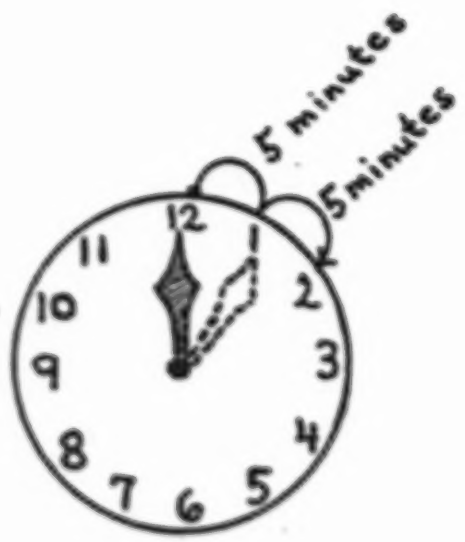
Lead the children to discuss reasons for measuring, recording, or knowing exact times during the day.

"If the school bell is broken how shall we know when to go out to recess?"

Tell the children that the short hand is the hour hand and that it goes all the way around the clock twice a day. Tell them that even without a minute hand you can show the time fairly well. Children need to see the clock hands one at a time to better understand their movement and function. Having both hands introduced simultaneously may lead to confusion. 12:438-447

A dittoed clock face can be glued to the paper plate.

The minute hand.



Display a clock with the minute hand only. Tell them the long hand goes completely around every hour. Show that it moves from one minute mark to the next in one minute, and from one numeral to the next in 5 minutes.

Have the children add the minute hand to their own paper clocks. Practice reproducing clock times together as a group. 61:91-93 190-191

Stay with the "o'clock" and "half-past" times in the beginning.

Both hands on the clock.

3 o'clock

3:00

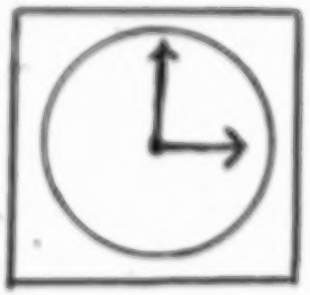
Play "Going to the Moon." Divide the class into two teams and have each team line up, one child behind the other. Put up two clocks with movable hands on the chalk tray. At blast-off time, hold up a card with "half past 7" written on it. The first child in each team will go to the chalkboard and set the hands of the clock to the indicated time. If they set the clock hands to the correct time, they are on the moon and may step aside into an area designated as the moon. If a child does not set the time correctly, he must go back to earth, that is , to the end of his line and await another turn. The game continues until all the children from one team have made it to the moon.

Writing clock times.

Children should learn to write both 3 o'clock and 3:00 as names for the same time, as well as half-past 1 and 1:30.

4:208

17:81-82, 88-89,91, 97, 100



3:00

3 o'clock

Play a matching game with three sets of time cards. One set will use the "o'clock" notation to indicate each of the hours. The second set will show each of 12 hours using a notation such as 12:00. The third set shows the 12 hours indicated by the hands on a clock.

Shuffle each pack of twelve cards and give them out. Have the child with a clock card come to the front and hold his card so all children can see. Have the child with matching times come forward and stand beside the first child.

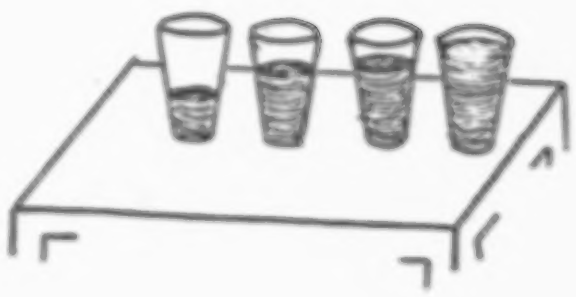
Volume

Seeing the need for standard units of measure.

Introduce the measurement of volume by having the children actually compare different containers filled with liquid. 13:49-54

Mix punch for the class in a large pitcher. Display many differently shaped drinking vessels. Tell the children you are going to fill all the containers with punch and that they may each choose one. Set up a system of taking turns to select their glass of punch. If the children, especially those who choose last, complain about the small amount of punch they get let them discuss their problem. (The problem here is how one compares the amount of liquid in various containers of different shapes, and it cannot be solved in this lesson.) When the discussion slows bring the lesson to a close by suggesting they drink the punch. The teacher's aim is to listen carefully to the level of sophistication in the childrens' comments concerning differences in volume and its relationship to the shape of the containers. From the clues the teacher picks up she should get some indication of how fast to proceed with lessons in volume.

Seeing quantity of liquid.

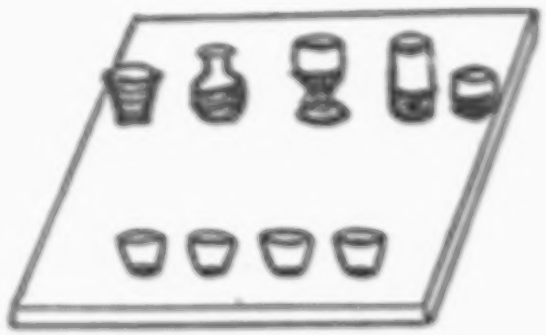


Divide the class into several teams. Place in front of each team four identical glasses filled with varying amounts of colored water. After the children note that the glasses are the same size and shape, unlike the punch glasses, ask them to place the 4 containers in a row, the one with the least liquid at one end and the one with the most at the other end. If some teams have trouble doing this, ask leading questions which cause them to re-examine their decisions. (Telling them they made a mistake will not develop the desired understanding of the relationship between height and quantity of liquid when the containers are the same.) Try adding a fifth glass of a different shape and have the children put it in order

according to quantity of liquid. Discuss its placement; then do a pouring experiment to see if if has more or less liquid than was estimated.

We hope the use of a differently shaped glass will lead to the idea that for identically shaped containers the height of liquid provides a means for comparing the quantity of liquid in the containers.

Using a standard of measure.



Now give each team of children a set of 5 containers of different shapes all filled to the same level with liquid. Also place 4 identical empty containers on the table. Ask: "Which of these 5 has the most liquid? How can we find out?"

The children hopefully, will suggest using one of the identical empty containers as a standard of measure.

Then let the teams experiment to find out the relative capacities of each of the 5 unlike containers.

Measurement using a standard 8 oz. cup.

Display a standard glass measuring cup. Ask 4:228-230 the children how we use it. Some of the answers may be-- "Mother uses it when she bakes." "Mother uses it to measure soap and bleach when she washes."

Ask the class why mother doesn't use <u>any</u> cup when she follows a recipe. (With some classes the best way to lead up to this lesson is to make apple sauce or cookies.)

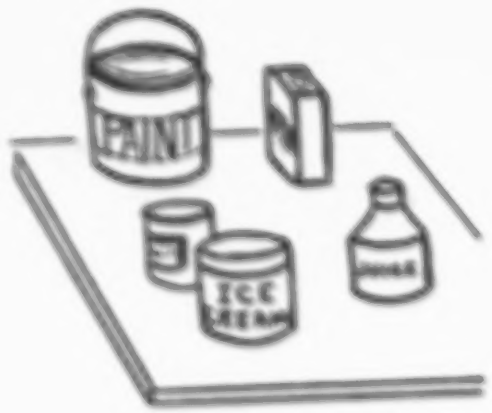
Fill the 8 oz. cup with colored water (food coloring) and pour this liquid into three or four containers of different size and shape. The children can soon see that the same amount of liquid can <u>look</u> different when in different containers.

One cup $\frac{a}{-}$

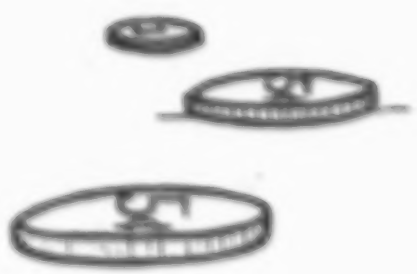
$\frac{1}{2}$ + $\frac{1}{2}$

cup cup

Pint, $\frac{1}{2}$ pint, and quart measures.



Money--measuring value



Display and experiment with a half-cup. Bring in a set of measuring cups and leave it on the sink where the children can experiment in their free time.

What can you buy in the store that is liquid? What size container does it come in? Have the children bring in containers of different liquid measure. (paint cans, milk bottles, cream bottles, punch bottles, juice cans, etc.) Perhaps this can be a homework assignment.

Also have containers on hand which represent the pint, half-pint and quart. Do several pouring experiments and clearly label all the containers. Keep them all near or on the sink so the children can experiment on their own. 6:103 11:59, 77, 111

Supply each team of children with play money--a set of coins which might include 100 pennies, 10 dimes, 20 nickels, etc. (Real money could be used to add more interest.) 12:101-104, 244-249

Combine your math lesson and social studies unit on the home and community. Organize a math team or social studies committee who will create a store (candy store, pet shop, ice cream parlor, toy store, etc.) 17:99 4:177-186

Put out play supplies and signs which display prices. The children can make these. Have class discussion concerning reasonable prices for articles which will be sold. 3:15-17, 55-57, 68,71,81, 95, 103,107

One group of children play the consumers. If the teacher makes up a very simple shopping list for each small group going to the store and gives each a set amount of money, they can plan their purchases together and add their own bills up. 11:53,85-87, 137,150,156

The teacher can check for accuracy after each "shopping trip." Did they get the correct change? Is their total purchase correct? 7:88-97

Discussion will arise as to the comparitive value of pennies, nickels, and dimes. Shall we use dollar bills?

Place 100 pennies on a table. Ask someone to separate the set of pennies into sets of 10 each. As he does this, put the piles of pennies in a row and place a dime beside each pile. "If you need a dime to buy something, how many pennies will it take to buy the same thing?"

Ten dimes can be traded for what piece of paper? A dollar bill.

What else could be traded for 1 dollar? (100 pennies)

A dollar in coins

DOLLARS	DIMES	CENTS

DOLLARS	DIMES	CENTS
1	3	5

Dollars	Dimes	Cents
1	3	5

How many dimes are worth the same as one dollar? (10)

This may be a point where some children will begin to see that the 100 pennies, 10 dimes, one dollar pattern is like the place value pattern of base 10. Put a chart on the chalk board like the one at the left.

Do not introduce notation $1. or $1.35 at this time. Call the amount one dollar, thirty-five cents and relate this amount to one hundred thirty-five.

Provide each child with a square of tagboard with the same place holder divisions. Let them place coins, bills, and numeral cards in place while doing various money experiments.

Let each child choose an amount of money (or give each one a card with an amount). Have them attempt to list 5 different sets of monies which make up that amount. Do this on the blackboard with the whole class in an example problem before presenting it to individuals.

36 CENTS

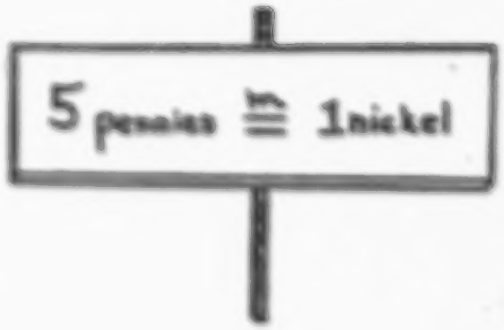
DIMES	NICKELS	PENNIES
3		6
3	1	1
		36
2	3	1
	6	6

Money measuring value

The symbol $\overline{=}$

1 dime $\overline{=}$ 2 nickels

1 nickel $\overline{=}$ 5 pennies



10 pennies $\overline{=}$ ____dime

1 dime $\overline{=}$ ___nickels

and ___pennies

3 nickels $\overline{=}$ pennies

Pictures of toys, candy, and food may be cut from magzines. These pictures should be pasted on cards with prices under each item. Have a child choose a card and tell what coins he would use to buy the item pictured. Ask if anyone can think of a different set of coins he could use to buy the item. Use real coins or play money to demonstrate. 4:180

This symbol, used with measurement of various kinds has a generalized meaning. It is read "is equal in measure to." When used with money it indicates that one amount has the same value as another amount.

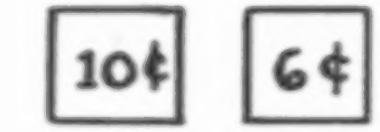
Read with the children--(5 pennies $\overline{=}$ 1 nickel) "Five pennies are equal in value to one nickel." Or it can be said that they are "worth the same." 4:180-181

The equality sign (=) can't be used, since five does not equal one. Now, write on the chalk board:

10 pennies $\overline{=}$ __?__ dime

Ask a child to write the correct numeral in the blank and read the completed number sentence. Do several other number sentences with the class.

Play the game "Who Will Trade?" The child who is "it" offers to trade a set of coins.



"I have two nickels and one penny. Who will trade with me?"

Someone may offer him one dime. If "it" refuses the offer, he can have another turn. If he accepts, he loses his turn because of his mistake in value judgment. The other trader then becomes "it," and the game continues.

Stage a treasure hunt. Hide cards, with various amounts of money written on them, in several 4:184-parts of the room. As each child finds one 185 card he takes it to his seat and puts out coins which have the same value.

The teacher is the official banker and checks to see whether he has the correct amount.

If you play this as a team game, score one point for each correct selection of coins. Another point might be added if the least possible number of coins is used.

Play "Who Can Guess."

"I am thinking of three coins which have the value of 11¢. Who can guess what coins I have in mind?"

Call on a child to give an answer. If he is correct, he gets a turn to ask a similar question. Vary the game.

"I am thinking of a toy that costs 15¢. What is the least number of coins I can use to buy this toy? The greatest number? (Make sure the children have their play money in front of them to manipulate while they are being questioned.)

Ask, "How many 1¢ candies can I buy with a dime?"

The calendar

The children will be introduced to the calendar to develop an understanding of a day, a week, and a month as units of time.

They will learn that seven days measure the same amount of time as one week. They will learn that each day of the week has a different name and that these days of the week can also be named with the ordinal numerals: Sunday is the <u>first</u> day of the week;Monday is the <u>second</u> etc.

The teacher will want to build the concept of the month in the children's minds by well planned classroom experiences using a workable daily calendar. She will want to help them understand that the month measures a longer period of time than the day or week. They will see that one month does not always measure the same amount of time as another month, and that months have no common day of beginning.

One week (seven days)

Ask the children if anyone knows the name of this day of the week. Record the name of the day on a large piece of paper such as chart paper. Continue to record the name of each successive day until the end of the school week. Discuss how many days they are in school. On the following Monday ask the children how many days they were not in school and what the names of those days are. Remind the children of the day they started the chart and explain that a week has passed. Help the children count the names of the days and conclude that one week measures the same amount of time as seven days.

Make a large set of cards showing the names of the days of the week. Say, "Will the person who has the name of the first day of the week please come to the front of the room?" Continue this procedure until all seven children have themselves 4:213

Calendar

S	M	T	W	T	F	S

in the correct order.

Help the children develop a feeling for the "month." Construct a large calendar and record the weather every day for a month. At the beginning of each day paste a picture or draw a picture showing the weather for the day - a yellow sun, a snowflake, etc. At the end of the month, discuss the month's weather. Ask the children how many sunny days there were, how many rainy days, etc.

Supply each child with a calendar form at the beginning of each month.

Each child can write the numerals on the calendar, mark the appropriate holidays, etc.

These calendars can be taken home for the child to use each day. The teacher can supply each of her children with a calendar page for the month in which the child's birthday falls. The child then draws the appropriate symbol on the day of his birthday, and writes his name at the top of the calendar page.

These pages can be placed on an arithmetic table or hung on a chart rack where the children can examine them. At appropriate times (such as the first day of each month) pupils who have birthdays during the month can show the class their calendar page. The other children can count the days to each birthday.

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O.C.S.E.I.P. SYLLABUS

Grade 2

SE006 069

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OFFICE OF EDUCATION

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PREFACE

The Orange County Science Education Improvement Program (O.C.S.E.I.P.) is sponsored by the National Science Foundation and hosted by U.C. Irvine. It is a cooperative venture undertaken by the University of California, Irvine, California State College at Fullerton, the Orange County Schools Office and local school districts throughout Orange County. This syllabus was written by O.C.S.E.I.P. to help teachers teach the best aspects of recent mathematics programs. It is not meant to be another textbook for a new program. Instead, it is meant to be a sharing and synthesis of effective teaching methods. The outline of topics is a minimum coverage which is common to all schools in Orange County. Topics adequately covered in the majority of texts in use are given a minimum treatment in the syllabus.

The first draft of this syllabus was written during an 8 week session at University of California, Irvine during the summer of 1966 by:

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NUMBERS AND NUMERATION

Grade 2

Grade 2

NUMBERS AND NUMERATION

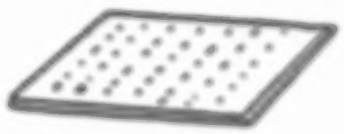
Counting from 0-100

For children who advanced rapidly in first grade only a brief review of fundamental ideas and basic skills will be needed.

Children who possess average and limited abilities will need to build up a feeling of confidence during the first weeks of work.

Children are able to count while they place the tee in the hole. This device may be used further to show tens and fives.

100's board using pegs or golf tees.



Daily stress different counting experiences. Count children in the room to see the number of children present and absent. Have objects to be counted. Count the number of books, straws, etc.

Look for different opportunities to count during the school day.

Teaching number sequence.

Build a chart like the following on the chalkboard or a large sheet of tag.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99
100									

Call attention to the tens on the left side and the ones across the top. Then fill in as the children count.

Give the children a ditto sheet of the chart with some of the squares empty. Ask them to fill in the missing numerals.

Repeat again another day but leave more empty squares.

Repeat again but without any numbers in the squares. Give help when needed.

Repeat from time to time, then ask children to write on regular lined paper and disregard the tens as set up on the chart. Write from margin to margin.

Numeral recognition out of sequence 1-99.

Number Party Game.

The number of player is from 6 to 12.

The materials needed are cards with numerals on them (one for each player) and chalk with a small chalk board.

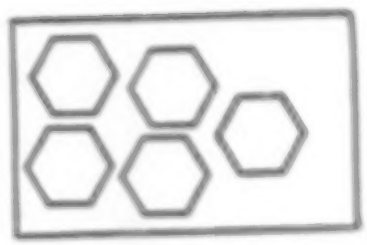
Several children sit in a corner. Other children have numbers as names pinned on them and wait in a group away from the corner. Each player who is standing in a group comes to the corner group one at a time, holding the chalk and chalk board. He says "May I come to your party?" The seated player says, "Yes, 5, if you can write our names." One by one they call out their numbers and if the outside player can write them all, he may take off his numeral and join the group. If he cannot, he goes back and waits for another turn for a second try. The numerals should be changed.

	Guess the Missing Number. This is a guessing game for teaching numbers in a series. The materials needed are a cardholder and large number cards. Place the cards in order as 11, 12, 13, 14, 15, 16. One child hides his eyes while another child removes a card. The child who is "It" then looks again at the cards and guesses which number is missing. If he answers correctly, he has his turn of removing the card and the game continues until all have a chance to participate. Note: Have variations of 2's, 5's, 1's, etc.
Counting in sequence using the numberline.	The teacher displays a numberline on the board. She uses a commercial numberline, creates her own with chalk, or prepares one using a felt-tip pen on adding tape. Each child can have an individual numberline on his desk. Using a character such as a frog, the teacher can "jump" the frog as the children count. Non-calibrated numberlines are available commercially.
Counting by "Ones".	Individual counters should be provided for each child. Poker chips, small blocks, colored disks, or milk bottle caps can be kept in individual containers. Handy containers might include cigar boxes, margarine containers, match boxes, envelopes, or plastic bags.

In the math interest center (a table with math aids for use as independent activities) there can be:

1. a large abacus.
2. beads strung on a piece of coat hanger.
3. a hundreds board.
4. a number line (0-100).
5. individual counters.
6. cross-number puzzles.
7. a ruler, pencil, and various geometric shapes to be measured.
8. a 3-dimentional tic-tac-toe game.
9. squares of some light-weight material (10 x 10½") such as pressed paper and a box of thumb tacks for counting. Children can make designs and keep track of the number of tacks used in each design.

Vocabulary development.



In order to teach the difference in the meaning of the words number and numeral, be sure you use them often properly!

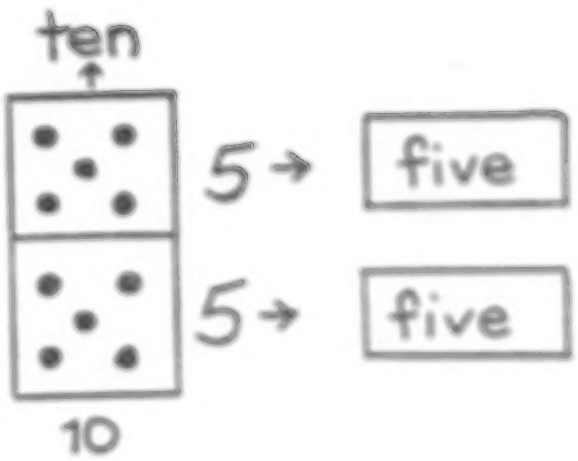
Show the children five objects on the flannel board. Ask them what number these objects make them think of (5). Tell them that we think of 5 as a number.

But when we write 5 (have a child do so on the board) it is called a numeral.

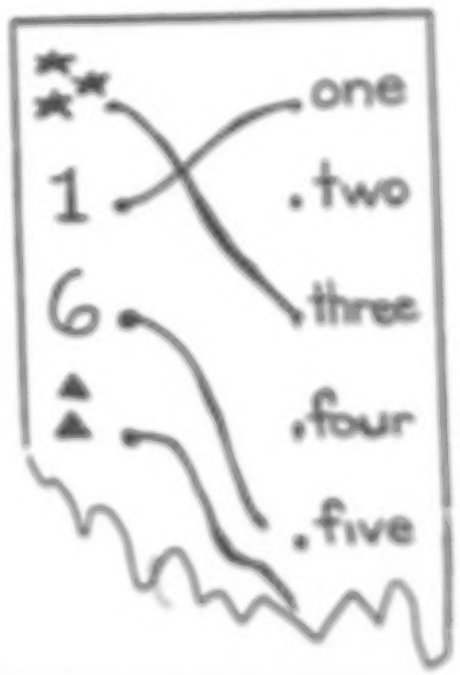
A numeral is a symbol that represents a number like the word "apple" stands for a certain fruit.

Recognizing the written numbers one, two, thre, four. . .nineteen, twenty.

This concept should be developed only after the children have mastered oral counting and can correctly write numerals representing given sets.



Cardinal numbers.



Show a flannel board with a given number of objects: 7 stars. Ask a child to write the numeral that shows how many stars they see (7). Then place the written number "seven" next to the set. Remind the children that all numerals have a word name.

Repeat this procedure with all the numbers from 0 to 10 and in a later lesson, from 10 to 20.

Related activity for teaching number vocabulary for groups or individual work: supply children with number sets, numerals and number vocabulary. Let them match the three variations.

Take a piece of cardboard 16" x 8". Punch ten holes, 3 inches from both sides. Tie shoe strings in the holes on the left side.

Write numerals' names on the right side in sequence with the numerals or sets on the left side out of sequence.

The children work in small groups and put each string in the hole next to the matching word.

Variations:

a. a laminated chart where children draw their own lines.
b. board work where children may erase their lines.
c. vocabulary can be changed to read 1st, 2nd, etc. In this case, the words must be out of sequence and the objects are labeled as to position on chart.

Ordinal Numbers thru 10th

Pocket Chart

You need ten elephants appropriately labeled.

To review an understanding of first, second, third, fourth, fifth, etc., read a story to the children. Then ask, "What happened first in the story?" Write first on the chalk board, then the sentence they offer. Continue with second, third, etc.

Have a pocket chart with ten pockets in it arranged in a horizontal position

Have ten elephants (or any other objects out of order) detached from the chart with the corresponding numeral on the back.

Give the elephants to the child and have him put them in the pocket chart in order of sequence.

This is a small group game, one child at a time.

Variations:

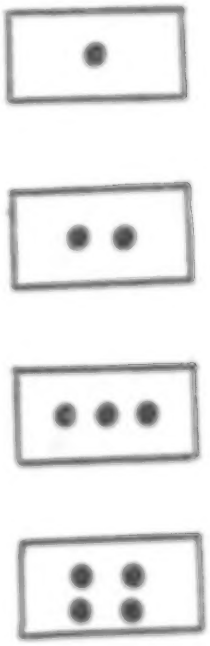
a. The elephants can be named by 10's or any sequential pattern desired.
b. Pass out the elephants and have the children come up to the chart when their elephant comes next.

Tell the children that you are going to give them some directions. Review the meaning of first, second, third, fourth, fifth. . .tenth.

Ask the third boy in Row 1 to touch the ground.
The fifth girl in Row 5 should jump up and down.

Or divide the group into two teams. Give each child on one team a card with a number in sequence (1,2,3,4,5). Give the children on the other team cards with first, second, third, fourth, fifth on them. Ask them to find their partners.

Odd and Even Numbers.



Through class discussion and participation, try to discover how you would divide 13 members of the class into two even teams (can't be done). Point out the difference between odd numbers and even numbers. Odd numbers cannot be divided into two parts evenly. This can be used in choosing teams for Physical Education.

Children will discover that:
Numbers ending in 1, 3, 5, 7, and 9 are odd.
Numbers ending in 0, 2, 4, 6, and 8 are even.

Utilise the patterns of dominoes as a convenient way to picture odd and even numbers.

The child can therefore discover that numbers with a dot left over are odd or uneven.

Place Value.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are symbols for numbers. When used alone they represent face value. When used in combination with any other numeral, they represent place value or face value depending upon the placement.

In the numeral 324, the four has face value of four one, (1111); the three has the place value of two tens; the three has the place value of three one hundreds.

A good basic understanding of place value and face value makes computation of any operation more accurate.

Place value order from 1 - 20.

This game should be used for small groups.

The teacher placed all the numerals in a pocket chart. She asks a child to find the numeral that comes before 6 or after 4. The child is to find the numeral and take it from the place holder. After checking to see if it is correct, he returns it to the pocket chart. A score is kept to see which child is most often correct.

Adaptions:

Two children can be speed tested for fast recall with a third person as score keeper.

Caution--definite rules must be layed down before game starts.

Counting--position of numeral ; (place value).

The number of players is from 20 to 30.

Two sets of large number cards (0 - 9) are needed.

Select two teams of ten players each. Give each team a set of number cards to be distributed among the members. The teacher calls a number between 1 and 99. If the number is called (37) the players who have the numbers (3 and 7) arrange themselves facing the teacher and the other players, holding their cards so that the correct number is shown. The team whose members make the correct response first, scores a point. No assistance may be given by other members of the team.

Small groups with a child as leader may be used.

Place Value.

See Addition and Subtraction section for further development.

Counting by 1's 2's 5's, 10's.

Bounce the Ball Game.
The numbers of players is from 2 to 12, or the entire class.
The materials needed is a ball.

Select a child to stand in front of the group.
The child states how he wants the children to count (i.e.: Leader says, "Count by two's.")

The leader bounces the ball any number of times (from 1 to 10 is suggested). (i.e.: five times--2,4,6,8,10).

The other children listen and try to discover the number that the last bounce made them think of.

The leader calls on someone. If they give the correct answer (i.e.: 10) they may be the leader and bounce the ball next.

Counting and Sequence.

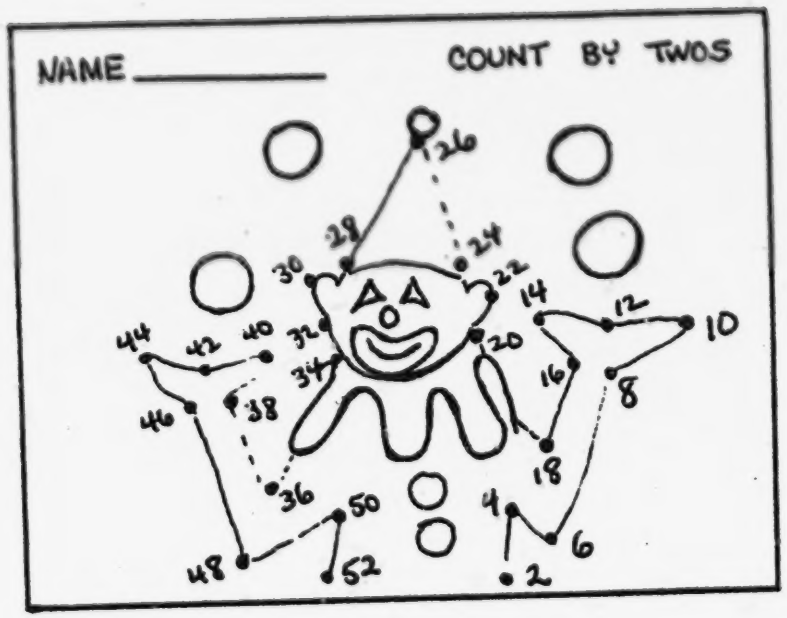
Direct the children to count by 2's. Example: the teacher says "24" and then the children count by 2's from 24 to 59, 80, 100 etc. This can also be used for counting by 5's and 10's.

Variations:

1. Go around the room with the children counting; as Johnny says 26; Mary, 28; Sue, 30; etc. until all children have contributed.

2. "Counting Bee"
   Divide the class into two teams. The teacher gives a number to a member on a team. He must count by 2's, 5's, or 10's for 3 numbers. Example: The teacher says 36. The team members count 38, 40, and 42. If a member misses, he must sit down. The teacher then goes to the other team and gives a member a different number.

   This continues until there are no people left standing. The last one down then becomes the leader.



Distribute a "follow-the dots" worksheet. These are easily made from coloring books. They could relate to a current social studies topic (i.e.: a community helper, a holiday, etc.)

Instead of each dot representing a number in ones sequence try having the dots count by 2's, 3's, 5's, or 10's!)

Extending the need for fractional numerals.

Pupils have by now discovered that the whole numbers of counting are inadequate to reflect all experiences in real life situations. List a few experiences that prove this, and ask your students to add more to the list.

Half of an object
Half of an orange or cookie
Half of a number
Half-way on the number line
Half of a dollar or dime
Half past two
Half price
First half of the baseball game

$\frac{1}{2}$

Each child got a third
One-third of the way around a circle
One-third off
One-third of a number of objects
One out of every three

$\frac{1}{3}$

A quarter of a dollar
A quarter of an inch
Quarter after four
A quarter of a gallon (one quart)
A fourth of a pie or an apple
A quarter of three

$\frac{1}{4}$

Three-fourths of the way around
Three out of four times at bat
Three quarters of an hour until lunch
Three quarters of a football game
Three quarters as large

$\frac{3}{4}$

Two-thirds of a pie
Two-thirds of the way
Two out of every three
Two-thirds off
Two-thirds of the number of objects
Two-thirds of the apples are ripe

$\frac{2}{3}$

Two half dollars
Two halves of a sandwich
Two out of two

$\frac{2}{2}$

Four quarters in the game
Four quarters in a dollar
Four out of four
Four fourths of a pie or an orange

$\frac{4}{4}$

Half for you and half for me
First and second half of a game
Half shaded and half unshaded
Half on each side of the ruler

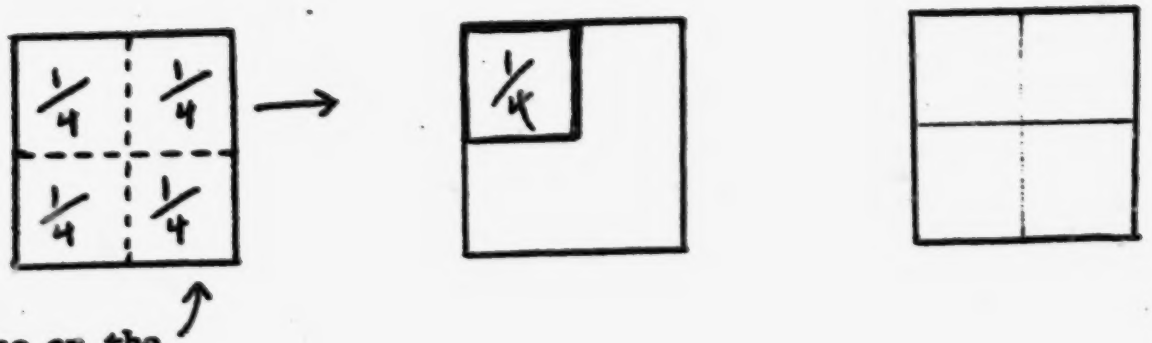
$$\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$$

One-third off
My sister carried one-third of my books
He has gone one-third of the way
He took one-third of the pie
Jump one-third of the way to 9

$$\frac{3}{3} - \frac{1}{3} = \frac{2}{3}$$

Fractions.

Give each child several geometric shapes (circles, rectangles, squares, triangles). Let them fold the papers to show $\frac{1}{2}$. Then label the parts and cut. They could use another shape to show fourths, thirds, etc. It is also helpful to have a whole object to fit the cut parts over.



Fit these on the whole square.

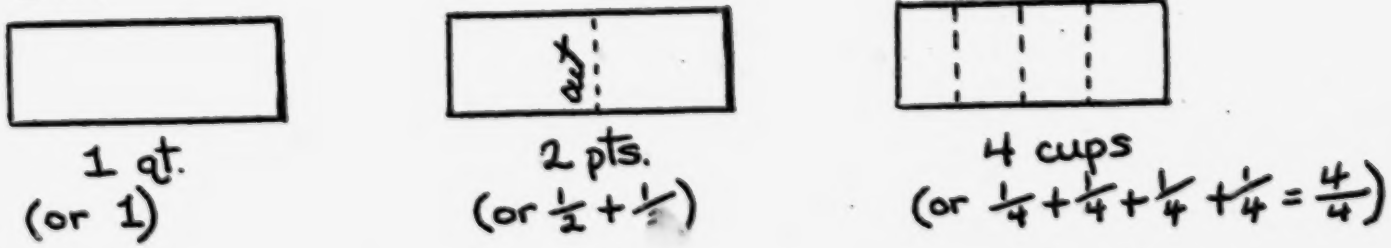
Show that: $1 = \frac{2}{2}$, $1 = \frac{4}{4}$, and $\frac{1}{2} = \frac{2}{4}$.

use also for:
1 quart = 2 pints
1 pint = 2 cups
1 quart = 4 cups

Give each child a set of the cut construction paper. Three pieces of 12 x 6. Let child fold and cut. Work out the concepts on the desks first. Then use the flannel board.

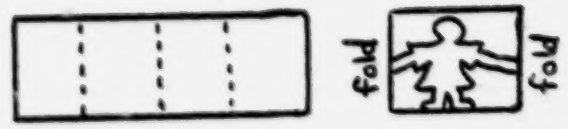
use 3 colors of construction paper:
12" x 6", 6" x 6", and 3" x 6".

The three different colors make it easier for the teacher to check at a glance and note which children need extra help.



This exercise is for a more advanced group as extra work.

Have the children fold a piece of paper into four parts.

Example: 

Have the children cut out paper dolls leaving their hands on folded sides uncut.

When the paper is folded out the child, can see four parts from one paper or one fourth of a set of four.

Variations: The child can fold the paper in 3rds and get three parts or 2 and get two parts, etc.

Fractions of a Set.

Divide a set of concrete objects into halves of a set, fourths of a set, thirds, etc.

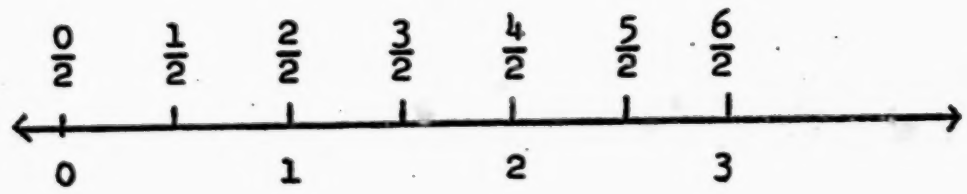
1. A set of 12 books can be divided into two sets of 6, three sets of 4 and four sets of 3.
2. Six pencils can be divided into two sets of 3 and three sets of 2 (halves and thirds).
3. Nine erasers are divided into thirds (3 sets of 3).
4. Sixteen pieces of chalk divided into two sets of 8 (halves), four sets of 4 (fourths), or eight sets of 2 (eighths).

Fractions--$\frac{1}{2}$ and $\frac{1}{4}$.

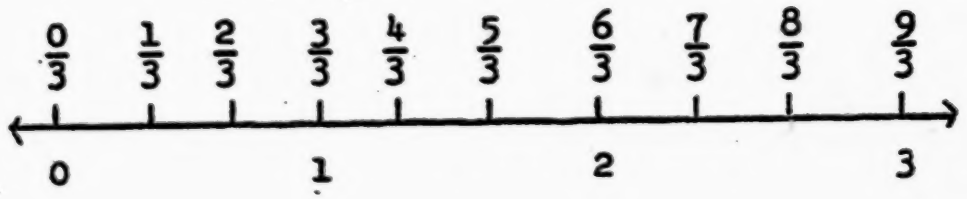
Ask volunteers to bring an apple. Work in groups of four. Provide ordinary kitchen knives as well as extra apples.

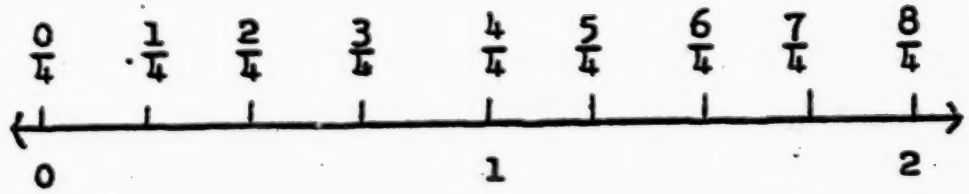
1. Discuss caution and proper use of knives.
2. Show an apple on the flannel board or chalk board.
3. Allow the children to cut the apple in $\frac{1}{2}$ and then again to make fourths. Put apple together again to make a whole.
4. Give each child in the group a fourth to eat.

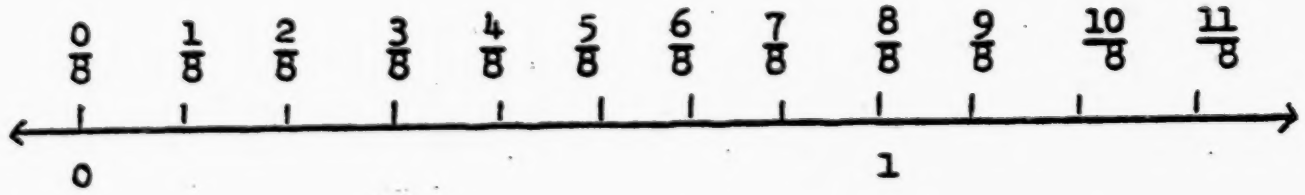
Extending the fractional number through use of numerals on the numberline.

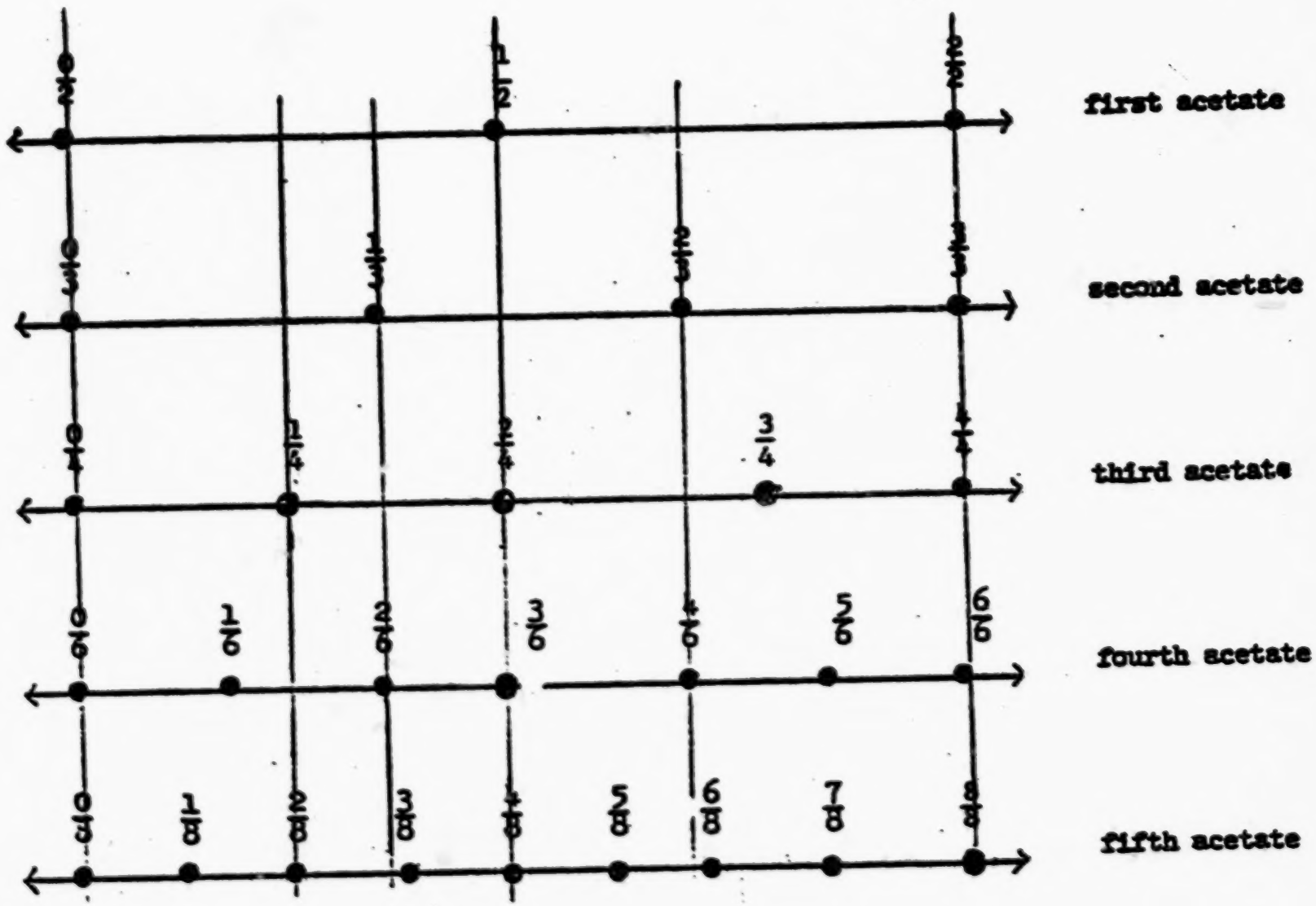


This would be particularly effective and easily taught by using a ruler.









Acetates of the above unit numberlines may be used as overlays for children to see that:

a. $\frac{2}{4}, \frac{4}{8}, \frac{3}{6}$, are other names for $\frac{1}{2}$.

b. $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{6}{6}, \frac{8}{8}$, are other names for one.

Roman Numerals

Tell the class a story about the origin of Arabic and Roman numerals. See if they can reason out why we use Arabic rather than Roman numerals.

Go over each Roman numeral's meaning, formation, and position from I to X.

Relate concept to the use of these numerals on clocks. Bring in a real clock with Roman Numerals.

Have the class make clocks from paper plates. Number these clocks with Roman Numerals. Practice telling time using these clocks.

If interest is high, continue process of counting and writing Roman numerals to 50 or 100.

To reinforce this, you could have the children do a "follow-the-dots" with Roman numerals replacing our Arabic numerals.

Have children add and subtract using Roman numerals.

III	XII	IV
+ IV	- IX	+VII

ADDITION AND SUBTRACTION - GAMES AND DRILLS

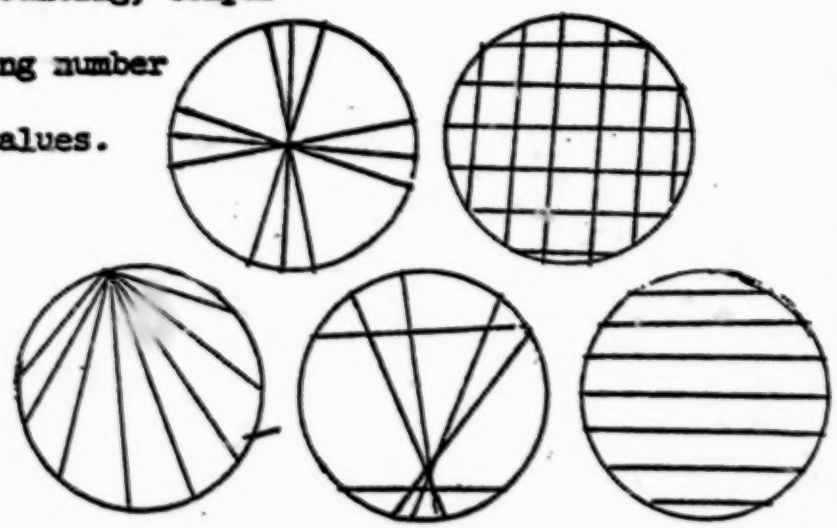
Grade 2

Addition and subtraction concepts and techniques redeveloped and extended: counting

Ask pupils to find an ear of corn that has 15 rows of kernels. Provide many ears of both sweet corn and field corn.

You may want to tell the pupils of the man who offered his boy a dollar for any fifteen-row ear he could find. See if pupils can discover this: all ears have an even number of rows; sweet corn, 8 to 14 rows; field corn, 14 to 20.

Counting, comparing number values.



Circles on the bulletin board are divided into "pieces"as shown. Each day, a question may be asked that calls for counting and making choices. "Which of the circles shown is divided into the largest number of parts?" "Which has the fewest number of parts?" "Are there any circles with the same number of parts?' (Yarn makes a good medium for circles and lines.) (Paper disks are also good.)

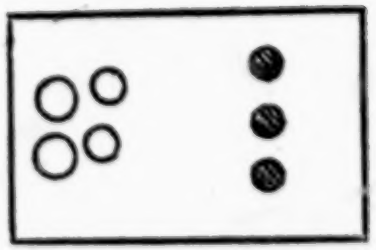
Counting place value:
Addition and
Subtraction

Supply a peg board with 100 holes in it. (10 across and 10 down) If possible supply one for each child in the group.

To teach counting, put a peg into the holes for each number and count at the same time.

This device may be used to teach many other concepts in a concrete way. Use with small groups.

Finding the missing numeral in a Subtraction equation.



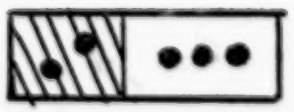
7 - 4 = 3

The teacher may use the flannel board to show a number sentence with some objects. (7)

Examples:

1. Show a set of 7. Take away 4. Ask the children how many are left. (3)
2. Show the set of 7 again. This time take 3 away. How many are left now? (4)
3. To teach the concept □ - 4 = 3, the flannel board should have 3 objects on it.
4. Ask them how many objects they see.
5. Ask the children to guess the number of stars that you are thinking of. (7) How many stars were there if we took 4 away and 3 were left?
6. Eventually they may conclude that if you add the two numerals together you will get the large number 7.

   4 + 3 = 7
   7 - 4 = 3



$\begin{array}{r} 5 \\ -\square \\ \hline 3 \end{array}$ or 5 - □ = 3

Use domino cards with one section blank or covered. Write the equation on the board.

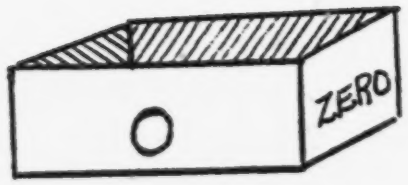
5 - □ = 3

Show 3 dots. Ask the children how many more dots they would need to make the 5. They will discover 2 more dots are needed. Pick a child to come up and write the missing numeral in the placeholder.

□ - 2 = 5

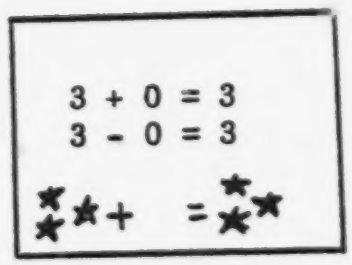
Put an addition equation on the board. 5 + 2 = 7 Ask the children which way they read this equation. (from left to right) Now put 7 - 2 = 5 on the board. Ask them to look at both equations and see what we have done. Let them discover that they are

turned around and have opposite operational signs. Therefore, in an equation such as □ - 2 = 5, we can do the opposite to find the numeral for the place holder. Have them try doing this two or three times to see if this is always true.



Show an empty box to illustrate an empty set.

Play "What's My Rule?" Teacher asks a pupil for a number between 1 and 10; explaining that she will "do something" to each number to get a new number. Pupils try to find what the rule for changing numbers is. They must not tell the rule, but may tell what the "new number" is for any given number. Children conclude that zero had been added to the number given and that numeral always remains the same when zero is added.



Show on a flannel board objects with zero added or subtracted. Show the equation and the objects for that equation.

Addition of numbers and joining of sets.

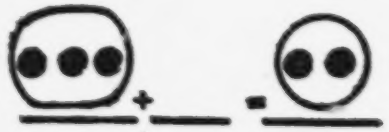
3 + 2 = 5

2 + 3 = 5

Review the joining of sets by using any type of manipulative device such as beads, flannel board and cut out objects, pocket chart with numerals on cards, etc.

Teach desired vocabulary in your text by means of charts, flash cards, matching games, discussion and everyday usage. 4:59

Reviewing addition facts--
union of sets.



Addition and Subtraction
by counting.

Ask students to write equations describing the sets pictured. (Use several examples of the type shown.) 7:162

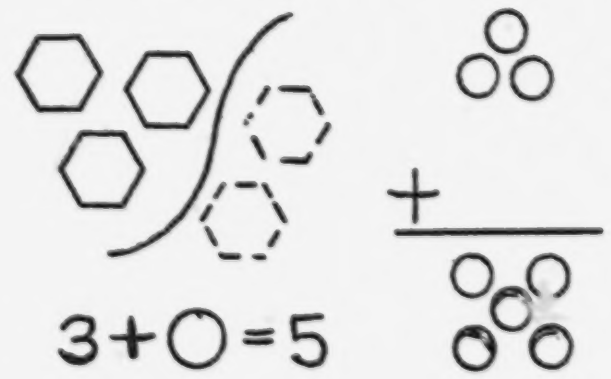
A dittoed page of "sets" of letters may be used employing words as the elements. Pupils count letters to find the number of each set, and count or add to find the solution. 17:31

Example:

	How many letters?
123 456	
N Ned ran =	6
N to school =	8
N Ned ran to school =	14

(The notation "N Ned ran = □" is read "The number of elements in the set Ned ran is What?"
____ Count the Letters.)

Missing Addends



Show the equation and the illustrations on the flannel board (3 + □= 5). Ask what number must be added to make five. Stress the fact that the equal sign shows that both sides are equal in the equation. Work on the flannel board completing equations with numerals and objects using signs and symbols. Review the principle of commutation (i.e., 2 + □= 5 or □ + 3 = 5). We may add in both directions. It is unnecessary to use the term "commutative" when discussing this property 4:8-9

Addition or Subtraction combinations. Less than 10

"Show-me" Pocket Charts.

Fold up the bottoms of brown envelopes and staple. Each child has a "Show-me" pocket chart and cards bearing the numerals from 0 to 9. These numeral cards are kept inside the pocket chart envelope when not in use.

The teacher states an equation or shows a flash card to the class. The children answer by placing the numeral or numerals in their pocket charts which they hold up so everyone may see.

Mailman Game.

Construct ten tagboard houses with a pocket stapled up at the bottom. Put corresponding numerals on them. Hang them somewhere in the room--from chalk ledge.

One child is the mailman. He draws a card from a designated pile. The card bears an equation. He solves the equation and "delivers" this "letter" to the right house by putting it in the pocket. If he delivers it to the right house, he gets to pick another mailman. If he does it incorrectly, the teacher picks a mailman to help him.

Flower-O.

The materials are as follows:

1. Flowers
2. Numerals (sums or differences) in a box
3. Paper covers (petal shaped)

The children or teacher may construct these flowers. Play the game as you would play Bingo. The teacher draws a paper with a numeral from a box (6). If a child has an equation with that sum or difference (3 + 3 = , 10 - 4 =, etc.) he covers that petal with a blank paper petal.

When all the petals are covered, he calls out "Flower-0" and wins the game. The teacher should check to see if the child is correct, by keeping a record of all the numerals that are drawn.

Drill

The number of players are from 7 to 21.
The material needed: Flash cards with addition combinations consisting of ten or less and other cards numbered from one to ten, two of each number.

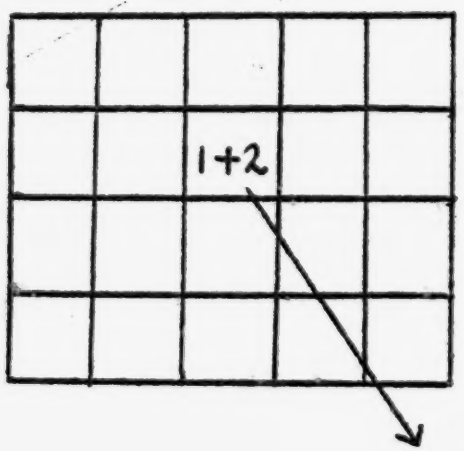
All of the players except one who is "It" sit on chairs or stand in a circle. The players are given number cards to correspond with the combination sums so that two of each sum goes to the children. The child who is "It" is given the flash cards whose sums are held by the players. "It" takes his place in the center of the circle and reads the combinations out loud (i.e., 2 + 3). The two players who have the numeral "5" change places and "It" tries to get the place of one of them. The one who becomes "It" is the displaced one. Change players' numbers frequently.

Going to Grandma's House.

The number of players is from 3 to 12.
The materials needed are: A flannel board or chalk board with boy, girl, combinations and Grandma's house.

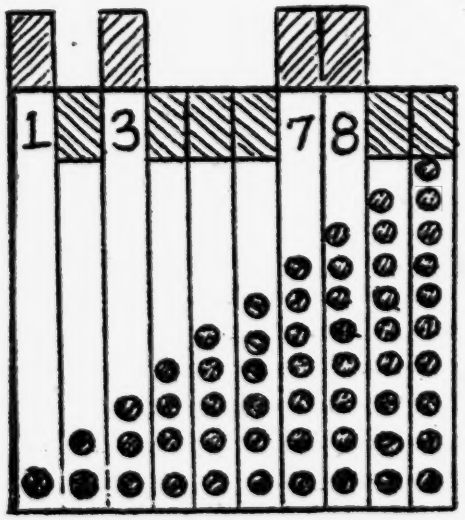
Choose a player to go to Grandma's house. He crosses the stream by giving the correct answers to the combinations of four stepping stones.

The teacher can replace stepping stones (equations) after each player participates.



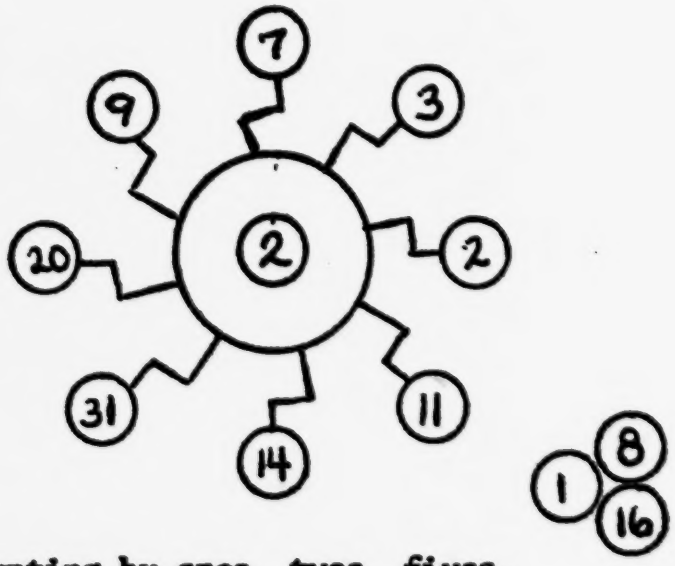
Bean Bag Game.
The number of players is from 2 to 4.
The materials needed are: One bean bag and chalk for marking the floor (masking tape will also work).

Mark the floor with 20 squares in 4 rows of five each. In each square put one of the addition combinations known by the group. Each player has 3 throws with the bean bag. He must give the answer to the combination in the square where the bean bag falls. If he is able to, he scores a point. If he does not or if the bean bag falls outside the square he gets no score. The children may or may not choose sides.



Game.
Number of players is 2 to 7.
The materials needed are: Tagboard backing, perception dots, and dice.

Each player completes his turn with the dice before the next player begins. With each roll of the dice, the player finds the sum, then covers the numerals on the game board that match the number rolles. (If a seven is rolled, the player could cover either 7, 5 and 2, 4 and 3, or 6 and 1.) The player rolls till he can no longer cover the number which comes up on the dice. He then records the numbers left showing (if any) as his score. Then the next player takes his turn. The player with the lowest score wins the game!

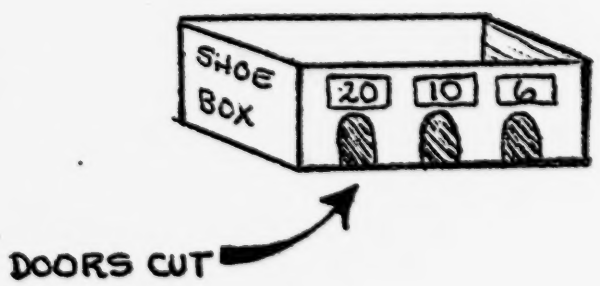


Counting by ones, twos, fives, and tens.

Game.

The materials needed are: Tagboard and disks with numerals for center of ferris wheel.

The children add the number in the center to each seat number on the ferris wheel. The teacher calls on one child at a time. A scorekeeper keeps track of the number of times each child answers correctly. The child with the greatest number of correct responses wins, chooses the next number for the center, and keeps score for the next game.



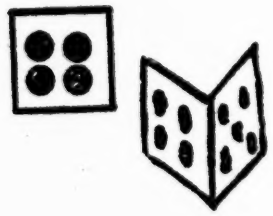
The materials needed are: a shoe box without cover with three doors cut, three marbles for each pupil playing, two or three empty boxes for the players, and a supply box full of material.

A player rolls all three marbles at one time toward the doors. He adds up his score and collects the appropriate amount of marbles and places them in his supply box (if there are no extra marbles the adding of points is sufficient or they could collect pieces of paper; one for each point.) Each player gets three turns. They then count up their marbles (or score on paper) by a method previously decided upon. The player with the highest number wins.

This game is adaptable according to the purpose set up by the teacher. Any numbers may be placed above the windows and any technique of counting can be used. Standards must be set before the game begins.

This game is more successful if the players can collect things and if it is definately a small group activity.

Subtraction is the inverse of addition.



4+5=9

9-5=4

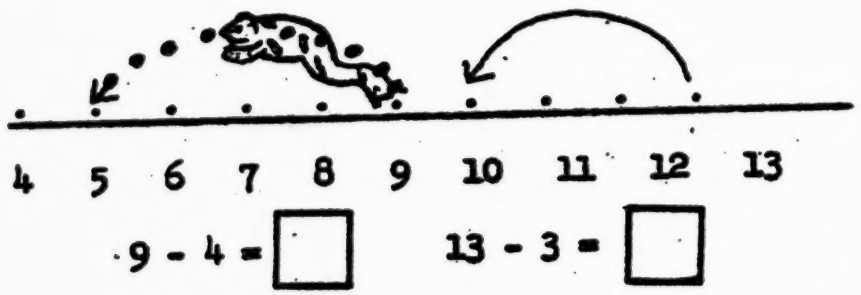
Have blocks with numbers and operational signs on them. Children see four possible combinations using these numerals and manipulating the blocks. Another child should write the equations illustrated by the blocks.

Use perception cards to show how subtraction undoes the addition. This is achieved through manipulation of the card.

Draw four dots on one side of a domino card and five dots on the other side. Fold the card in half so that only four of the dots are in view. Show the class the side of the card with four dots. Have someone tell how many dots he sees. Unfold the card so that the children can see the five from left to right and the joining of the sets described. Ask someone to give the addition equation corresponding to this activity. Now tell the children to watch as you fold the side of the card with five dots back out of sight. Ask how many dots they saw when both sides of the card were in view. Ask how many dots remain. Have the action described in terms of sets and have a child give the corresponding subtraction equation. Continue in this way with other combinations.

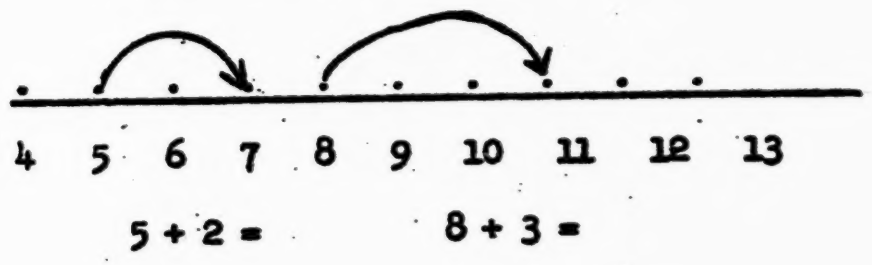
4:15

Numberline and "cross-number puzzles" to illustrate addition and subtraction.

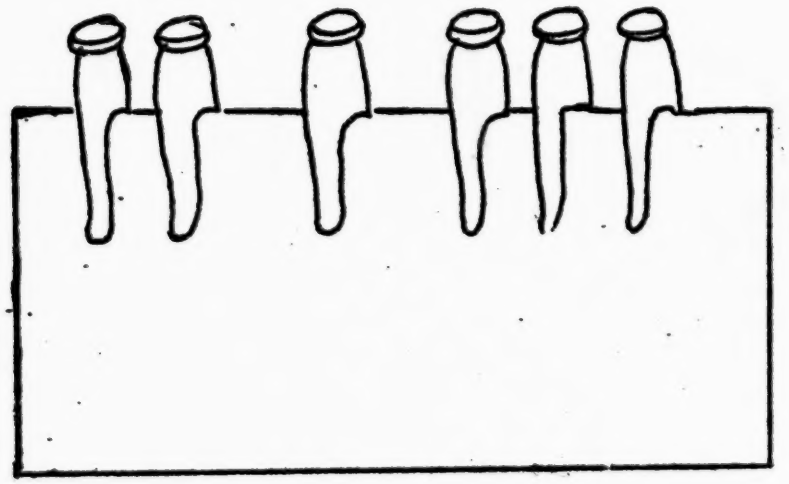


If pupils have not developed a background of familiarity with the numberline, it may be necessary to go back to using a floor-model numberline (masking tape), and let pupils step off "jumps".

18:7



Parentheses to show order of operations in addition.



2 + (1 + 3) =

(2 + 1) + 3 =

(1 + 3) + 2 =

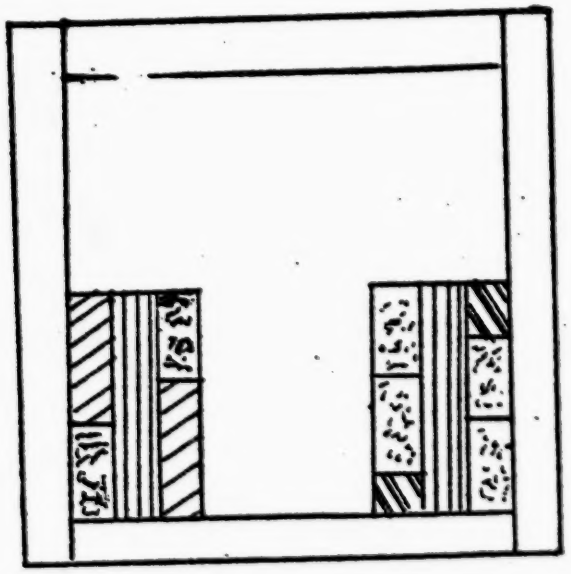
1 + (3 + 2) =

Associativity and commutativity may be spoken of as "the in-any-order" rule, or as "scrambling." To illustrate the idea of changing orders, a clothespin card may be used. Thus (2 + 1) + 3 = 6 and 2 + (1 + 3) = 6 can be shown by actually moving one of the pins to show the first addition. 4:31

Pupils may use these clothespin cards to assist their work on dittoed work pages. Individual flannel boards are also extremely useful.

Pupils may make "Ten-Frames," using 1" square-ruled graph paper and a construction paper or chip-board frame. As an alternate, the teacher may make more durable ones, using two 12" squares of chip-board per pupil. One square is a backing and the other is cut into four frame-strips and 19 number-strips as shown. 4:21

Commutativity and associativity shown through use of "Ten-Frame."



Make cuts with the paper cutter. (It may help to trim the insides of the frame strips to allow 1/8 clearance inside.)

The Ten-Frame may be used to show and work simple combinations, or to show the "in-any-order" rule (If students are grouped by 3's or 4's, pooling their "Ten-Frames," several equations may be shown at once.)

Again, the "contents" of the Ten-Frame is as follows:

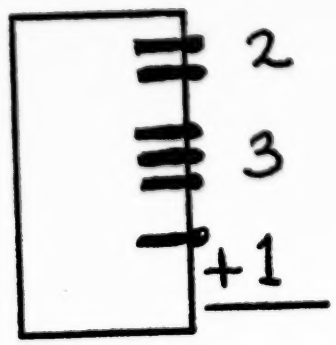
1 - frame & backing
1 - strip of 10
2 - strips of each of the numbers 1-9.

If the teacher finds it to advantage, an envelope for storing the Ten-Frame kit may be made from tagboard as a construction activity.

$2 + 3 = \square$ $\quad$ $(1 + 2) + 2 = \square$

$3 + 2 = \square$ $\quad$ $1 + (2 + 2) = \square$

Column Addition thru Vertical Notation (Associative Property)



Have available a cardboard strip and clothespins. Group the clothespins on the card to correspond with the equation $(2 + 3 + 1 = \square)$. Select a child from the class to join one group of pins to another.

$2 + 3 + 1 =$
$(2 + 3) + 1 = 5 + 1$

Then the child joins the last group of pins (1) to the group of 5. This will enable the child to see that only 2 numbers may be added at a time.

Put the clothespins back as they were originally, turn the cardboard upside down and repeat the process.

$1 + 3 + 2 =$
$(1 + 3) + 2 = 4 + 2$

Continue this procedure until the children discover that the order of the addition does not affect the sum.

$$\begin{array}{r} 5 \\ 2 \\ 1 \\ 3 \\ +4 \\ \hline \end{array} = 7$$

Let a child come up and begin by adding the first two numbers and then add that sum to each of the other numbers in succession. Show the children that they can find the same sum by adding up from the bottom.

$4+3+2=\square$

Select some children to come up and represent a given equation. (i.e.: 4 + 3 + 2) The children should be grouped into a set of 4, a set of 3, and a set of 2.

Have the set of 4 join the set of 3 and see how many are in the new set (7). Then the group of 7 joins the set of 2 to find the total sum of 9. Show this operation on the board using numerals and parentheses.

$(4 + 3) + 2 = \boxed{7} + 2 = 9$

Now have the children return to their original sets and ... The set of 2 joins the set of 3 to get a new set (5). Then the set of 5 joins the set of 4 to get 9.

Again show this operation on the board.

$4 + (3 + 2) = 4 + \boxed{5} = 9$

Discuss the fact that the sum is not affected by the order of addition.

Column Addition using Vertical Notation.

	Think:	
2		
3		
9	9	6
4	+4	8 =10
+ 1	13	4
	+3	+ 2 =10
	16	20
	+2	
	+1	
	19	

Show the children that they can find the largest number and add that to the other numbers. Add that number to the next largest and so on. Another idea to make column addition easy and fun is to have the children find groups that add up to 10 and then add the remaining numbers to that 10.

Place Value

The Countingmen may be used to show the children how they count beyond ten.

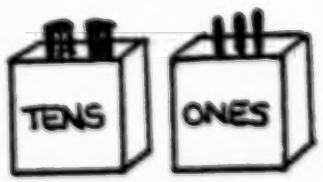
The children can learn the following poem:

Everytime I count to ten,
I stop, and start the count again.
(This poem enables them to see that they can count on and on by keeping track of each ten.)

23=20+3

The teacher provides glass jars labeled Tens and Ones. She puts straws in the jars. In the Tens jar, the straws are tied in bundles of ten with yarn or rubber bands.

To show 23, the children would put 2 bundles of 10 straws in the Tens jar and 3 straws in the Ones jar.

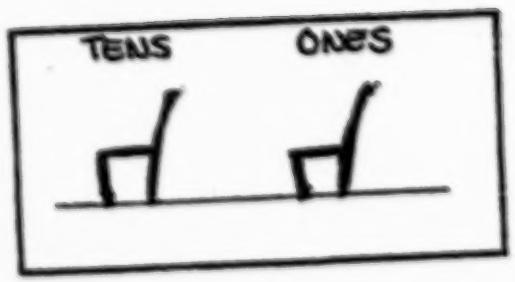


23

Use sticks in milk cartons that have been covered or painted and labeled Tens and Ones. The sticks can be bundled in 10's and unbundled as needed.

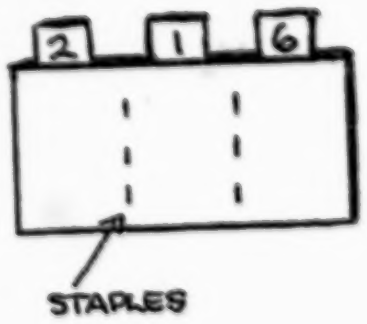
The teacher can easily make 10's or 100's Wheels. The children read the numeral shown and tell how many 10's and ones the numeral has.

23 means 2 tens and 3 ones.



On a piece of tagboard, glue two pictures of two chairs. Label each chair with "Tens place" and "Ones place." Pretend that an invisible person sits in each place or chair. The children should be careful not to let the invisible man sit in the wrong place.

For example, in the numeral 23 the 3 is in the ones place and cannot sit in the tens place or the number would mean 32. A hundreds place and chair may be added when the concept of hundreds is studied.

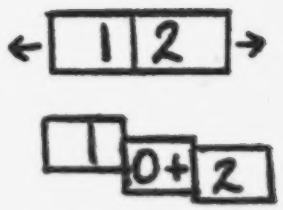


This aid is to be used in small groups to help in understanding place value. Each child should have his own packet.

The teacher writes a number on the board and the children restate the number in their packet tag holders. This works for any three-place numeral. The children hold their pocket charts up and the teacher can help individuals. After the game becomes familiar to the children, the teacher will want to call out the numbers to see if they understand the place value concept.

Expanded Notation

$12 = 10 + 2$

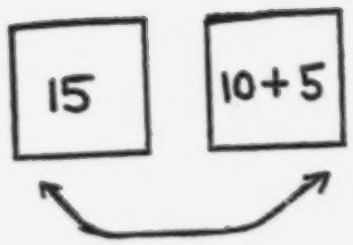


Fold the paper so that only the numeral 12 shows. As you unfold the paper the children see the numeral expanded to say 10 + 2.

<u>"Find Your Partner" Game.</u>

Distribute cards with numerals 10 thru 19 to various children. Also distribute cards with the expanded form of these numerals.

Pick a child with a numeral. That child goes to the front of the room and shows the class his card. His partner with the expanded form must come up immediately and stand beside him. Failure to come up causes the child to lose his card.



To help children understand the term "expand" use a child with a name that is longer than what he is called. For example: A child called Joe can expand his name to Joseph; Tom is expanded to Thomas; Judy is expanded to Judith, etc. The name and person are the same, but the name is made longer.

Also the teacher can expand the name Joe to Joe Smith.

<u>Number Families</u>

15 = 10 + 5
15 = 10 + 3 + 2
15 = 10 + 4 + 1
15 = 10 + 2 + 2 + 1

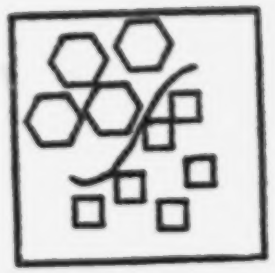
Distribute cards with various number combinations. One child stands and holds the key number up, 15. Those holding correct combinations which belong to the "15" family come up and stand by the player holding Number 15. The other children check to see if the family is correct and if all are present

Addition Sums of Ten and More.

"Operation Big Ten"

6 + 8 =

6 + (4+4) =
(6+4) + 4 =
10 + 4 = 14



Review concept of associative property--numbers in parentheses are added first. Remind the children that adding 10 plus any number is very simple.

10 + 1 = 11
10 + 2 = 12
10 + 3 = 13

Tell the children that there are many ways to solve the equation 6 + 8 = __. "Today, we can learn one way to solve this equation."

1. How many do we add to 6 to make 10?
2. Where are we going to get this 4?
3. If we take that 4 away from 8, what will be left of the 8?
4. Show the new equation 10 + 4 and how easy it is to solve now!

IMPORTANT: At the same time, these steps are being developed, work the equation out on the flannel board using 6 objects and 8 objects.

Sums and differences less than 20 through regrouping.

Problem: 8 + 7 = ___
8 + □ + 5 = ___
10 + 5 = 15

"String" coat hanger with 20 beads.

Straighten wire to desired length depending on size of beads. This game helps the children drill on regrouping first the tens and then the ones. Count off eight beads. Note that there are seven more beads needed to be separately grouped to complete this problem. If we are regrouping to make ten, we must borrow two of the group of seven beads. After we have borrowed two beads from the group of seven, there are five left. Therefore, we write 10 + 5 and arrive at the sum of 15.

Reinforcement of Addition and Subtraction. Sums through 18.

Many games can be played that encourage practice in the number facts while keeping the pupil interested. Among these are number puzzles, "What's my rule?", circle puzzles, and many point-scoring games.

"What's My Rule?"

Your Number	1	3	9	4
New Number	7	9		

(The rule for this is "add 6.")

Your Number	9	4	3	5		2		
New Number	11	6			3		2	

(The rule for this is "add 2")

Teacher asks a pupils for a number between 1 and 10; explaining that she will "do something" to each number to get a new number. Pupils try to find what the rule for changing numbers is. They must not tell the rule, but may tell what the "new number" for any given number is. Chart the numbers, and encourage pupils to look for patterns. After a short explanation, remain silent. Let pupils do as much discovering "on their own" as they can. Try not to rebuke those who blurt out "rules," rather than giving a "new number" response. (Many will start by saying, "you add 2!" instead of saying that the new number for 4 is 6.)

Addition and subtraction practice.

With little or no instruction, pupils can begin to find answers to a puzzle made of 3 concentric circles. Indicate by a sign whether to add or subtract. Let a child put in any answer he knows. One by one, fill in the blanks.

"Add"

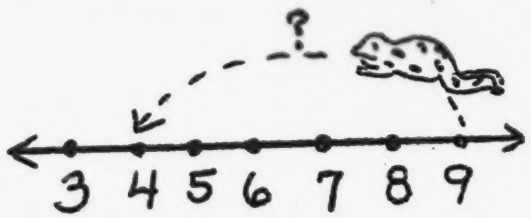
+

Begin with center numeral, add one in the second ring, and then write the sum in the outer ring. (6--2--8) Put another numeral in the second ring. Let pupil answer.

Children may use the number line for solving an equation.

$9 - \square = 4$

Subtraction Missing Placeholder.



They start at 9, move the frog left to 4, and count how many spaces the frog jumped. That number is the numeral that goes in the placeholder.

Repeat this procedure with several similar equations until the children discover that the missing number and the difference (4) added together make the largest number (9).

This could also be done by using other insects such as Mr. Grasshopper, Mr. Beetle, etc.

Problem: $\begin{array}{r} 14 \\ -\ 9 \\ \hline 5 \end{array}$

For the advanced student use bottle caps, plastic counters, etc. to show counting process back to ten from a greater number.

Use tagboard with 20 squares--color in as many as the numeral shows.

$14 - 9 =$
$(14 - \square) - 5 =$
$10 - 5 = 5$

$14 - 5 =$
$(14 - 4) - 1 =$
$10 - 1 = 9$

Combinations Drill.

$\begin{array}{rrrrrrrrr} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \\ \hline 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array}$

$\begin{array}{rrrrrrrrr} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \\ +2 & +2 & +2 & +2 & +2 & +2 & +2 & +2 & \\ \hline 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & \end{array}$

$\begin{array}{rrrrrrrrr} 1 & 2 & 3 & 4 & 5 & 6 & 7 & & \\ +3 & +3 & +3 & +3 & +3 & +3 & +3 & & \\ \hline 4 & 5 & 6 & 7 & 8 & 9 & 10 & & \end{array}$

$\begin{array}{rrrrrrrrr} 1 & 2 & 3 & 4 & 5 & 6 & & & \\ +4 & +4 & +4 & +4 & +4 & +4 & & & \\ \hline 5 & 6 & 7 & 8 & 9 & 10 & & & \end{array}$

$\begin{array}{rrrrrrrrr} 1 & 2 & 3 & 4 & 5 & & & & \\ +5 & +5 & +5 & +5 & +5 & & & & \\ \hline 6 & 7 & 8 & 9 & 10 & & & & \end{array}$

$\begin{array}{rrrrrrrrr} 1 & 2 & 3 & 4 & & & & & \\ +6 & +6 & +6 & +6 & & & & & \\ \hline 7 & 8 & 9 & 10 & & & & & \end{array}$

$\begin{array}{rrrrrrrrr} 1 & 2 & 3 & & & & & & \\ +7 & +7 & +7 & & & & & & \\ \hline 8 & 9 & 10 & & & & & & \end{array}$

$\begin{array}{rrrrrrrrr} 1 & 2 & & & & & & & \\ +8 & +8 & & & & & & & \\ \hline 9 & 10 & & & & & & & \end{array}$

$\begin{array}{r} 1 \\ +9 \\ \hline 10 \end{array}$

The above chart can be extended through facts to 18. It is especially effective for use with independent activities such as card games using combinations or equations. It's use keeps interest from lagging when answers aren't known and provides drill without teacher assistance.

Reverse this chart for subtraction facts.

1	2	2	3	3	4
+1	+1	+2	+2	+3	+3
2		4		6	
4	5	5	6	6	7
+4	+4	+5	+5	+6	+6
8		10		12	
7	8	8	9	9	
+7	+7	+8	+8	+9	
14		16		18	

Children seem to learn the doubles in addition more quickly than the odd combinations. To make them easier, do the doubles, then show the children that the odd facts come between the even facts.

The Ladder Game--
Build equations for specific numerals as for 10.

Make flash cards. Have the children place them under a specific numeral on the flannel board, or have different children come up and write the facts on the chalkboard as a ladder.

Ladder of 10

5 + 5
6 + 4
4 + 6
3 + 7
7 + 3
2 + 8
8 + 2
9 + 1
1 + 9

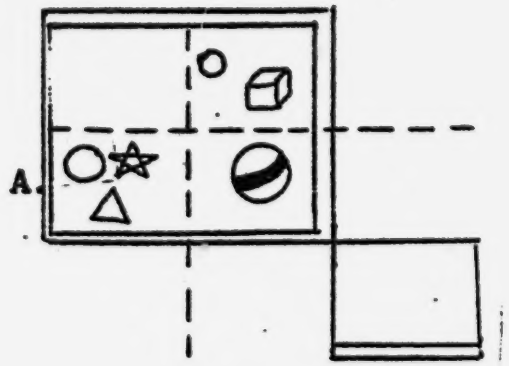
Addition Combinations.
Using the Cross-Number Puzzle

Cross-number puzzles give repeated practice, with the benefit of "rewarding" the pupil by checking his answer while working the puzzle.

A picture puzzle (a, in the examples) may be shown on the chalkboard. The number "record" for this (b) is worked out orally, or by student volunteers with chalk, to correspond with the picture. Solid objects may be more effective than pictures.

(complete the record)

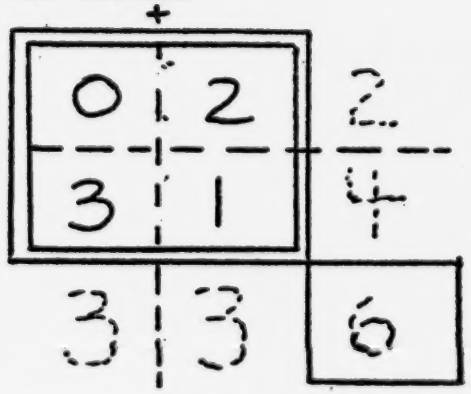
A.



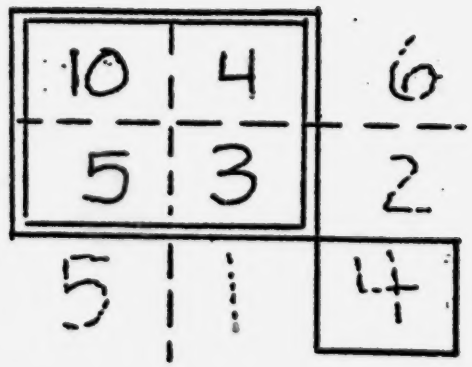
Subtract the smaller number from the larger one

Add the numbers listed in each row and column

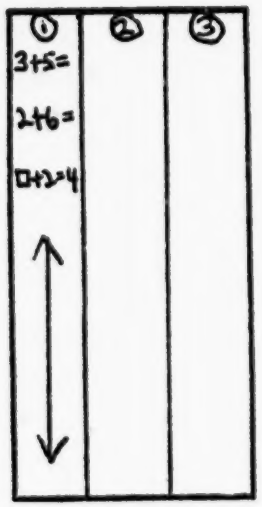
B.



C.



Combination Drill.



The number of players is from 2 to 12.

On a blackboard or a chart, write the known equations in a column each with one missing part. Arrange them in irregular order. The harder combinations can represent the most dangerous places. The pupils climb the mountain or ladder by giving the missing part in each combination beginning at the bottom of the column.

Children take turns until they miss. The first child to the top wins.

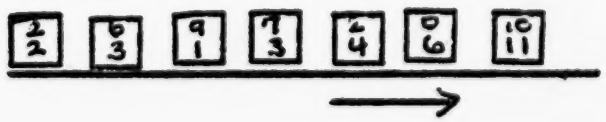
Variation--draw mountains on board. Put equations or combinations in vertical notation from in the same manner or above. The first one to the peak wins.

Tens and Ones Drill.

The number of players is from 2 to 10.

The children take turns standing before group and make a statement regarding place value of a numeral. Example: "I am thinking of a number that has one 10 and three 1's." Those who think they know what the answer is, raise their hands. The leader calls on them and that child come forward and writes his numeral (13). If what he wrote is correct, he becomes the leader. If he misses, the leader gets another turn.

Combination Drill.



There should be two players for each game.

Flash cards of all known combinations to be used on chalk tray. Answers to be written with chalk.

Place flashcards on chalk tray. One child starts at each end, awaiting a starting signal. Upon hearing

signal, each player proceeds toward the other, writing the answers on the board above the combinations. They work toward each other until they meet. The one with the greatest number of correct answers wins the game.

Combination Drill.

Streetcar.
The number of players is from 4 to 12.

The teacher arranges 12 chairs to represent a streetcar. One child is chosen to be the streetcar conductor and holds the flash cards. Each player must answer correctly the equation on the card shown to him before he can board the streetcar.

The next conductor is the player who has "won" by solving the most equations.

Addition and Subtraction Combinations.

The Fireman Game.
A flaming house is placed against the blackboard. A player or the teacher places equations on the board.

The children take turns solving the equations. If a child solves the equation within a certain time limit, he becomes a fireman and wears a paper firehat.

Each fireman helps put the fire out by folding a flame down behind the house.

This game is played till all the flames are out. It is a good small group activity.

Addition and Subtraction Practice.

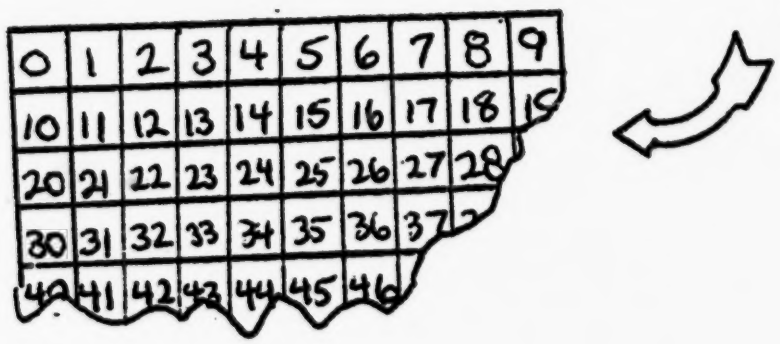
Target games using numbered corks or beanbags for adding up a score may be played in small groups, even on the playground.

A variation might be used that involves a beginning score of, say 20. Pupils subtract any points scored from this. The lowest scorer wins.

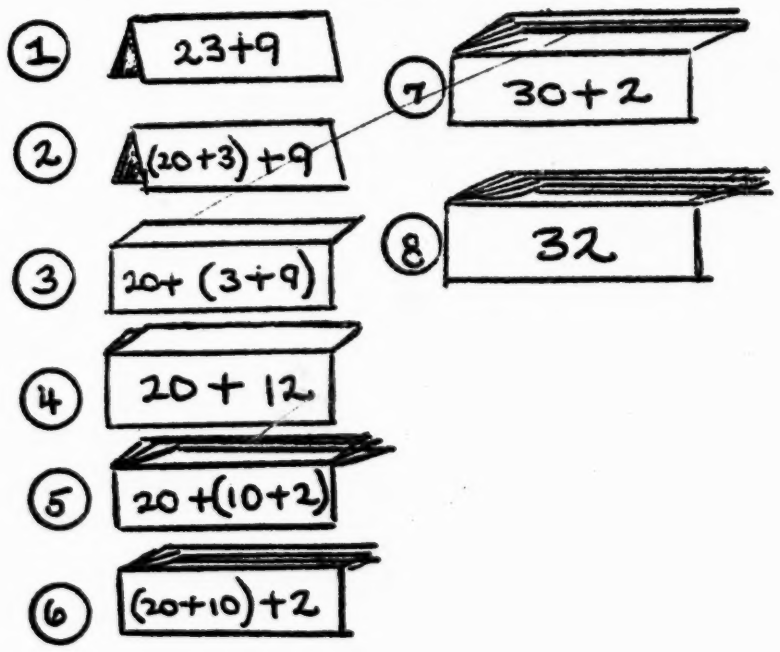
Addition and subtraction of 3-place numerals, involving regrouping.

A numberline may be divided into "chunks" for a device to show subtraction or addition. 18:65

Use 15 strips numbered 0-9, 10-19, 20-29, etc., to 149. Every jump down is equivalent to adding 10; a jump to the right adds one; left one space subtracts one, and a jump upward subtracts 10.



Regrouping (carrying).



Introduce the concept of regrouping, using the aid pictured in the example column. To make the aid, fold four sheets of paper in half so that you have 8 sides for the examples with only one showing at a time. Each time you fold over a side, you discuss the process involved.

There are 8 steps to this problem, but it is unnecessary for the child to write out all the steps if he has a basic understanding of the concepts. It is important that the original equation by kept in sight using flannel board or chalk board.

Addition and Subtraction of 2-place numbers involving regrouping.

$$28 + 31 =$$

$$\begin{array}{r} 28 \\ +\ 31 \\ \hline \end{array}$$

$$\begin{array}{l} 20+8 \\ 30+1 \\ \hline 50+9 = 59 \end{array}$$

Review the concept of adding the tens and the ones separately with emphasis on the ones first.

Solve the equation using both horizontal and vertical equations. Now change the equation to involve regrouping of ones to tens place. Allow the children to manipulate devices such as bundles of sticks or pencils to show visually how 10 ones are regrouped to be 1 ten.

Example: Have straws bundled in 10's and 1's. Ask a child to show the numeral 28 by placing two bundles of 10's in the tens jar and eight 1's in the ones jar. Ask the second child to do the same with the numeral 34.

Ask the children how they can regroup 12 straws into 10's and 1's. Note that another name for 12 is 10 + 2. Have the children bundle ten straws with a rubber band

and place the bundle in the tens jar. Ask the children to find how many 10's and 1's we have.

Repeat this procedure several times with various equations. If possible, allow each child to manipulate the same device at their seats. Show the equation and how to solve it as the concrete experience is developed.

Borrowing (regrouping).

The following steps should help develop an understanding of two-place subtraction with borrowing.

1. Solve the addition equations 26 + 34 = 60 by the addition steps used previously.

2. Review the concept of inverse relationships by using a simple equation such as:

$$3 + 2 = 5$$
$$5 - 2 = 3$$

(3) $62 - 34 =$

(4)
$$\begin{array}{r} 60 + 2 \\ -\ 30 + 4 \\ \hline \end{array}$$

(5)
$$\begin{array}{r} 50 + 12 \\ -\ 30 + \ 4 \\ \hline \end{array}$$

(6) $20 + 8 = 28$

3. Have the children expand the equation 62 - 34; one numeral at a time.

4. Discuss the necessity for regrouping to enable the children to be able to subtract because 4 from 2 is impossible.

5. Discuss the regrouping of only the top numeral.

6. Solve the new equation and find the sum of 20 + 8 (20 + 8 = □)

Two and three-place addition and subtraction with or without regrouping.

Three-place addition and subtraction is developed in the same manner as two-place addition and subtraction. The technique is used until it is apparent that children understand the place value of each of the numerals and the concepts of carrying, or regrouping into the next column.

It is not necessary to complete all these steps with the children in depth if they grasp the operation.

$$\begin{array}{r} 348 \\ +246 \\ \hline \end{array}$$

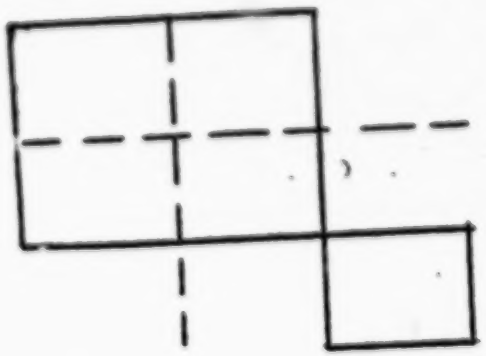
300 + 40 + 8
200 + 40 + 6
500 + 80 + 14

500 + (80 + 10) + 4
500 + (90 + 4)
500 + 94 = 594

Subtract the smaller number from the larger number in each row and column.

143	127	16
95	16	79
48	111	63

("You will need to to write these examples out on paper, and find the answers to fill in the blanks.)



Cross-number puzzles may be used that require computation on paper. They offer a means of checking that provides "build-in" reinforcement to the student. Beginning puzzles will be much simpler, and later ones may involve the leaving out of some figures in the large rectangle.

Let students make up their own puzzles by putting numerals into dittoed puzzle grids.

Make a blank puzzle on a sheet of Styro-form. Use pencils as markers.

Comparison of Numbers and Equations.

$10 = 10$ $\quad 5+3 = 0+8$

$7-3 < 8+2$

$30+8 > 40+10$

1. Show the operational signs $<$ and $=$ on the flannel board.
2. Name each symbol.

   (greater than)
   (less than)
   (equal to)

   Discuss the meaning of each.
3. Display 3 objects and 5 objects and compare their values.

Three is less than five.

4. Have a child place the correct sign in the circle placeholder. (Mention that the small side of the sign "<" points to the smaller number and the larger opening side paces the larger number.)
5. Reverse 5 and 3 and repeat the procedure. ($5 > 3$)
6. Add 2 more to 3 and repeat $5 = 5$.
7. Continue procedure (see examples).

Relations and Mathematical Sentences.

1. is not equal
   $3 + 2 \neq 4$
2. open sentence
   $5 + 2 =$
3. closed sentence
   $3 + 7 = 10$
4. true sentence
   $4 + 5 = 9$
5. false sentence
   $3 + 8 = 10$

Write a mathematical sentence on the board. (i.e., $4 + 5 = 9$) Discuss this sentence discovering that it is a true sentence; that is, 4 things and 5 things are 9 things together. Then, using the examples listed on this page, discuss the different mathematical sentences and their meanings. Show that the mathematical sentences 1 and 5 are the same type of sentences shown in two different ways. The same holds true for sentences 3 and 4, although a closed sentence could be false.

MULTIPLICATION AND DIVISION

GRADE 2

GRADE 2

MULTIPLICATION AND DIVISION

GAMES AND DRILL

Multiplication defined through set union

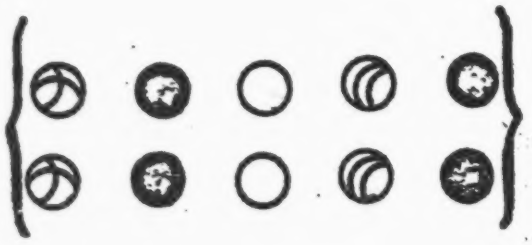
Multiplication and division may be introduced when the children are able to recognize equivalent sets and know addition facts through 10.

Understanding the operation and the ability to manipulate sets of objects is more meaningful than rote memorization of the basic facts (at first).

The teacher may introduce multiplication by showing five "sets"of one (one flannel board object in each of five rings of yarn.)

Loop a piece of yarn around all 5 objects and ask the number of this new set.

Display five sets of two objects. Ask for the number of objects in each set. (2) Then circle the five sets with yarn and ask how many objects are in the big new set. (10) Lead pupils to count by twos. Ask what objects come naturally in sets of two. (hands, feet, shoes, gloves, earrings, etc.)



Class Participation

Call a student up to the front of the room. How many eyes has he? (a set of 2)

Call up 4 more children, one by one, and ask the same question.

Now we have 5 sets of two. How many eyes are there in all? (10)

After class understands combining 5 sets of 2 into 10 use the multiplication sign (x).

$$5 \times 2 = 2 + 2 + 2 + 2 + 2$$

On the blackboard or flannel board place objects to illustrate repeated addition.

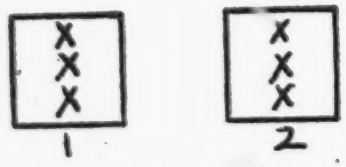
There are 5 sets of 2 balls. How many in all?

$$5 \times 2 = \square$$

Multiplication defined through union of sets

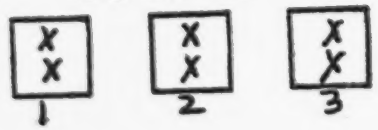
2 x 3 = 6

Two sets of three.

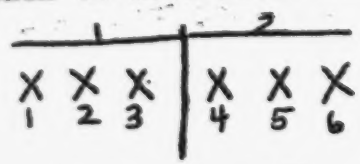


Two, three times

3 x 2 = 6

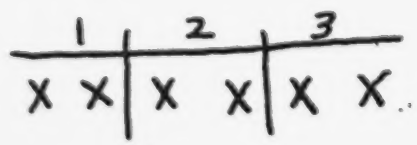


Partition to show division by 2



6 ÷ 2 = 3

Partition to show division by 3



6 ÷ 3 = 2

Multiplication and division of combinations is best introduced to second graders with manipulative devices. The understanding of the operation and the ability to write the combinations becomes more meaningful when approached in this manner. Memorization will come later and will be easier if the child has a proper background in manipulative devices.

After manipulation the child should write the fact horizontally and vertically.

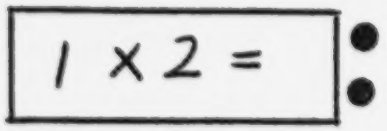
Suggested objects for manipulation: discs, chips, bottletops, jacks, beads, small blocks, sticks, buttons, paper clips or seeds.

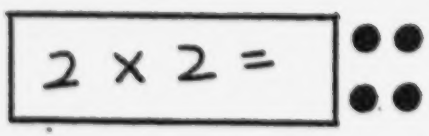
Write the facts after manipulation

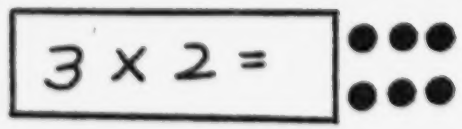
$2 \times 3 = 6$
$3 \times 2 = 6$

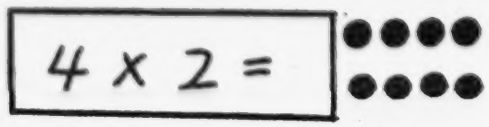
$\begin{array}{r} 2 \\ \times 3 \\ \hline 6 \end{array}$ $\begin{array}{r} 3 \\ \times 2 \\ \hline 6 \end{array}$ $6 \div 2 = 3$
$6 \div 3 = 2$

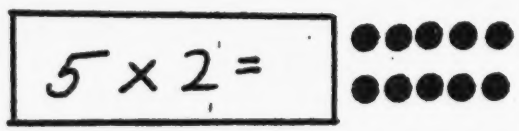
Using the mathematical sentence
(Multiplication algorism)











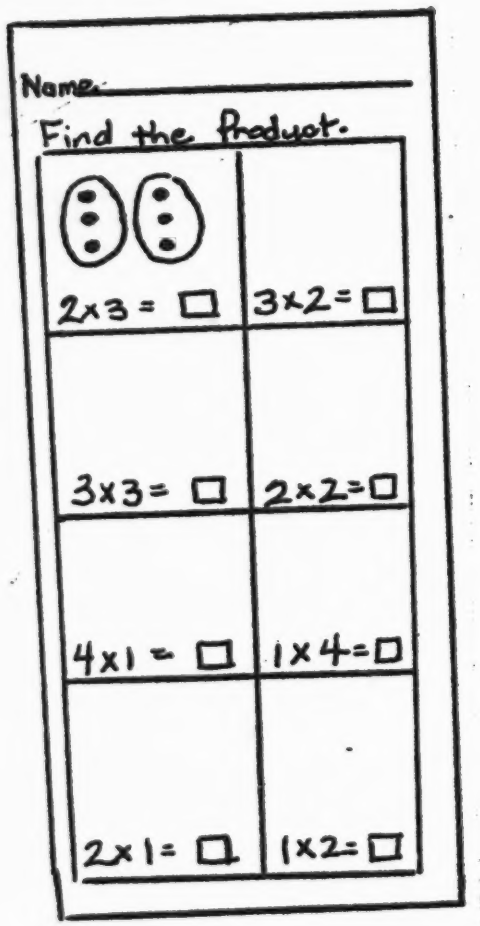
Small group:

1. Provide each pupil with a set of counters and cards (about 2" by 3") labeled with 5 facts - 1 x 2, 2 x 2, 3 x 2, 4 x 2, and 5 x 2.

2. Show an open equation such as 3 x 2 ☐ . The children solve this equation by placing groups of counters on their desk tops.

3. Ask them to put the card that shows the equation next to the counters.

Multiplication defined as set union
(Repeated addition)



Multiplication; a repeated addition

2 x 4 = ____

4 x 2 = ____

Drill

1. Use after the set union concept has been taught.
2. Give children dittoed worksheet.
3. Instruct them to find the product by illustrating the number sentence.

2 x 3 = □ means to draw 2 sets of 3.

1. State the problem on the chalk board or flannel board.
2. Redefine the problem stating that 2 x 4 is two sets of 4 or 2 4's. Also that 4 x 2 is 4 sets of 2 or 4 2's.

Show on board

$$\begin{array}{r} 4 \\ +\,4 \\ \hline 8 \end{array}$$ 1st set, 2nd set

$$\begin{array}{r} 2 \\ +\,2 \\ \hline 4 \\ +\,2 \\ \hline 6 \\ +\,2 \\ \hline 8 \end{array}$$ 1st set, 2nd set, 3rd set, 4th set

Game - Buzz Counting

Children are to count out loud to a numeral agreed upon, each multiple of that number to be left out. The student is to say the word "Buzz" instead. An integer is chosen as "it." Example: Today 3 is "it," and asks a student to begin counting. One, two, buzz, (three) four, five, buzz, seven, eight, buzz, ten, eleven, buzz. Or -

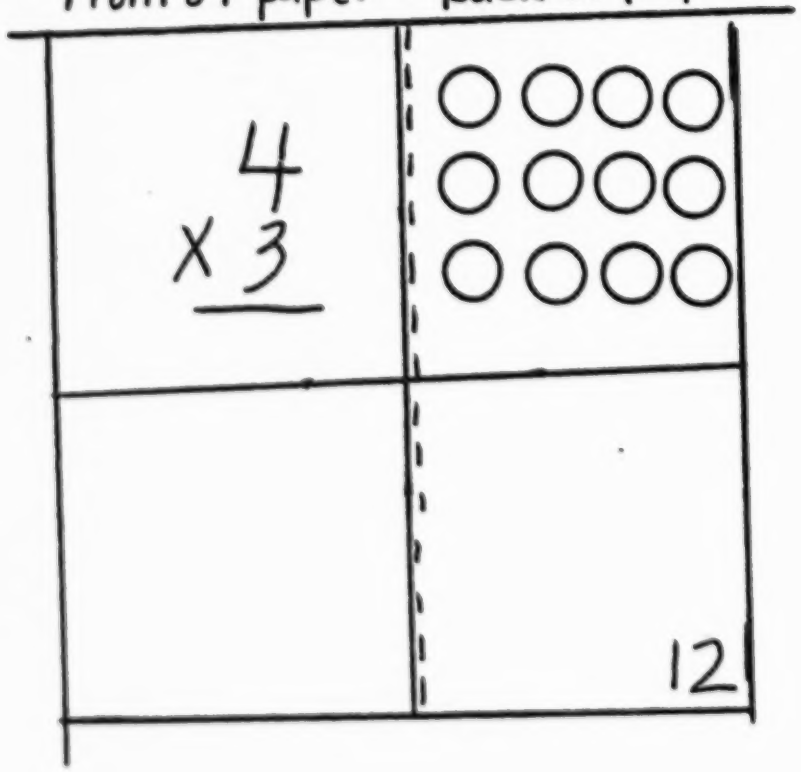
One, two, three, four, five, six, seven, eight, nine, buzz, eleven, etc.

6:183

Multiplication and division combinations, products through 25.

Pupils may make study cards for discovering and practicing their "combinations." Each card should be about 2" x 3". One side shows the symbols. The back of the card shows an array of dots, arranged in lined groups. Pupils may work with these individually, or in partners.

The partner would show the symbolic representation 4 (x 3), and check the pupils' answer by looking at the array. Both profit by the game.



Multiplication drill

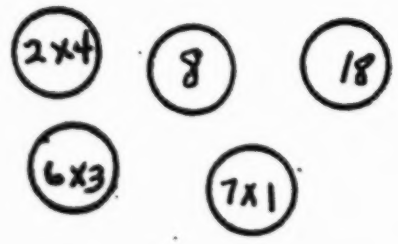
Problems could be illustrated with dots or objects, on paper or at a desk with columns and lines.

Example: 2 x 3 - Illustrate

	x	3
2		ooo
		ooo

Treasure Hunt Game

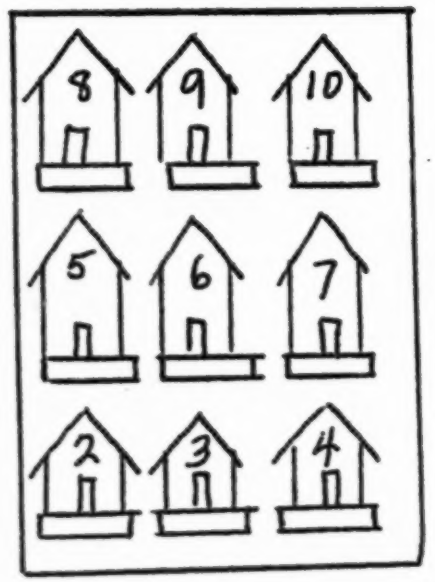
Children are given small "treasure boxes" at the beginning of the game. "Gold coins" are placed in a larger treasure box. Children are to take coins from the large box and place them in their treasure box.

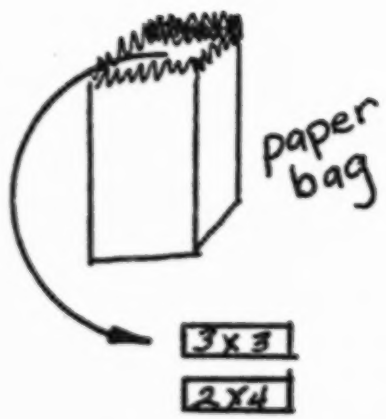


Coins with all equations.

On each coin multiplication or division facts are written. The child must give the correct answer, which is written on the back of the coin. When a correct answer is given the child keeps the coin. Child accumulating the most coins, wins.

This same idea may be used for addition and subtraction equations.





Mail to be delivered to the houses.

Postman

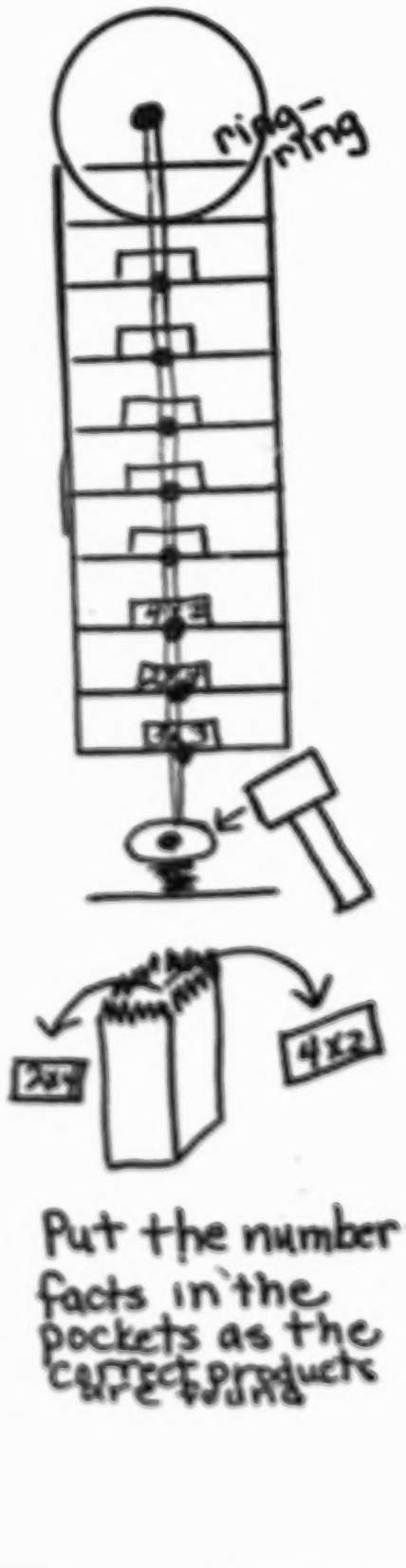
1. Have a small group of children work around a chart with a number of houses on it. Under each house there should be a pocket and on each house a numeral. The number of houses should correspond with the products needed.

2. Each child takes turns being a postman who delivers the letters to the different houses while a score keeper keeps track of a dead letter box.

3. The postman determines the answer to each equation on the letter and puts it in the house that is labeled with that numeral.

4. The postman gets 3 chances to deliver letters. If he makes a mistake he does not forfeit his turn. His score would reflect his errors.

5. The child who has delivered the fewest dead letters wins the "postman of the year" award.



6. This may be used when teaching any computational math concept.

Strength Test Game

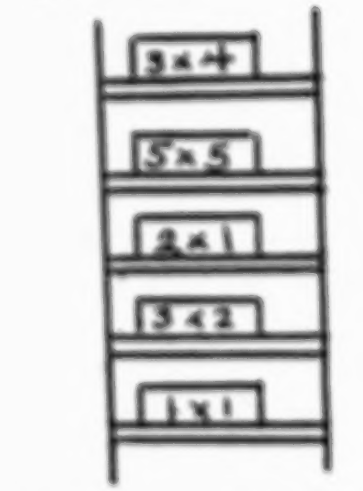
1. Choose a small group of children to play a multiplication drill game.

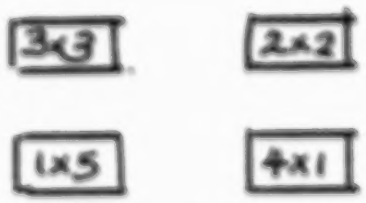
2. One student holds the hammer and draws out multiplication facts from a paper sack located below the bell.

3. He continues until he misses one;when he does he gives his turn to the next player.

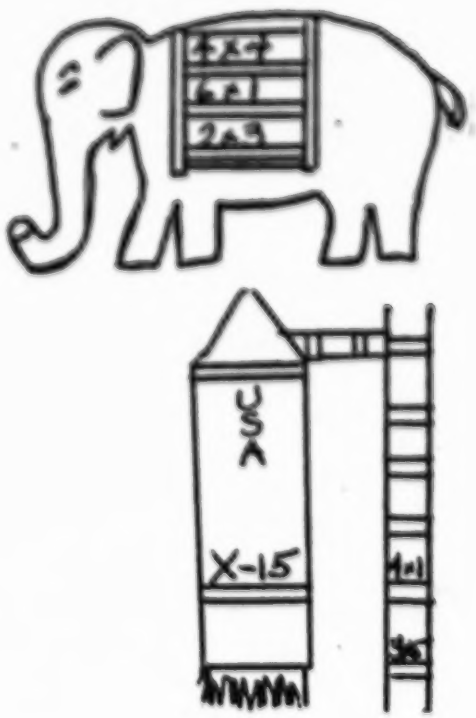
4. The winner is the student who is able to ring the bell. He is the strong man of the class.

Multiplication





Extra Cards



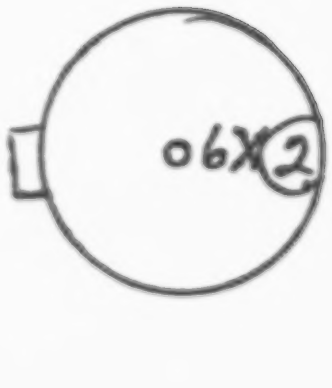
Climb the Ladder

1. This is a small group activity
2. The children sit in a circle around a pocket chart in the form of the ladder.
3. They take turns trying to climb the ladder to the top. The child that reaches the top wins.
4. The teacher should change the combinations every so often to keep the group alert.

This game can also be adapted to:

1. Ride the elephant
2. Climb aboard the space ship

Multiplication facts

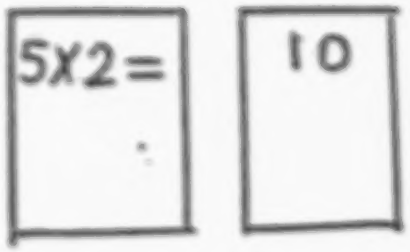


Hole for numeral to show

Place larger wheel on top and fasten with brass fastener.

Using tagboard make two wheels, place numerals around one of the wheels. Place a numeral and the multiplication sign on the other, cutting a round hole where the second factor will appear. Place the two wheels together, fastening with a brass fatener. As the lower wheel is turned, different numerals appear in the placeholder. The child must give the correct answer. The wheel is then rotated and a new numeral will appear.

Make additional wheels for all numerals and factors desired.



A chart of combinations should be available for children to find answers when needed. As

0	0	1	2	3
x 0	x 1	x 1	x 1	x 1
0	0	1	2	3
4	5	6	7	8
x 1	x 1	x 1	x 1	x 1
9	0	1	2	3
x 1	x 2	x 2	x 2	x 2

A Game, Merry-Go-Round for drill on multiplication. May be used on flannel board or large cardboard disc.

Cut out animals for the Merry-Go-Round and put numerals on them. Place these around edge of circle or disc.

Place the multiplier in the middle and let the children have turns "riding" around. If a child misses he "falls" off, and another child has a turn.

Numerals may be used instead of animals if desired.

A card game "I WIN" for teaching and drill of multiplication.

The answer deck has numerals placed so that the complete equation can be shown.

Rules:

Two or four players (not more than 5) deal 4 or 5 cards one at a time clockwise.

The answer deck is placed face down in the center of the table. The player to the left of the dealer plays first. Player draws an answer and if he can complete the equation he lays his card and the answer in front of him. If he doesn't draw an answer he can use, he turns the card face up beside the answer deck. The next player may draw this discard or from the face down deck. Proceed until a player is out of cards and calls out "I Win." Shuffle cards and repeat. Suggest chart be made through nines as shown to facilitate finding answers quickly, otherwise the game slows down.

When pupils have learned one-half of the basic facts in the table, they will know the other half because of the commutative property of multiplication.

The identity element of multiplication

$3 \times 1 = 3$
$4 \times 1 = 4$
$5 \times 1 = 5$

One serves as a unique number in multiplication because any number by one is always the number. 7:176

The multiplication property of zero.

$0 + 0 + 0 = 0$
$3 \times 0 = 0$

The product of 0 and any other factor is 0. 7:171

The impossibility of division by zero

$5 \div 0 = \square$

The solution is impossible since there is no number which, when multiplied by zero ($\square \times 0 = 5$) gives five. 7:174

Level B of Math Workshop for children has many, many innovative methods for intuitive learning for multiplication and division. The author Robert Wirtz believes that:

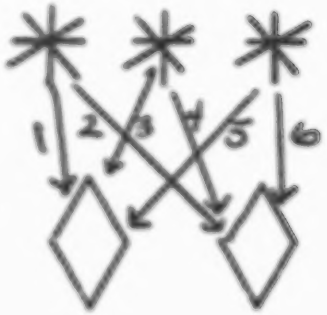
> "Our first and almost only concern is that a problem should illustrate the relationship between mathematics and the world of events, a relationship that becomes more obvious if the mathematical statement is constant ly brought to the front of the stage." 17:44

1. State the problem on the chalk board or flannel board showing (instead of just the numerical equation) the objects in sets.

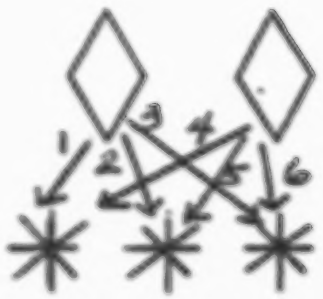
Multiplication in rows and columns
(cartesian products)

3 x 2 or 2 x 3

3x2



2x3



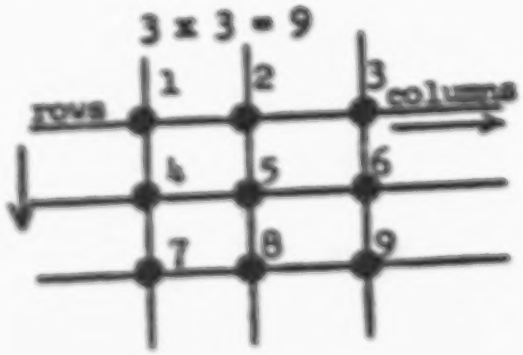
2. Proceed to pair every diamond with every star. The class will find that after each star is paired they have used six different combinations of the figures; the only combinations possible.

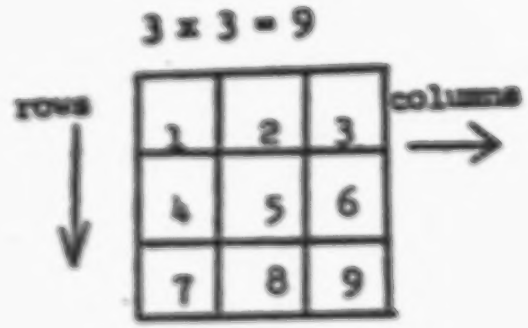
3. Put other problems on the board for follow-up to reinforce the lesson.

<u>Definition of the Cartesian Product</u>:

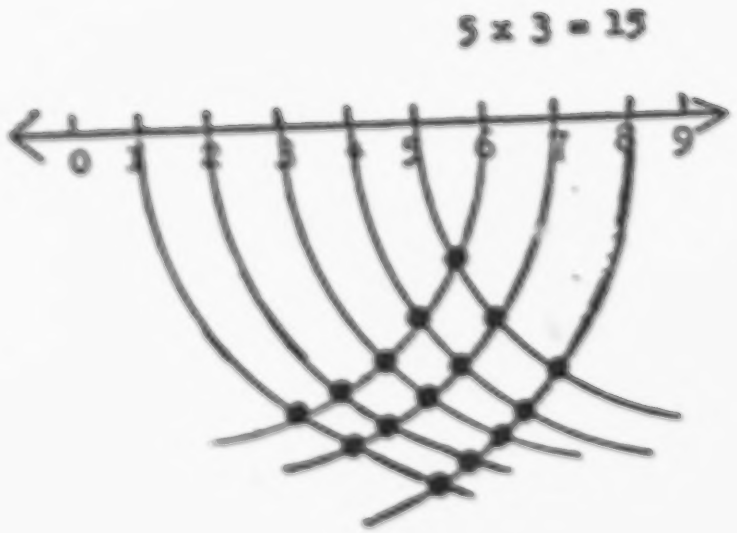
The cartesian product of A and B consits of all possible matchings of each element of set A with each element of set B.

Multiplication arrays

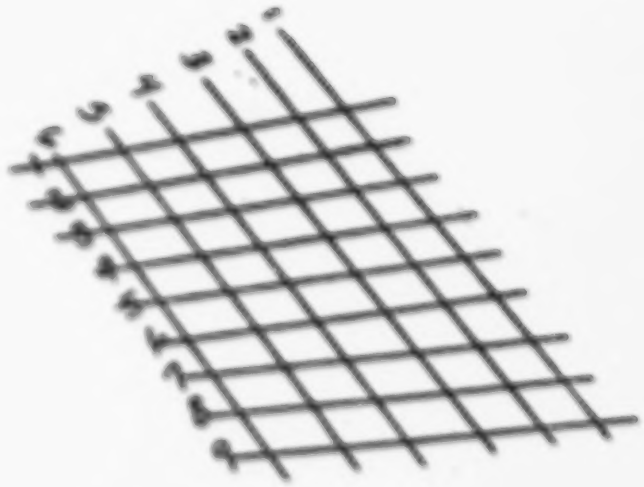




The column is the up and down "street." The row is the across "street." Where the streets intersect makes a multiplication array. It is customary to name rows first. 18:104 7:170



Parents and teachers are prone to quickly supply the unknown product when a child just can't remember "a times problem." This is unfair to the child because he depends on being <u>given</u> the answer and does not depend upon himself for finding out. If, instead of the answer being given him, he is instructed to "make a picture"and count the intersections he will probably get tired of counting intersections and try to remember the basic facts.



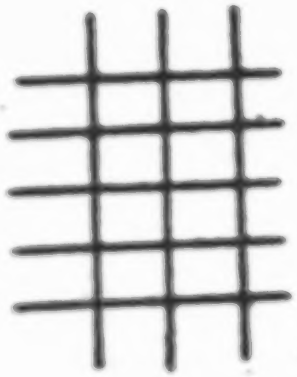
A quick picture of 9 x 6

"Lazy" learners will find it easier to memorize that 7 x 8 = 56, than to count the fifty-six intersections.

Multiplication in rows and columns--arrays and cartesian products.

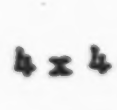
"How many cans are there in each row? (4) How many in each column? (2) How many altogether? (8) What multiplication sentence tells this? (2 x 4 = 8) Is there another sentence that also tells this? (4 x 2 = 8)"

To help pupils see what happens in putting numbers together by multiplying, an arrangement of objects or symbols into rows and columns may be used. Use solid objects at first such as pencils stuck into a sheet of styrofoam, groups of blocks or cans, or pictures arranged on the bulletin board. Pupil pages may use pictures of animals, geometric shapes, and crossing line segments.



How many "lines" go down?
Left and right?
How many times do you find
they cross (intersect)?

"Draw an array that has the same number of lines across as down. "How many crossings are there? "Write the number sentence for it. Can you think of another square array?" (Pupils may draw arrays of this sort.)

1 x 1 = □ 

2 x 2 = □

3 x 3 = □

4 x 4 = □

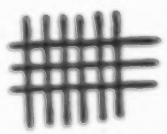
___ x ___ = ___

If they are still "with you," ask for "square arrays" for such numbers as 1, 4, 9, 16, 25.

a. 3 x ____ = 18

b. ____ x 5 = 15

Have pupils construct line segment arrays for several equations, with any of the three numeral positions empty. Give some problems without, and some with grids. Ask pupils to "supply the missing numerals." Keep the game varied, and quit before they tire.

a. 

3 x 6 = 18

b.

3 x 5 = 15

c. 4 x 3 = ___

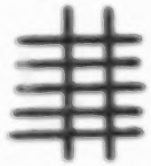
4 x 3 = 12

d.

d. 4 x 2 = 8

e.

e.



___ x 5 = 10

2 x 5 = 10

Ask students to find or name several things that are arranged in arrays, such as muffin pans, window panes and file drawers.

Arrays of 18 (Pupils made patterns and equations)

9 x 2 = 18

6 x 3 = 18

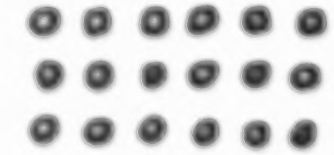
Ask pupils to describe the arrays telling how many objects are in a <u>row</u> (left-to-right) and in a <u>column</u> (top-to-bottom). Work sheets with arrays and number sentences give good, meaningful practice of "times tables."

Ask pupils to drew an array to show the factors of, say, 18. (For now, stay with <u>rectangular</u> arrays- same number of members in each row or each column). Ask if there are others.

1 x 18 = 18

2 x 9 = 18

2nd Grade



3 x 6 = 18

Division as set partitioning

6 ÷ 2 = 3

2 ⟌ 6

Hold up an arrey card. See who can tell how many elements there are the fastest. (Hold it still.)

Give pupils sheets of 1" or ½" ruled graph paper. Let them cut out various sizes of squares and rectangles, and write the numeral on the back of each to tell how many squares there are. Pupils may exchange cards and "quiz" each other (perhaps with a short time limit).

It cannot be taken for granted that children thoroughly understand concepts even though they compute for correct answers. Therefore, in the sentence 6 ÷ 2 = □ the children and teacher should verbalize, manipulate objects, use the number line, and make up stories that can be told by the math sentence.

1. Gather 6 books, pencils, or other objects.
2. Have two children go to the front of the class.
3. Discuss the fact that you are going to divide these six objects between the two children. "How shall we do it?" We give one book to the first child, then one to the second child. We do this till all the books are distributed between the two children.
4. "How many books does each child have?" (Each has 3)
5. Repeat this process with different children, objects and varied numbers of objects.
6. When the concept is understood then introduce the number sentence for division

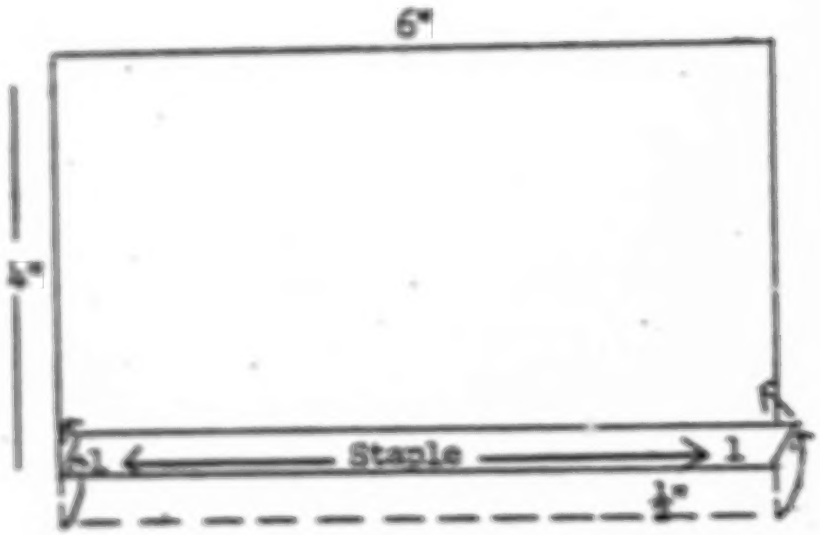
Multiplication and division combinations to 10

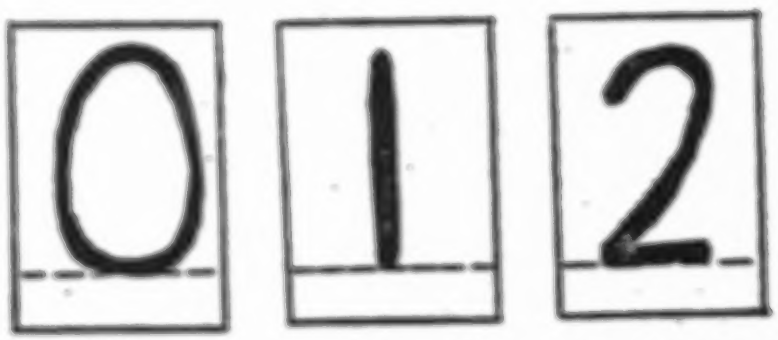
1. Show a flash card or state the equation 3 x 2 = □
2. The pupils respond by holding up the correct number of fingers. (6)
3. If quite a few miss one fact, the teacher may say, "That's right, the answer is 6."

Multiplication and division combinations, products through 25

"Everybody show" cards provide a manipulative aid to facilitate oral drill. Pupils should be provided with an answer frame and twelve numeral cards. The frame is made from tagboard or construction paper, 4" x 6", with one 6" edge folded and stapled to provide a ½" pocket for cards. The cards should be 2" x 3", with numerals printed ½" from the bottom. As the teacher gives a problem, the pupils find the numerals for the answer, and set them into their frames. At the call "Everybody show," or, if you wish, whenever they have the answer, pupils hold up their answers. 6:427

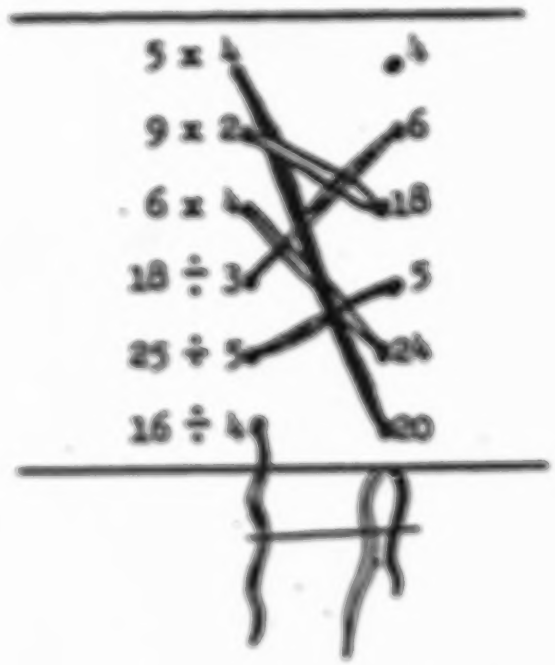
A brown envelope may be stapled up on the end. The envelope may then hold the cards too!





Make 2 - 1's, 2 - 2's, 1 each of 3 thru 9, and 1 - 0.

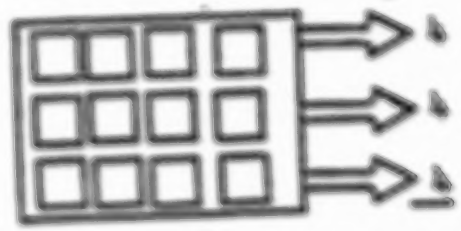
Practice of combinations



Matching games may be made out of chipboard and shoestrings. Any desired combinations may be used, with the difficulty level suited to the class.

Several such games may be kept always ready for number practices at a math center in the classroom.

Multiplication as repeated addition, and division as repeated subtraction



12

10	
10	10
+ 10	x 3
30	30

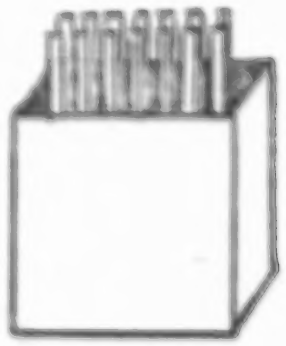
Ask students to find the number of figures in the picture. Then ask how different pupils found their answer. List the ways: 3:4-6

"We can count them."
"We might say there are 4 squares in a row, and we have 3 rows."
"Or, we could say that because there are 3 rows with 4 squares each, we can write

4
x 3

In this addition problem, how many 10's are being added? (3) What is a way of writing this as multiplication?"

Division as repeated subtraction



How many times can we take 2 away from 14?

Division as repeated addition

$8 \div 2 = \_\_$

$8 \div 4 = \_\_$

Students may do several problems of this semi-abstract type, and then move on to the numeral problems, such as rewriting addition problems as multiplication examples.

After the introduction of division show that we may find the quotient by repeated subtraction. Have available a group of counters, given the set (14). Ask how many sets of 2 are in 14? Take sets of two from the original set. How many times did we subtract?

Each child should have materials to manipulate, or they can work in pairs.

1. State the problem on the chalk board or flannel board.

   $8 \div 2 = \_\_\_\_\_$

2. Redefine the problem stating $8 \div 2$. Tell the children we are interested in learning how many sets of 2 are in a set of 8. We can find out by subtraction. Division is a repeated subtraction.

and

8		8	
- 2	1 set	- 4	1 set
6		4	
- 2	2 sets	- 4	2 sets
4		0	
- 2	3 sets		
2			
- 2	4 sets		
0			

3. From the above examples we found 4 sets of 2 in eight and 2 sets of 4 in eight, therefore $8 \div 2 = 4$ and $8 \div 4 = 2$

Division as the inverse of multiplication

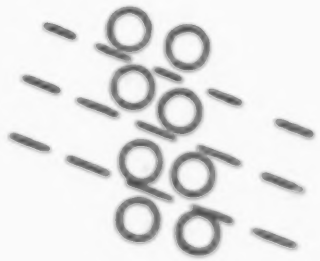
Readiness for division may be developed through work with missing factors in multiplication. Using counters, pupils solve problems which the teacher puts on the chalk board, such as $\square \times 2 = 8$. Ask pupils to lay out 8 counters, and to find how many sets of 2 there are in 8. Help them develop the number sentence $4 \times 2 = 8$. Repeat with other multiples of 2. 4:217

As a variation, direct pupils to lay out ten counters. Ask pupils to make 5 sets, with the same number of counters in each set. Assist pupils in writing the equation that tells this story ($5 \times 2 = 10$). Continue with groups Of 8, 6, 4, 2, and 0. Help them realize that the same number of members must be in each set.

$\square \times 2 = 8$

There are 4 sets

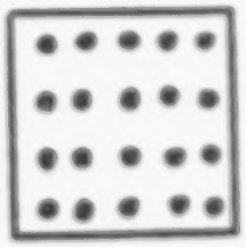
2 in 8.



$4 \times 5 = 20$

$20 \div 4 = 5$

$20 \div 5 = 4$



1. The inverse relation may be described as being the "opposite," or the "undoing" of what multiplication does.

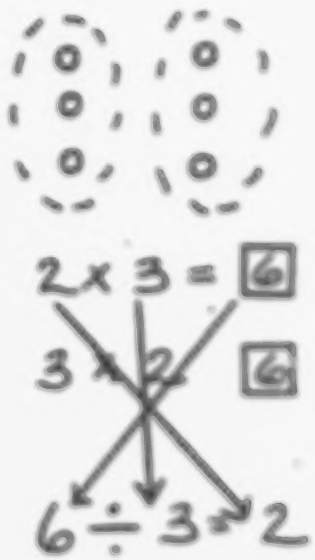
2. It is helpful to start with a known multiplication fact, such as 5 x 4 shown as an arry of dots.

3. The five columns and four rows make altogether 20 dots. In division the same 20 dots can be divided into sets of four. How many sets of four will there be?

4. If the child has a card of his own he can fold the card to find the answer

or

Putting yarn around sets of discs helps point out the inverse operation.



Multiplication facts

For a Small Group:

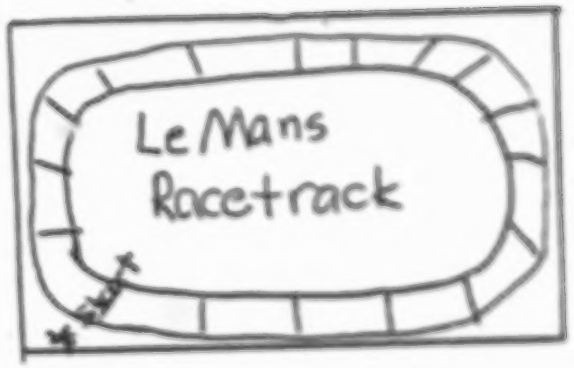
1. Show a set of 15 jacks.
2. Ask a student to throw the jacks out on the floor or a table.
3. Show the equation 15 ÷ 3 = on a flash card on the board.
4. Ask for a volunteer to divide (partition) the 15 into sets of 3.
5. Complete the equation

   15 ÷ 3 = 5
6. Repeat this 15 ÷ 5 =

Pick a petal

Have a tagboard flower with removable petals. Write a multiplication or division equation on each petal. The children pick petals and give the products or factors. Children accumulate petals. New flowers may be made with new equations. Flowers may be placed on the bulletin board.

Multiplication and division facts



Tagboard race track - colored and laminated

Race track game

Children are given small objects or cars to place on the race track. They will proceed around the track from square to square after giving the answer in response to a flash card.

Have flash cards in a stack. If a child does not answer his card he does not go on. When his next turn comes he tries again.

You might have a Big 5 race and go around the track five times, or set a time limit and whoever goes the furthest wins.

Multiplication, division and addition review

1. A small group of children gather around a sturdy clown figure with holes that are labeled with multiplication or division facts.
2. The children take turns tossing a ball of yarn or a bean bag at the holes in the clown.
3. Each child gets three turns.
4. When the ball of yarn goes through one of the holes, he calls out the answer to the equation and writes it down.
5. After each child has his turn, the child with the highest total wins the game.
6. This game can be adapted to the needs of the children.

"Standing for Symbols"

Large numerals and signs for the operations are made on larger cards. Several students are given the large numerals and operational signs. As a problem is given, (6 x 3) the students with the symbols needed arrange themselves in front of the class.

Multiplication and division facts

Drill for multiplication and division

Put up a bulletin board with a cowboy with a lasso. Place equations on cards within.

Children may find a partner. While one child tries to say all the sums or differences, the other student holds the answer card (found in the lower right hand corner). If he does this correctly he is a cowboy. (He could win a paper cowboy hat) The other student has his turn. Change the equations frequently as you cover new concepts and combinations.

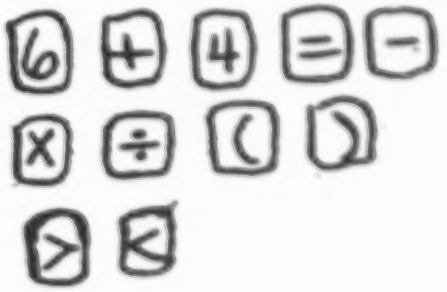
Multiplication and division facts

Place numerals and operation signs on small squares of tagboard. Laminate them to keep them from wearing out.

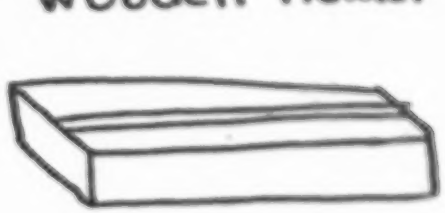
Have a wooden 2 x 4 with slotted center.

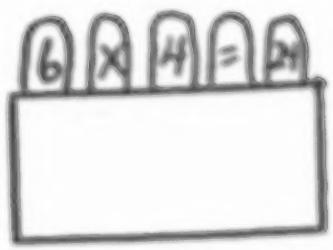
Children are to make equations with available numeral and symbol cards.

Laminated
numeral and
symbol squares



Slotted
wooden holder

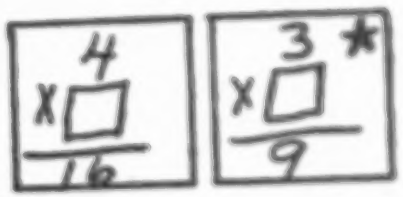


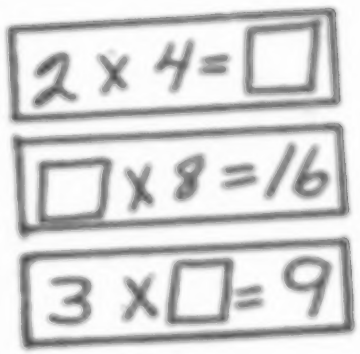


They may draw cards until an equation can be made.

Children may make equations as dictated by the teacher or by other children.

Games and drill - Multiplication and Division

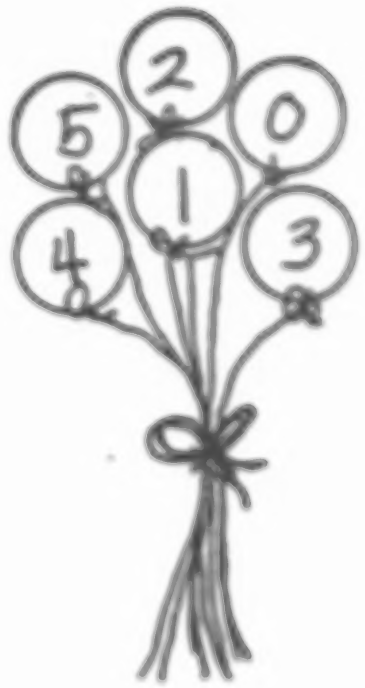




Red Star

1. Red Star is a small group activity.
2. Children sit and take turns drawing the flash cards. They answer the equation on the card and put it in one central pile.
3. When a child draws a red starred flash card and answers it correctly, he gets to have the central pile of already answered flash cards.
4. If a child answers with a wrong answer, he forfeits all his collected flash cards.
5. The object is to get as many flash cards as possible.

Multiplication and division fact - drill for fast recall



1. Have the children drew six large balloons on the blackboard or cut them from colored paper and fasten them to the chalk board.

2. Write one numeral on each balloon beginning with zero.

3. Choose a number and tell the children that the number represented on each balloon must be multiplied by the number you selected.

   For example: Tell the children that each number is to be multiplied by 5. Point to each balloon in turn. The child whose turn it is must try to pop the indicated balloon by quickly telling the product of five and the number indicated on the balloon.

4. If a child successfully pops all six balloons he may choose a number for the next game and point to the balloons.

Baseball

a. Divide the class into two teams.
b. Teacher is the "pitcher" and uses multiplication flash card.
c. Establish places in the room for 1st Base, 2nd Base, 3rd Base and Home Plate.
d. A child from Team A goes to home plate. He decides how many cards he will try to answer. If he picks 1 and answers one, he goes to first base. If he picks 2 and answers them correctly, he goes to 2nd base, etc. To make a home run, he must choose 4 and answer all of them correctly.
e. The team scores for each run made.
f. A miss is an out. Three outs make an inning and then Team B is "Up to Bat."

PROBLEM SOLVING

GRADE 2

2nd GRADE

PROBLEM SOLVING

In teaching elementary mathematics the discovery approach is very effective. It is wise to stress patterns and relationships as the child moves from concrete experiences to abstract ideas and as he applies these ideas to new situations. Basic mathematical concepts are developed, then tested and extended again and again. The child may learn that not all problems have ready solutions and that problems may have more than one possible solution. He learns to test alternate approaches.

Children can be led to find problems in individual experiences. Let them exchange papers and work each others problems. When the answer is formal the children should check and see if it is a reasonable solution.

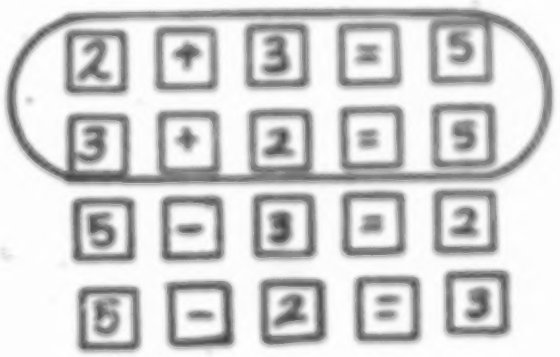
Ask these questions:

1. What must we find out?
2. What facts does the story tell?
3. How can we use the facts to find the answer?
4. Is the answer reasonable?

Many math sentences from the same picture.

Tom has three blocks, show four equations he can make.

Find the problem, analyze facts given, decide which operation to use and solve the problem.



Numbers could be made on small blocks or cards for use at the math table or desk. The pupil is then asked to make as many numerals or names for 5 as he can. This procedure can be used for finding the sentence that solves the problem, (e.g., Ted found two rocks at the river. He found three more at the lake. How many does he have in his collection?)

Uses of Tables in problem solving
(functions and relations)

Standard textbooks are full of questions like this: "If candy bars are 4¢ each how much would three bars cost?"

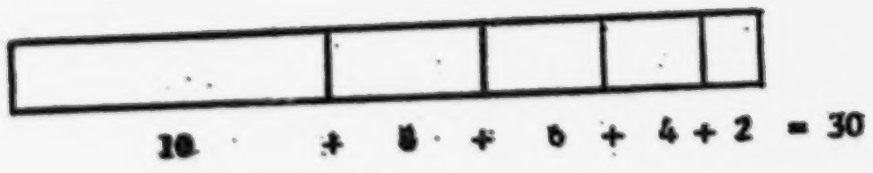
We should be less interested in the answer 12¢ than in problems the clerk might face. Suppose a customer wants four bars or five bars or only one. The clerk might find the table helpful.

Teacher's records of many kinds--attendance, height, and weight, scores, etc.---can be used as a basis for pictorial record.

Using shapes to show relationships of numbers

1. If the size of the smallest shape suggests 2, then the size of the other shapes suggest 4, 6, 8, or 10. The shapes placed end to end suggest 30. We will call the group a train. 16:53

No. of candy bars	1	2	3	4	5
Total cost @ 4¢ each	4¢	8¢	12¢	16¢	20¢



If we were to change the order of the grouping would the train still suggest 30? (yes)

2. How many different numbers can you suggest using 1, 2, 3, 4 or 5 cars of the train? (2 + 4 + 6 or 12; 2 + 4 + 6 + 8 or 20; etc.

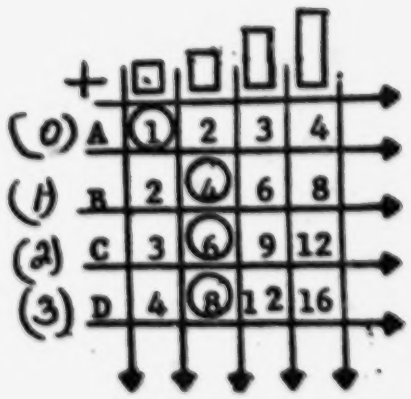
3. How many odd numbers could you suggest? (None)

4. "What numbers could you suggest if you had two of each of the cars instead of only one? What if you had three of each?

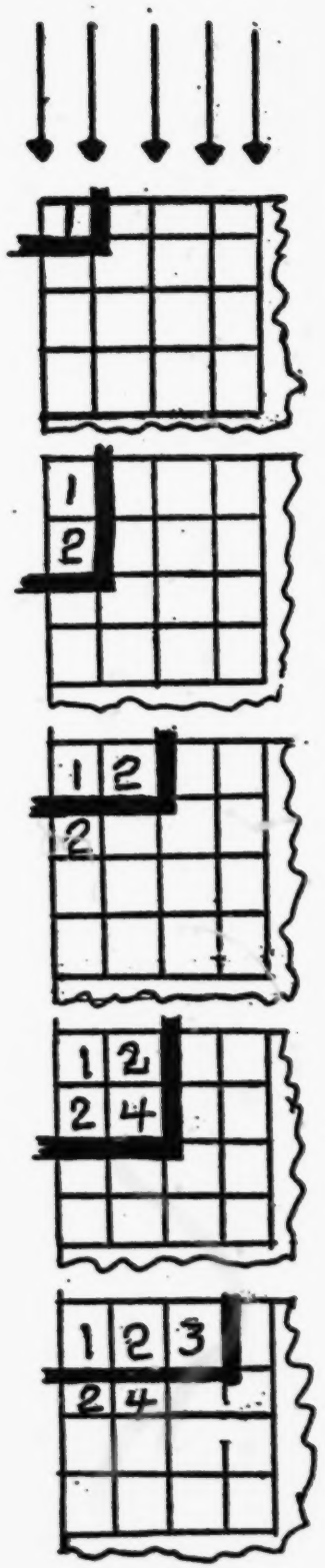
5. Discuss the problems and draw pictures to illustrate your conclusion."

Number relations

Make the drawing at the left on the board but mark in only the numbers that have been circled. Encourage pupils to help you finish the chart. 17:54



If you use the alphabet in the left column, lead the children to discover what number each letter stands for.

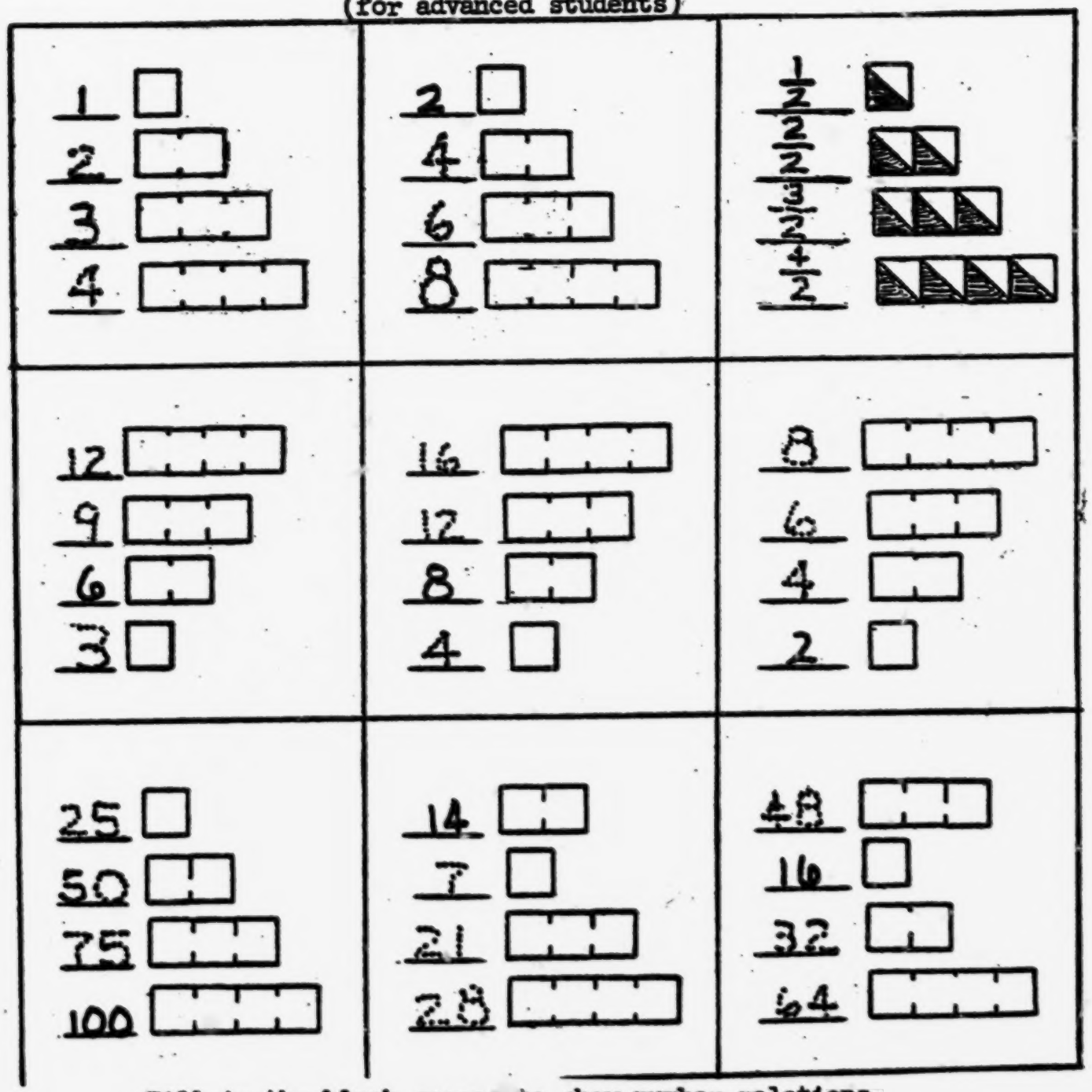


It's a simple matter to extend the chart in both directons if time permits. Give the children opportunity to discover familiar patterns.

Let the children use graph paper and a replica of a carpenter's square cut out of cardboard. Starting at the left-hand corner of the paper, pupils are to mark off square by square in successive order, and to number each square as they proceed. In this activity the students will be creating exactly the same chart as the one you drew on the board. Have pupils make the comparison.

"Are the patterns the same? Do the patterns become different as the numbers get bigger?"

(for advanced students)



Fill in the blank spaces to show number relations.

Problem solving without numbers and telling the operation used to find the solution.

1. On Marie's birthday a certain number of her friends came to her party. There were as many girls as boys.

   How many boys were at the party?

2. The boys had fun searching for peanuts. Each boy found the same number of peanuts.

   How many peanuts did they find?

3. The girls liked the pretty paper hats. After Marie gave a hat to each girl she had some hats left over. How many did she have to start with?

4. Each of the guests brought a gift for Marie. Grandmother, Mother and Father gave her some gifts. How many gifts did Marie receive?

5. When it came time for the children to go home Marie gave each of them the same number of lollipops. How many lollipops did she give her friends?

Verbal problems

Mary, Mary, quite contrary
How does your garden grow?
With ten bells and eight
cockleshells

(How many flowers all in a row?)

Ask the children to think of rhymes, and help them make the necessary changes in wording, and substitute numerals in order to change the rhyme into a story problem. Each child may say or read his rhyme and call on others to say the answer. 4:96

Humpty-Dumpty sat on a wall.
Humpty-Dumpty had a great fall
Eight king's horses and seven
king's men
Couldn't put Humpty together
again

(How many tried to put Humpty
together again?)

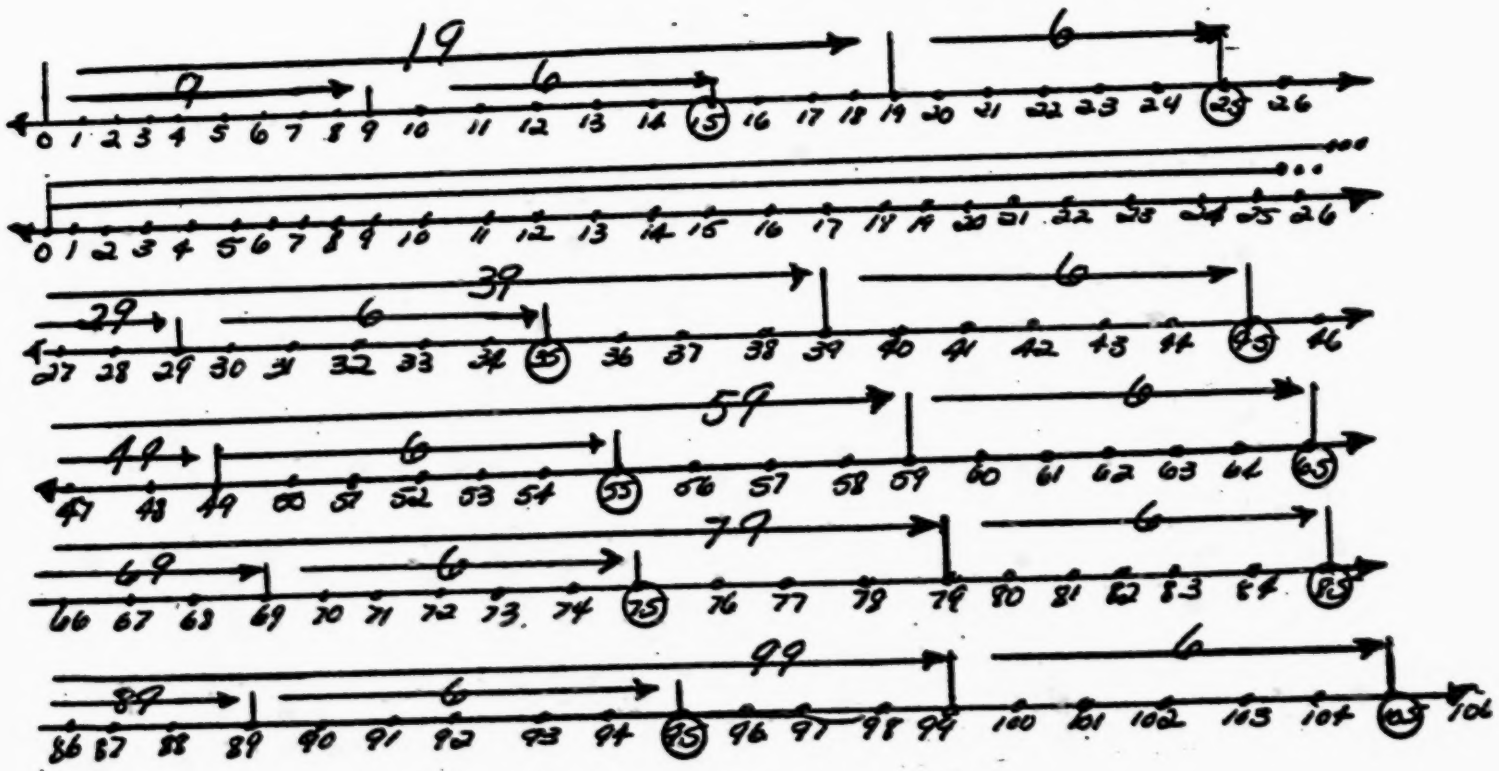
1. Make use of patterns in helping children learn to think quickly. Ask them to give the sum as you call out number combinations:
   9 + 6, 19 + 6, 29 + 6, 39 + 6 and so forth through 89 + 6.

2. Again call out " 9 + 6," but vary the pattern by following with 9 + 16, 9 + 26, 9 + 36, and so forth.

3. We could chart these patterns on a number line to show the relationships of the additions.

4. Subtraction patterns may be shown by "undoing" the addition; 105-6, 99-6, 89-6, etc.

5. Call out subtraction combinations and have children give the differences; 12-8, 22-8, 32-8, through 92-8.



A chart of drawings similar to this was used in one classroom to introduce a new activity to the class. 12:2-22

Price Limit - 15¢

In calling attention to the chart the teacher said, "These drawings refer to an activity that other classes have liked very much. The first person says, 'I went to the store and bought a pencil for 5¢.' The next person, as you can see, makes the same purchase, adds one purchase, and gives the total. This goes on until someone makes a mistake. You really have to pay attention to make a long list of purchases. The record in my classroom last year was twelve purchases. I wonder whether we will equal or pass that before we give up."

This activity requires that pupils concentrate their attention on each previous statement, as it is introduced to the class. This, plus the experience in adding and stating the previous purchases, gives some practice in verbal problem solving. The difficulty of the additions may be partially controlled by the price limit.

This activity could be altered by allowing some subtraction to be inserted. (i.e. "I lost 5¢ so I only had 19¢.")

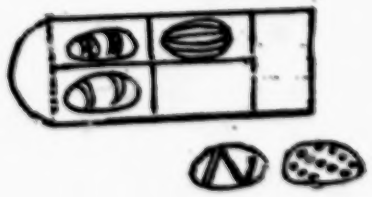
Illustrating given math sentences by telling and writing story problems.

Eddy had 5 eggs and gave 2 to his friend. How many did he have then?

$5 - 2 = \square$

I put two eggs in the crate. How many do I have now?

$2 + 3 = \square$



Sketches and diagrams as problem solving aids

What coins did Tom have?

One day at recess Tom was jingling some coins 16:4 in his pocket. He turned to the other boys around him and said: "I have thirty cents in my pocket. I don't have any pennies. What coins do I have?" The other boys made five guesses, naming coins that equaled 30 cents. Can you tell what their five guesses were?

$25 + 5 = 30$
$10 + 10 + 5 + 5 = 30$
$5 + 5 + 5 + 5 + 5 + 5 = 30$
$10 + 10 + 10 = 30$
$10 + 5 + 5 + 5 + 5 = 30$

Before introducing written Problem Solving, it would be well to teach the vocabulary key words, and their meaning.
Such as:

How many
How many in all
How many more
How many less
How much
How much more
How much less
Find the difference
Find the product
divisor, divide
dividend, quotient
multiply, multiplier
multiplicand, product

It is also suggested that much practice be given in oral problem solving before attempting written work. Children should be encouraged to do story problems of their own. "Playing Store," giving parties, taking trips, and other activities making problem solving more meaningful.

Story problems

Often story problems use vocabulary that is too difficulty for children to read, or the expected solution is too difficult for the second grader to determine.

In such cases the problem may be restated more clearly to make it more easily understood.

The problem below was rewritten to make the story problem more interesting and meaningful to the student.

1. How many s are there? A class saw a baker put 18 s on the table. Then the baker put 24 more s on the table.

Ask for fitting stories for mathematical equations. The response must include an indication that the storyteller knows how to complete the math sentence.

Completion of mathematical sentence

Headline Story

$7-3=\_$

$8+\_=10$

$\_+6=9$

A	B	B
7	baseball	25
18	a party	15
(5)	astronauts	6
12	a trip	9
20	setting a table	10
4	animals	(12)
24	on a ranch	10
16	cookies	14

The following idea is to encourage application of language to math sentences.

Children are asked to fill in a chart with numbers in two columns, and words in a third that could be used in a story. Missing numerals and/or names may be shown as given in the example.

A variation of this idea is to have the child choose any three terms and make a story with them. Write the headline with a number equation.

Example: A trip - Twelve boys started on a hike to the lake. On the way five of them stopped to rest. How many boys hiked to the lake first?

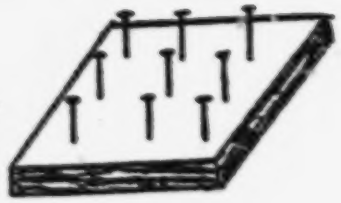
12 - 5 = 7

GEOMETRY

2ND GRADE

Constructing geometric figures

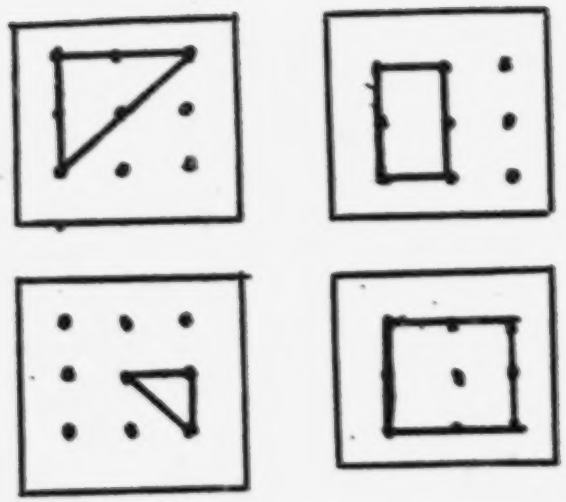
square
triangle
rectangle
and other
quadrilaterals



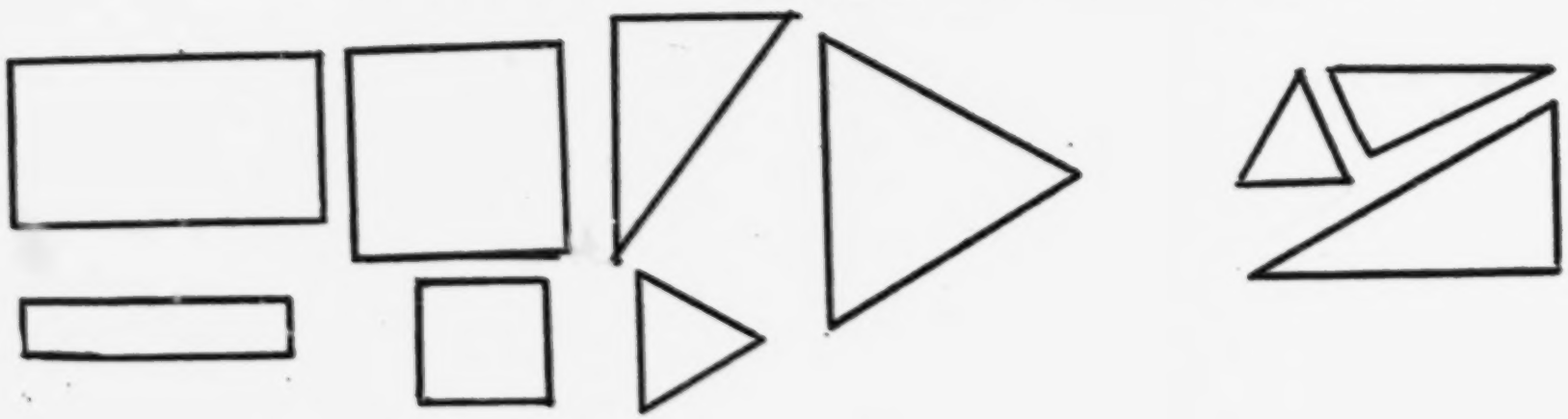
Geo - Board

Make geo-boards with 9 points represented by nails equally spaced. 1/4" plywood cut in 6 inch squares works well or use the 12 inch acoustical tiles usually found in classroom ceilings and drive golf tees in the holes of the tiles. Give the boards with various colors of rubber bands to the students. Allow them several sessions of free play with the boards, experimenting to see the various figures they can construct. Encourage verbalization of vocabulary: point, line segment, side, square, rectangle, triangle, and corner.

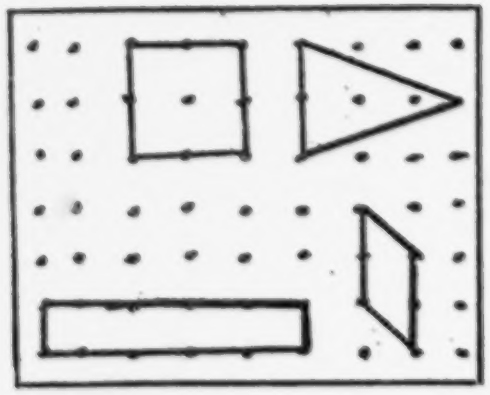
After freeplay ask students to represent specific figures: Show a small square, show a larger square, show a "tall" rectangle, show a large triangle, etc. Teacher can show student examples on board for class discussion.



Encourage children to make these same geometric shapes with tagboard or construction paper. Make different kinds of triangles. Show various lengths of sides of triangles. Proceed in the same manner for rectangles.



Constructing geometric figures

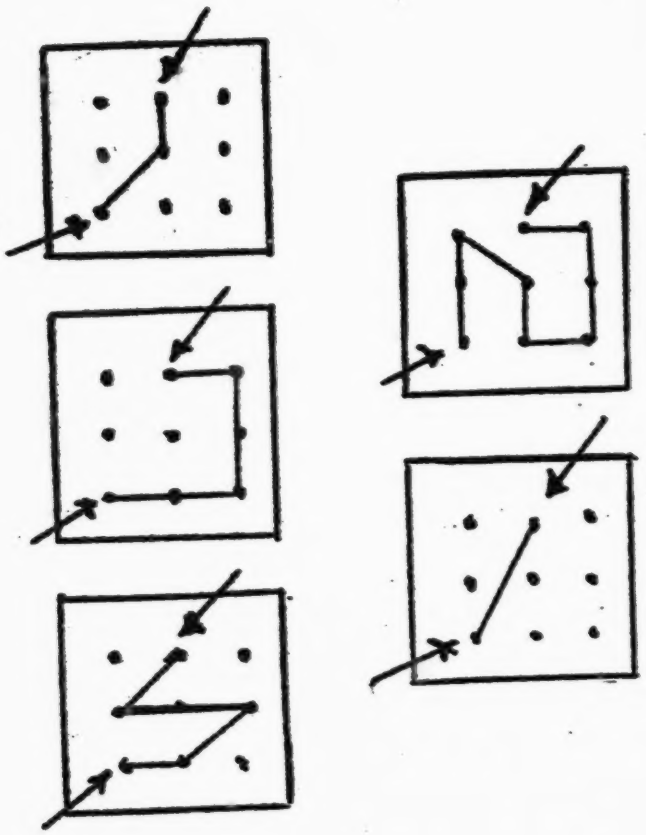


Give students paper marked with equally spaced dots to represent points. (Easily mark by placing onion skin paper over a sheet of 1" or ½" graph paper and marking dots over the intersection of the lines. Trace dots on master and ditto off. (Do not give graph paper to students as they have difficulty thinking of line intersection as points while you're asking them to construct figures.)

Let students experiment with the dotted paper, drawing whatever geometric figures they wish using a pencil and straightedge. Insist only that all paths drawn be straight and pass through at least two points. When the class is comfortable with this activity have them draw specific figures according to detailed description:

1. Can you make a triangle with one side?
2. Can you make a triangle with two sides?
3. Make a three-sided triangle.
4. Construct many different triangles.
5. Construct a three-sided square. (impossible)
6. Construct a four-sided square.
7. Construct many different sized squares.
8. Construct many different rectangles.
9. Construct a four-sided figure that is not a square or a rectangle.
(many possibilities)

Concept of line segment



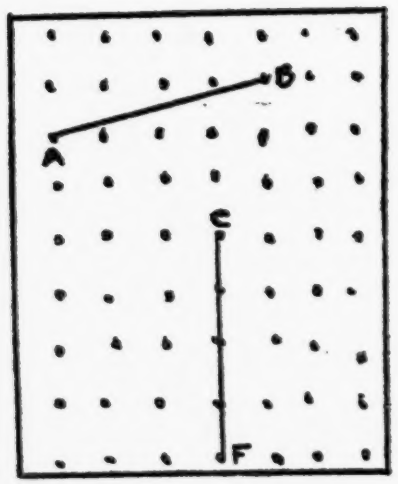
Concept of line segment

Students choose two points on a geo-board and use rubber bands to show different paths of points between the two selected points. Some possibilities are seen at left. Ask:

1. Which is the shortest path between the two chosen points?
2. Which is tne simplest path between the two points?
3. Which is the straightest path between the two points?
4. Is it a line?
5. Does the line have a beginning point?
6. Does the line have an ending point?

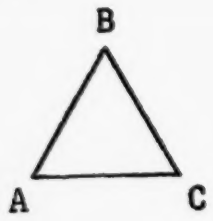
Generalizations: The shortest path between two points lies along a (straight) line. This line including its end-points (beginning point plus ending point) is called a line segment. Use the term, line segment, in talking with the children. Acceptable definitions for seven year olds include: "A line segment is the shortest path from one point to another," "A line segment is stopped on both ends," or "A line segment is a piece of line."

Let children look for examples of line segments in the classroom (edges of books, desks, chalktray, crayon box, intersection of front wall and ceiling, etc.). Have them name the "beginning" and "ending" points of the segments found.



line segment AB
line segment BA
line segment CF
line segment FC

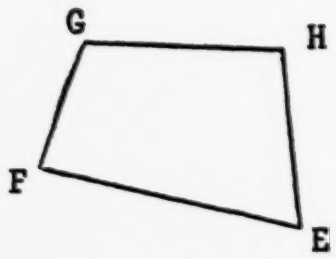
Give sheets of paper marked with dots to students. Have them choose a pair of points (dots), label them with capital letters, and connect the dots using a straightedge. Ask "what name can we give a line segment so that others know which segment we're talking about?" Ask them to name a line segment in two ways ie: line segment <u>AB</u> and line segment <u>BA</u>.



$\overline{AB}$, $\overline{BA}$, $\overline{BC}$,

$\overline{CB}$, $\overline{CA}$, $\overline{AC}$

Have students draw triangles on the dotted paper, labeling their endpoints. Ask them to name the line segments in one triangle. Help them to see that there are three line segments, each having two names: line segment AB, line segment BA, line segment BC, line segment CB, line segment CA, and line segment AC. Say "line segment AB." Do not write symbolism $\overline{AB}$ at this level. Refer to line segments verbally.



Have student draw squares, rectangles, and other quadrilaterals (4-sided figures) on the dotted paper. Ask them to label the endpoints of the line segments. Place figures on chalkboard and discuss the eight names for the four line segments of each quadrilateral.

Points, lines, and points on a line

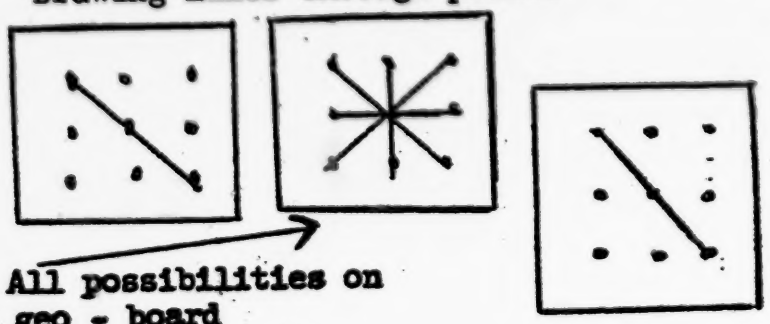
Ask students to think of a place in space and to put their finger on it. "Are you touching the same points? (no) How many points are we touching? (A number equal to the number of students in class) Now touch two points in space. Now ten points. How many points are we touching? (ten times the number of students in class or just "lots and lots of points.") Suppose every student in our school was touching ten points. (Wow!)

Yarn representing a line segment and a line

Place two students on opposite sides of the room where they can be easily seen by the class. Have them hold and pull taut a long piece of yarn or rope. Ask a student to think of a point along the string and to place his finger on it. Have him touch two points. Ten points. Keep adding students to the string. Continue to ask questions: "How many more points can we touch? If we touched the points with our pencil points instead of fingers, could we touch more points? Suppose the points continued beyond the endpoints of our line segment. Where would the points end?" Generalization to be reached informally: We can count the points in a line segment or a line forever or how many points are in a line segment or a line? (more than we can count) This activity can be performed by asking students to mark points along the string with clothes pins, opened paper clips or safety pins.

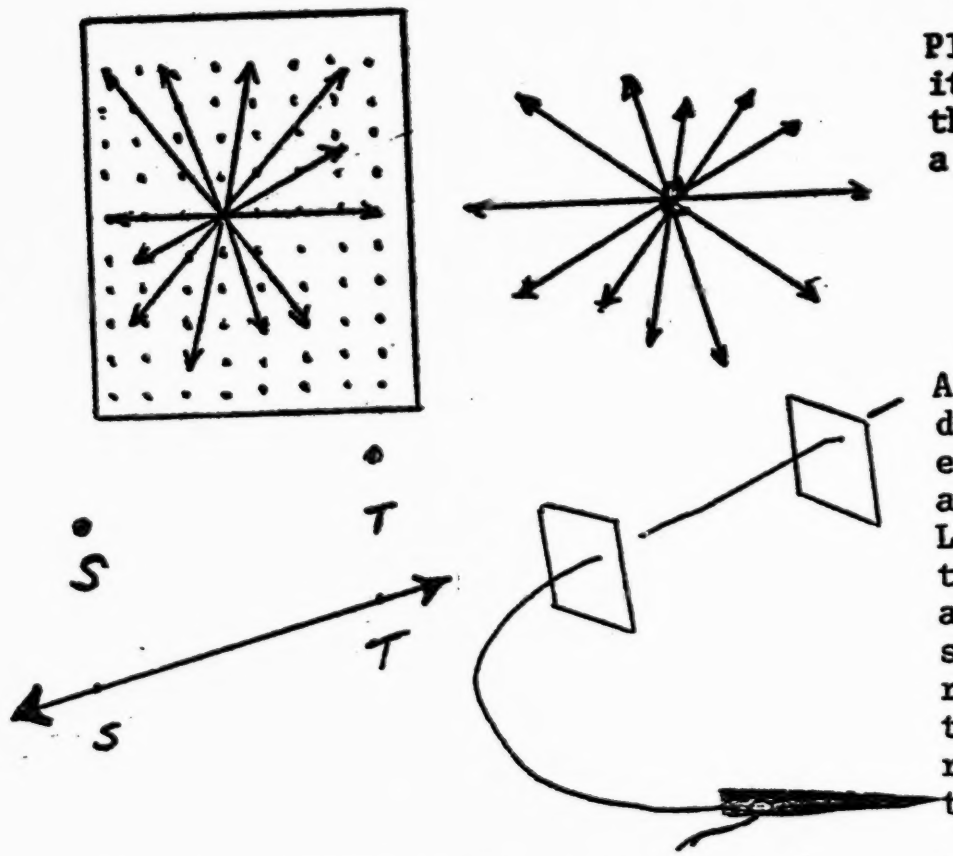
Drawing lines through points

Give students dittoed sheets of lines shown in various lengths and positions. What do the arrowheads tell us? (The line goes on forever in both directions.) Tell them to label any two points A and B. Mark with dots as many points as you can on line segment AB. "How many points are contained in line AB?" (more than we can count) Sharpen student pencils to fine points and repeat exercise with other lines.

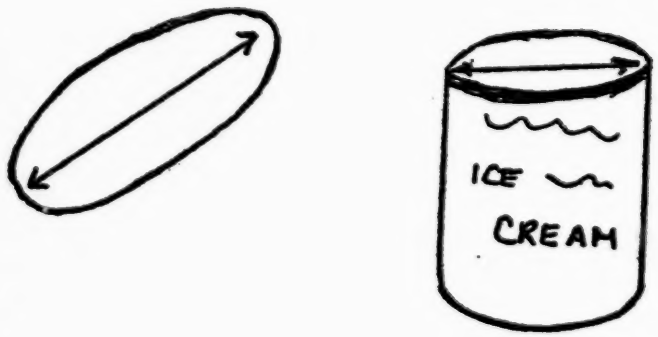


All possibilities on geo - board

Use a 9 point geo-board and rubber bands to show lines passing through a point. "How many lines can pass through a point on this board?" (four) Give the student dittoed sheets of dotted paper. Select a given point and using a straightedge draw lines through the point. Remind them to include arrowheads. "How many lines can you draw?" (many-answers vary)



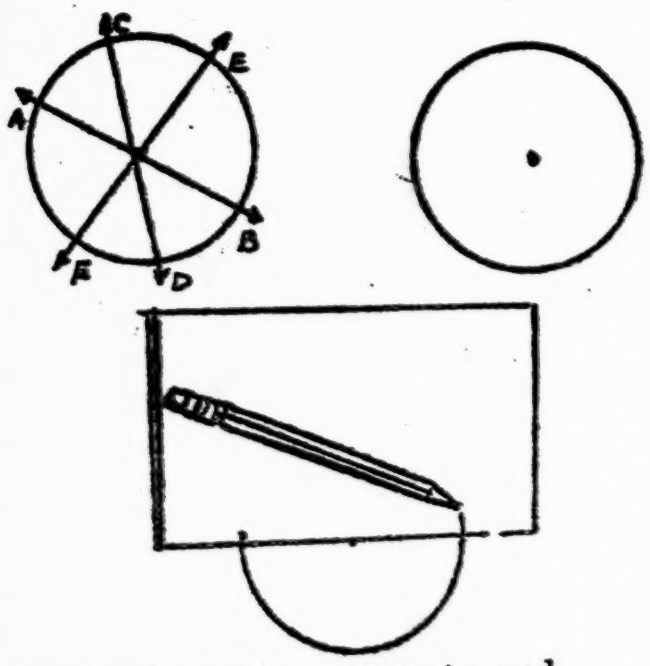
Diameter concept (without use of term)



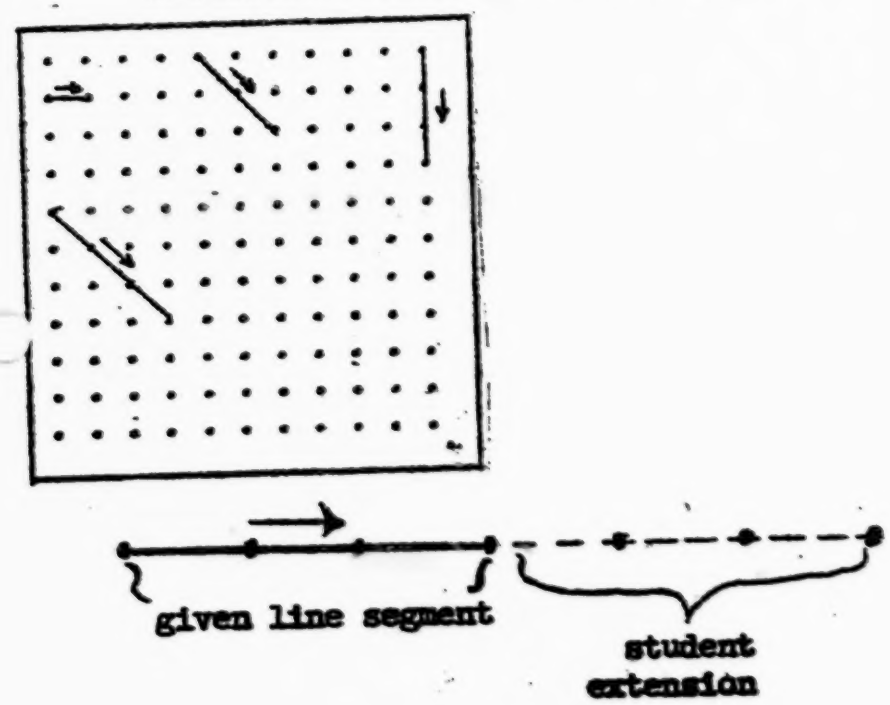
Place a dot on the chalkboard and draw one line through it using a ruler. Invite children to draw more lines through point C. Ask "How many lines can be drawn through a point?" (more than we can count)

Ask students to mark two points on a paper. (Make the dots tiny.) Label the points S and T. Using a straight-edge, draw a line through S and T. Ask—"Can you draw a different (or another) line through S and T?" (no) Lead children to see that there is only one line through two points—Demonstrate by using two students for points and a piece of yarn or rope representing the line. If the size of the dot or mark used is too large, confusion may result. If necessary, use a needle and black thread and two sheets of white paper to illustrate, i.e. the line represented by the thread and the points represented by the tiny needle holes in the two sheets of paper.

Provide three dimensional objects which represent circles such as bicycle tires, hula hoops, embroidery hoops, 3 gallon ice cream containers (cylindrical), etc. Have students measure the width of the circle shape at its widest point using non-standard units such as pencil lengths, string lengths, etc. "Does the widest path across a circle always pass through the center of the circle?" (yes) Prove visually with rope or easily seen, brightly colored yarn.



Extensions of line segments and reflections of curves and figures



Give students dittoed sheets containing circles of various sizes with their centers marked. Ask them to draw several lines across a circle that pass through the center point. Label with capital letters the points where the lines intersect the circle. Have them measure the line segment AB by placing a sheet of paper along AB and marking its length. Place the paper ruler along CD. "Are line segment AB and line segment CD the same length?" Measure line segment EF. "Is line segment EF the same length as the others?" Continue exercise with circles of various sizes. Generalization to be reached: The line segments of a circle that pass through the center of the circle and whose endpoints are points of the circle are equal in length. Let students verbalize this concept in their own way and in words which have meaning for them.

Give students sheets of dittoed paper with line segments of various lengths drawn on them. Ask student to extend each line segment i.e. "Make each line segment twice as long. Use your pencil and straightedge. Draw small arrows alongside each line segment so that student will know which direction to extend the segment. Be sure to arrange the segments to allow students to draw them without causing the "new" line segments to touch each other. Work sequentially: student should extend a line segment of one unit length first, then two units, then three units, etc. Extensions should be made right to left as well as left to right. If students have trouble, encourage them to count the spaces between the dots of the given line segment. A given line segment with four spaces between its endpoints can be extended by drawing another line segment with four spaces between its endpoints. Exercise may be preceded by demonstration on chalkboard or overhead projector. Also, a large demonstration size geo-board with 100 points may be useful

Extension of line segments and reflections of curves and figures

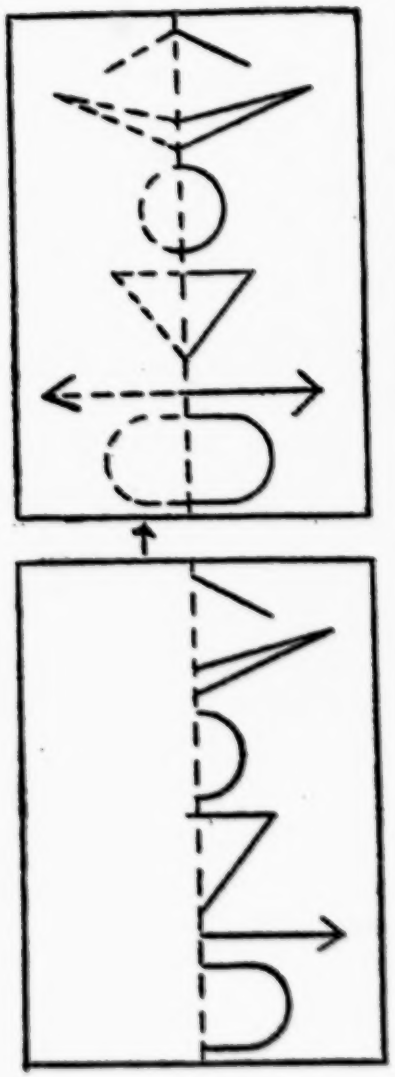


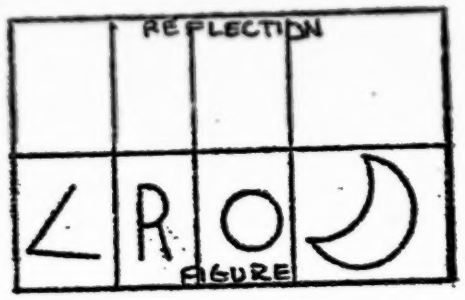
FIGURE	REFLECTION

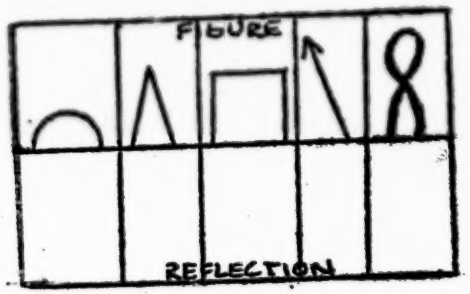
REFLECTION	FIGURE

Give students dittoed sheets showing pictures alongside a dotted line which divides the sheet of paper in half. Explain that half of each picture is missing. "What would the pictures look like if we could see both halves together?" Have students trace the given halves with crayon, fold the paper along the dotted line and then rub the folded paper with a straightedge. When opened, the given halves and their maps will be seen. Have students trace over the copied halves with a crayon different in color than the first one used. Give students another ditto sheet. Have them draw the reflection of the figures directly without folding the paper or referring to the completed pictures of the first exercise.

Give students dittoed sheets of paper like examples at left. Allow them to use small hand mirrors. First have them guess what the reflected image should look like. Then, use the hand mirror to test their guess. If necessary, students should be encouraged to continue using the mirror while drawing the reflection. Provide opportunity to reflect in both left-to-right and right-to-left directions about a vertical line in a plane. Keep figures simple and appropriate for both perception and hand skills of seven-year olds.

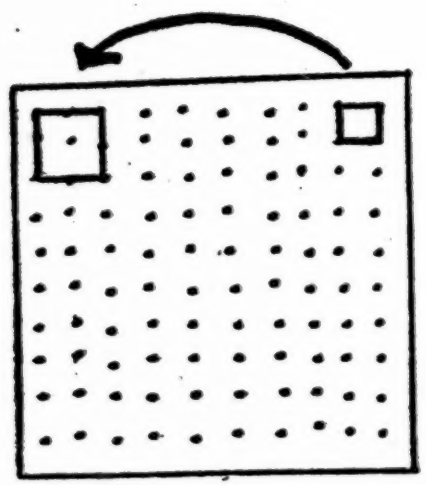
When students are successful with reflections about a vertical line, introduce reflecting about the horizontal line. Redevelop concept using the same steps as those outlined in reflection about the vertical.





Similar polygons

square
rectangle
triangle



Use a demonstration size geo-board and rubber bands (36 " x 36" plywood board with 100 points represented by nails) show how we can construct similar figures by doubling the lengths of the line segments of a given figure. Develop sequentially starting with a square whose side is one unit in length. Ask--"How can we copy this square so that the sides of our new square will be twice as long?" (Double the length of the line segment) To double line segments encourage students to count the spaces between the end-points of a given line segment and then double the number. Continue using larger and larger square until student sees the pattern.

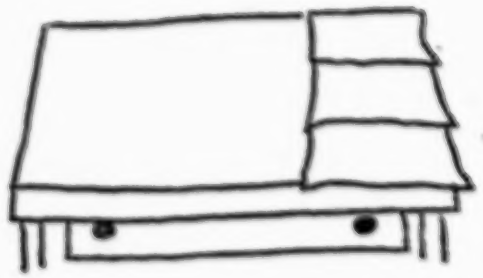
Give students dittoed sheets of paper with points marked by dots $\frac{1}{2}$" apart. On the top left corner show squares, rectangles and triangles of various sizes. Label a vertex in each figure with a capital letter. (A vertex is a point where two sides intersect.) Ask students to "copy the figures so the line segments in your drawings are twice as long as the line segments in the top left corner." To make sure that their figures will be properly spaced have a vertex of each figure they are to draw already marked and labeled with the same letter given in the model i.e. point A in the newly drawn figure should correspond to point A in the model. It need not be pointed out but some students may notice that each new figure is four times the size as the model. That is, the new figure has twice the perimeter of its model and four times the area of its model.

MEASUREMENT

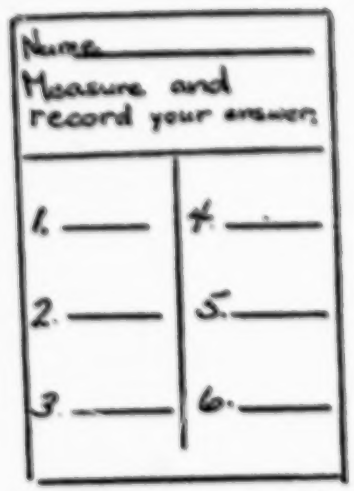
GRADE 2

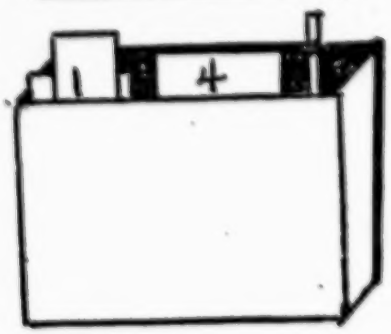
GRADE 2

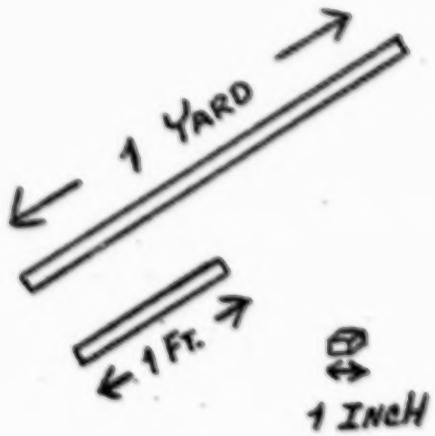
Linear measure



Length







For review, have children measure their desks with various "units" of measure -- strips of colored paper, straws, tongue depressors, the width of the hand, etc.

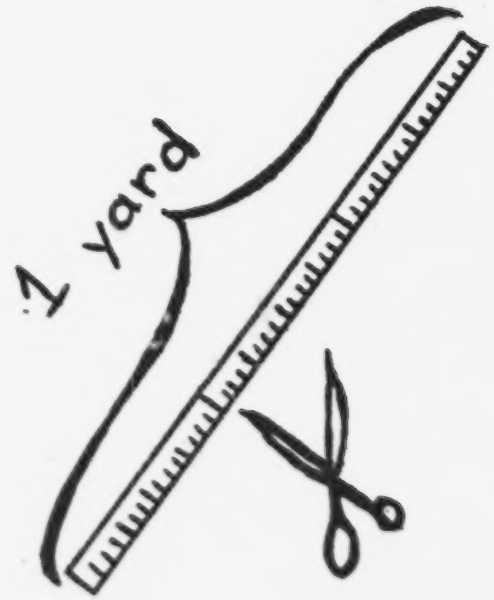
When their results are listed on the board it will be apparent that a standard unit of measure is necessary.

Discuss the use of the inch, the foot, and the yard in different situations.

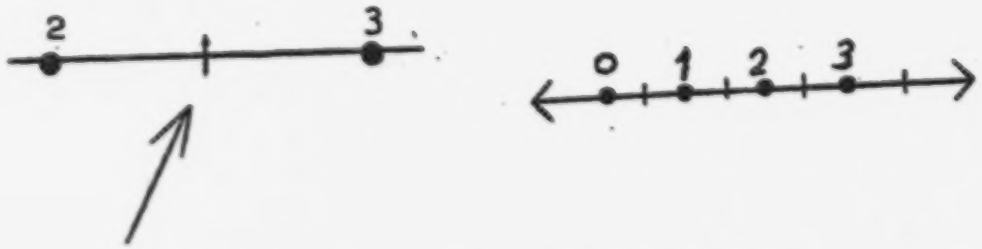
Have available the uncalibrated yardstick, the foot ruler, and the one inch cubes. Have three children measure the chalk tray with these three instruments. Compare to see the idea of selecting an <u>appropriate</u> unit of measure.

Provide many experiences to reinforce the relation between one inch, one foot, and one yard units. The understanding of this relationships in linear measurement is the basis for success in future abstract kinds of measurement.

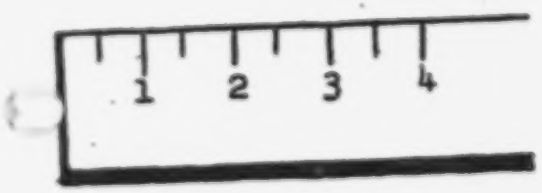
For individual work, fill a box 18 x 12 x 4" with different sizes of pieces of wood or cardboard and number each piece. Add twelve and six inch rulers, a pencil and a pad as shown.

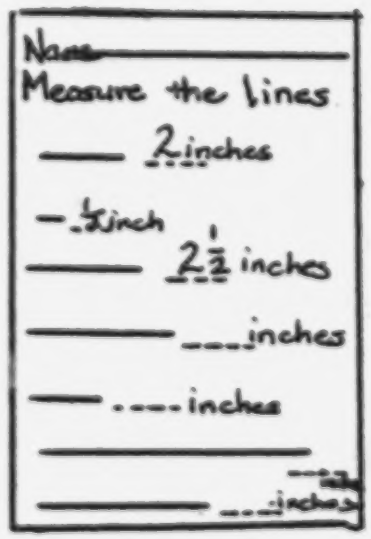


Inches and half inches on a foot ruler



A line segment on which a unit length has been laid off and marked some number of times, as shown, is called a linear scale (or ruler).





Cut strips of paper 1 yard long for each child from tagboard. Have them use their foot rulers to mark off the feet. Then let them cut their paper yardstick into 3 foot lengths.

Using their wooden foot rules, let them mark off the inches and $\frac{1}{2}$ inches, then cut one of the foot lengths into inches, and another into $\frac{1}{2}$ inches. Let them compare and discuss.

Have a very large segment of number line on the board--as well as an enlarged drawing of a foot ruler. Have a child hop, with their finger, from numeral to numeral on the line. Have him stop midway between two numerals. Where is he? He is halfway between 2 and 3. We call this $2\frac{1}{2}$. How far has he gone? He has hopped $2\frac{1}{2}$ units of measure.

A frog may be used to hop the distance too. Repeat this with others -- $1\frac{1}{2}$, $\frac{1}{2}$, $3\frac{1}{2}$ etc.

Point to the drawing of the foot ruler. Have a child hop using the frog, or his finger, on the ruler, then on a foot ruler at his desk.

To reinforce this -- have the children measure things in their desks which will involve $\frac{1}{2}$ inches. Give a worksheet involving simple measuring of lines.

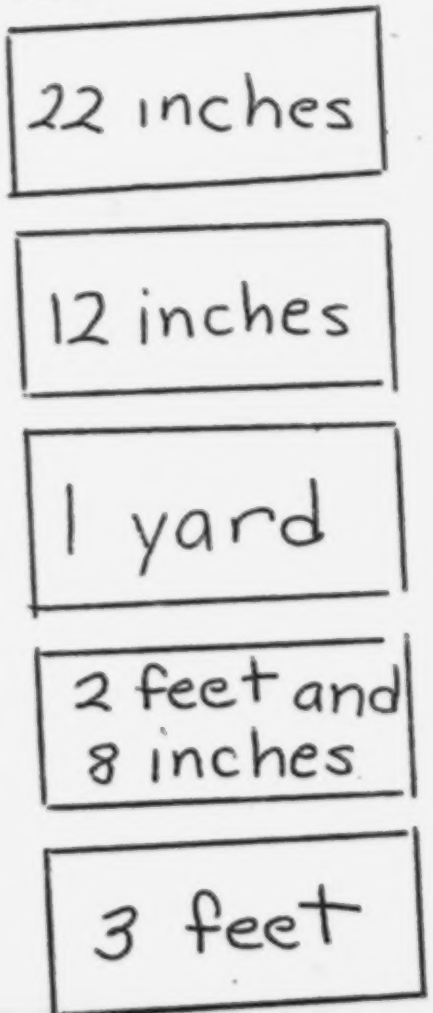
12 inches $\frac{m}{-}$ 1 foot

7 inches

4 inches

8 inches

Various measurements



4:200-205

Twelve inches is <u>not</u> exactly the same as one foot. Twelve inches <u>measures</u> the same <u>amount of</u> length as one foot. (You can have 12 one inch line segments which measures the same as a 1 foot line segment, but is obviously not the same.)

Have a child draw a four inch line on the chalkboard. Ask another to draw a line 2 or 3 inches longer than the first line and label its length. Have the children check each others lines as the activity continues. Teachers should remember that you can add and subtract numbers, but not feet and inches.

4:204

1. Distribute cards on which are written various measurements.

2. Ask the children oral riddles.

3. The child who holds the correct answer on his card stands up and answers the riddle. Example: "I am thinking of something that measures the same as one foot.

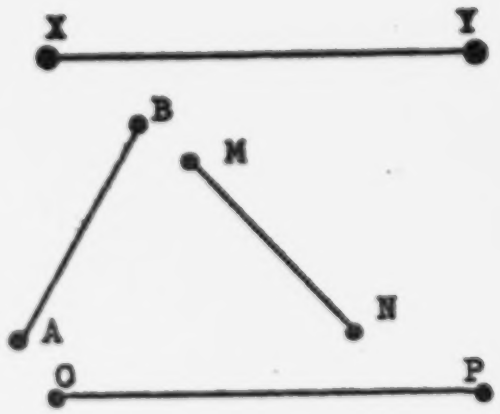
   The child who holds the card that says 12 inches stands up and says, "12 inches measures the same as one foot."

4. Continue the riddles: "I'm thinking of something that measures the same as 36 inches."

   "I'm thinking of something that measures 4 inches smaller than 3 feet."

5. When children have played for a while have them change cards and start again.

Term "Line Segment"



Draw line segments on the board and label the end-points with letters. Find out which is longer, shorter, etc. Have the children measure with string, foot ruler, and yardstick.

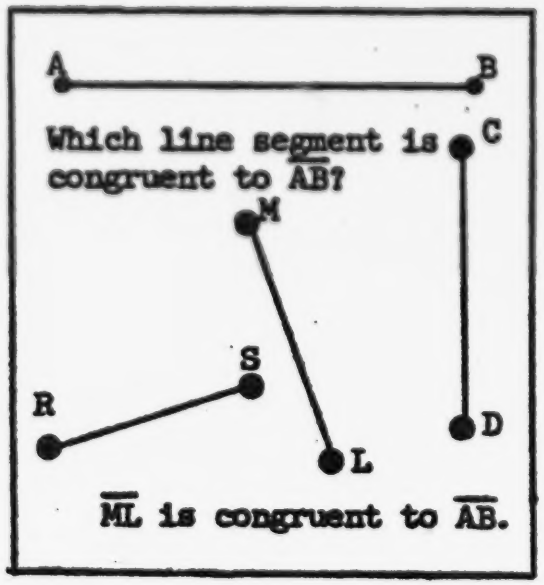
Have the children draw lines eleven inches long and one foot long on the board and again discuss which is greater and which is lesser. SRA p 201

Ask other questions: "Would it be better to buy 2 inches or 2 feet of ribbon for a hair bow?" "Why?"

"Tommy had 2 sticks, one seven inches and one six inches long. If he put them end-to-end, would they be longer or shorter than a foot ruler?"

"If I have a piece of string 1 foot long, how many ways can I cut it so that each of the 2 pieces can be measured in whole inches?"

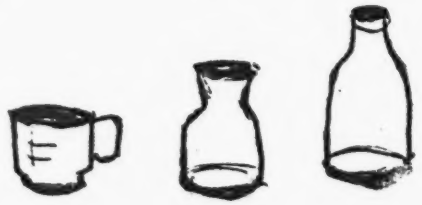
Term "Congruent"



Provide dittoed work papers with line segments. Ask the children to use their rulers to check the lengths of the segments. "Congruent" means that 2 line segments are the same length. (One will cover the other perfectly on tissue paper.)

Liquid measure

Cup, pint, and quart



symbol $\underline{\underline{m}}$

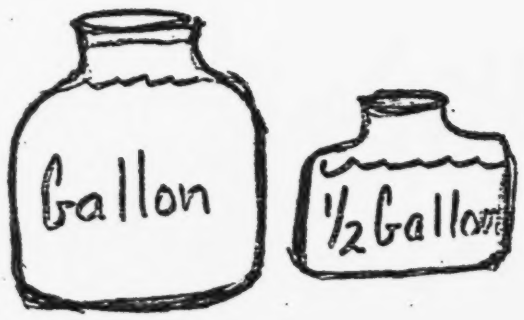
2 pts $\underline{\underline{m}}$	1 qt.
4 pts $\underline{\underline{m}}$	2 qts.
2 qts $\underline{\underline{m}}$	4 pts.
etc.	

Have the children bring to class measured containers for liquid. (cups, pints, quarts, gallon and half gallon containers, etc.) 8:37-40

Experiment with cups, pints, and quarts of colored water making individual charts to record the results. Have them work in teams.

Remind the children that $\underline{\underline{m}}$ means that this is read "two pints measure, the same amount as 1 quart." (2 pts $\underline{\underline{m}}$ 1 qt.)

The gallon, the half gallon



Line up a measuring cup set, a pint container, $\frac{1}{2}$ pint, gallon, quart containers and $\frac{1}{2}$ gallon containers on a table. Make sure you clearly label each container.

Have the children experiment pouring liquid to find out how to line them up in order of amount of liquid (greatest amount to the least amount or vice versa). Discuss the gallon and half gallon and their relative volume when compared with the quart measure (1 gal $\underline{\underline{m}}$ 4 qts, $\frac{1}{2}$ gal $\underline{\underline{m}}$ 2 qts, gal > qt, $\frac{1}{2}$ gal > qt, qt < gal, $\frac{1}{2}$ gal < gal, qt < $\frac{1}{2}$ gal, etc.) What do we buy at the store in gallon and $\frac{1}{2}$ gallon containers? Make a list.

1 quart = 2 pints

1 gallon = 4 quarts

half gallon = 2 quarts

1 gallon = 2 half gallons

1 cup = 1 half pint

1 pint = 2 half pints

1 pint = 2 cups

1 quart = 4 cups

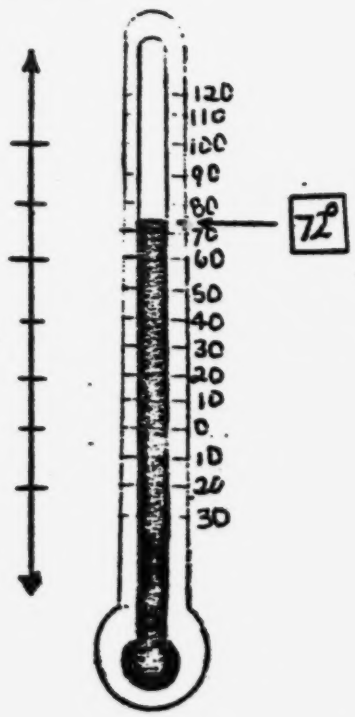
Also make a table which, when finished, should look something like the one at the left.

Sometime during the year (possibly during a study of the market) call attention to the way things are sold. Bring in samples:

eggs - dozen
milk - quart, half gallon, etc.
cream - ½ pint, pint
butter - ¼ lb., lb.
cokes - oz.
etc.

For a culminating activity plan to make refreshments for a party. (Butter, bread, cookies, punch, etc.)

Temperature

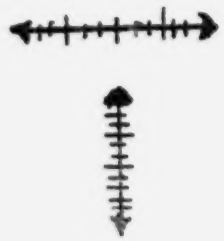


The thermometer is an up-and-down (vertical) number line. What do we use it for? (To tell how warm or cold it is.) Have a large demonstration thermometer with a movable ribbon to represent the mercury column. 10:348 355

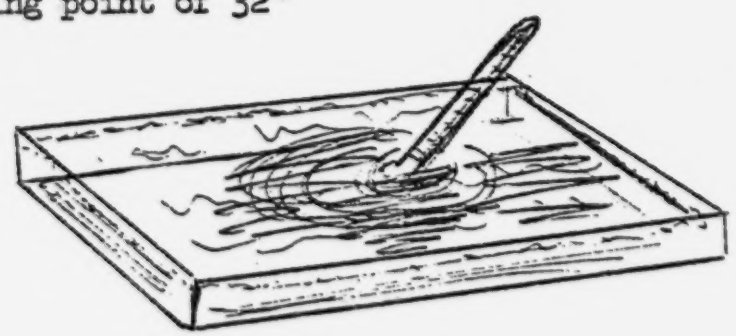
Tell the children that both mercury and alcohol are used in thermometers because both of these liquids expand and contract easily. (This is a science lesson in itself.)

What is the average temperature inside the classroom and outside? Keep a chart of the temperatures two or three weeks before going into the subject more deeply.

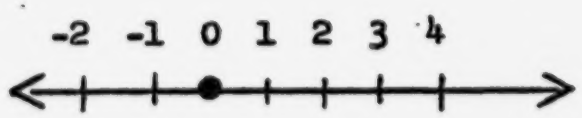
degree = °



The freezing point of 32°



0 ° zero degrees



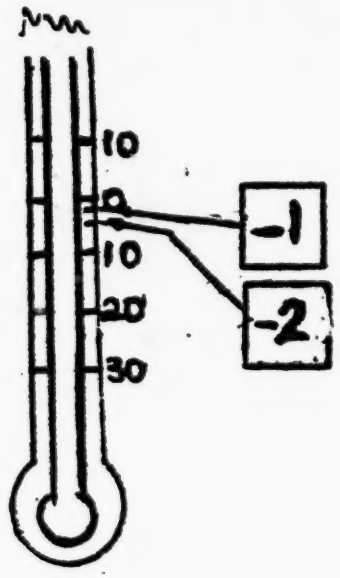
After discussing the outdoor temperature which the class has been recording point out that most temperatures anywhere in the world fall between 0 ° and 130° °.

Just as the number line is divided into units of measurement so is the thermometer. We call each of these equal units of measure a degree. However, on a Fahrenheit thermometer there is mark on the scale for every two degrees. The halfway point between two of these marks indicates one degree.

Take a large shallow pyrex baking pan and fill it with a mixture of rock salt and crushed ice. In its center place a shallow glass pyrex dish filled with water. Then insert a thermometer into the water and set this apparatus in the freezing section of the cafeteria refrigerator. (The teacher should experiment with this herself to insure a well-timed experiment in the classroom.) Just before the thermometer reads 32° bring it into the classroom to see the water freeze. Note the point of 32° (Do not leave the experiment in the freezer too long.)

Ask the children to find out at what temperature a large home freezer usually stores food. (The answer is generally 0°.)

Discuss again the number line with the concept that it can go on forever in both directions. How then shall we name the numeral to the left of zero?



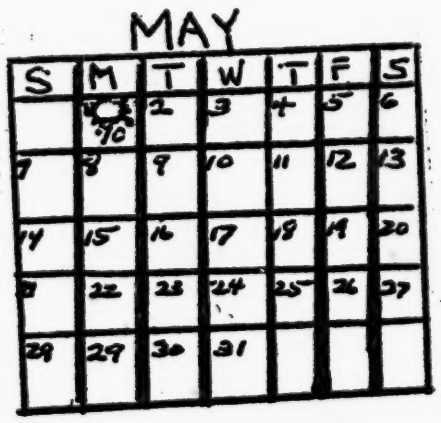
If 0 is less than one, it must be one less than zero. We show this by writing a minus sign, a little higher than usual, and call this negative one and negative two, etc. Turn the number line to a vertical position and you have your thermometer.

Bring in as many types of thermometers as possible to put in the math interest center. 14:12,13

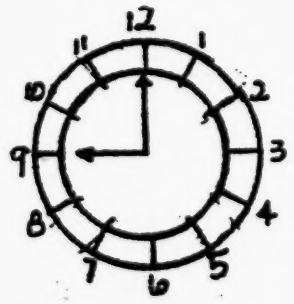
clinical thermometer
centigrade thermometer
candy thermometer
oven thermometer, etc.

Prepare worksheets to follow up "reading" a thermometer. 8:144

Make a calendar for the month. Each day choose one student to illustrate the kind of a day it is and record the temperature below.



Time and the number line

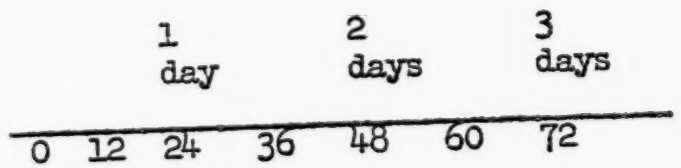


9 o'clock

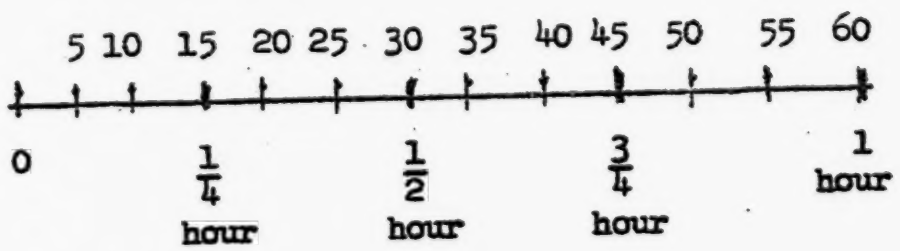
Ask the children to bring to class everything they can find which can be used in measuring, or pictures of measuring devices. For example: thermometer, light meter, scales, ruler, measuring cup, tape measure, egg timer, etc. Have them notice that the scale on all these instruments is much like a number line. 18:7

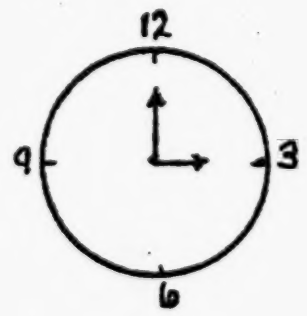
3 hours ago	Now	3 hours from now
7	10	1
8	11	2
9	12	3
10	1	4
11	2	5
12	3	6

There are 24 hours in a day



Telling time to the nearest minute



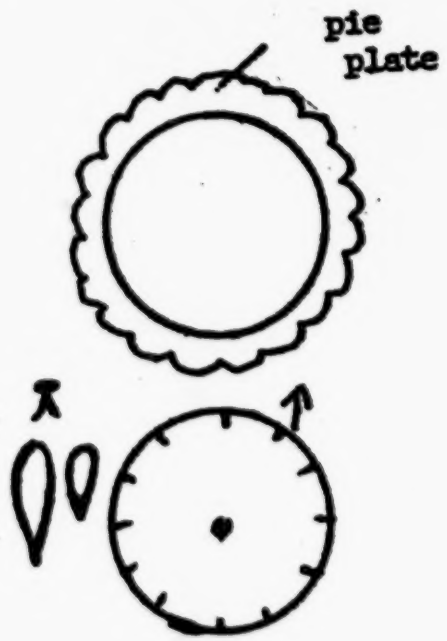


The clock face scale is a number line, but jumps on this number line do not always represent ordinary addition and subtraction. Put three headings on the blackboard. Record the time in any column. Then ask the children to complete the row. Let them use a clock face with moveable hands in this activity.

Construct a blackboard size number line. Have the children help you finish marking points on it. 17:84

Discuss, with the use of the number line, quarter hours and 5 minute intervals. Work with a toy clock at the same time. Have the children stand beside their chairs and then sit down when they think a minute has elapsed. Then check the clock in a "second try."

Time

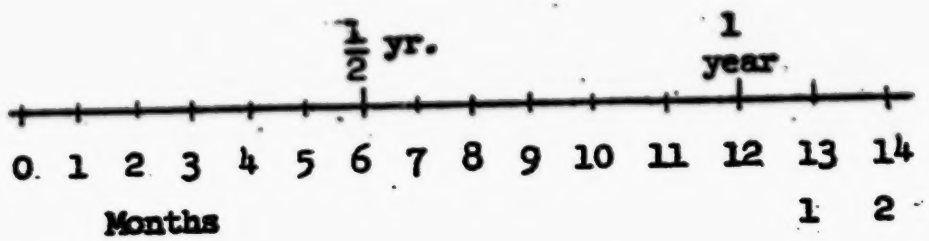


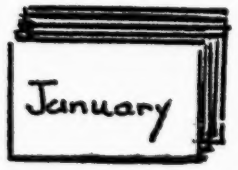
1. Give each child a paper plate - a circle worksheet marked off with points, and a paper fastener.
2. Let the children make their own clock to manipulate during future lessons.
3. Bring an old clock which the children can manipulate. Set an alarm for 5:00. Ask a child to move the hands so that it will go off.

Children mark the numbers on the face and put on the hands of the clock.

Time---with the calendar

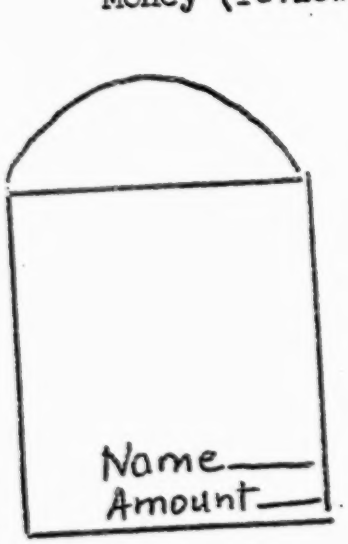
Construct a large number line for months and years.

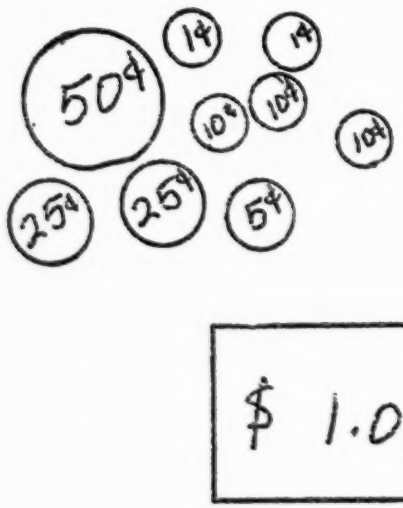


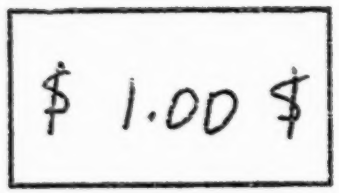


Have the children make their own calendars each month. Ask leading questions: How many Mondays does this month have?
How many days in all? Is this more than last month?
What is the first day of this month?
What month follows this month?

Money (review)







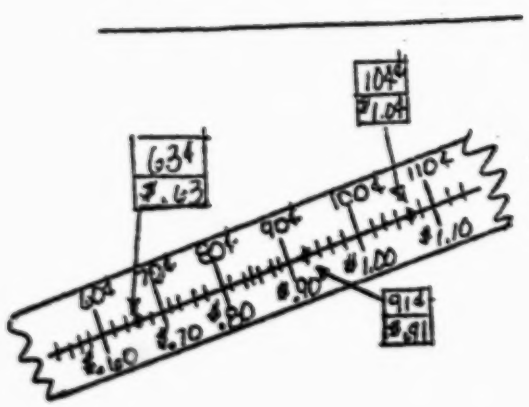
1. This is a small group review activity using real money, introduce the value of a penny and show that 10 pennies equal a dime or 10¢ and 5 pennies a nickle or 5¢.

   The first day's lesson could go up to $1.00, talking about the different coins.

   Pass out construction paper with different sized circles representing the different coins.

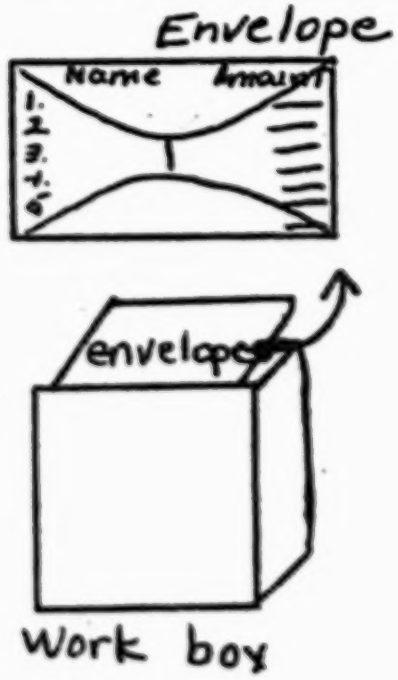
   Have children cut them out and put them in an envelope. The children should count their play money and put the total on the outside of their envelope. These are to be kept at their desks for individual work

Money and the number line



Discuss a dollar bill and the number of pennies, nickels, dimes, etc. which are equal in value. 18:77

Construct a large money number line on wide adding tape which carries two notations. (90¢ and $.90) Pronounce both labels in the same way. When you exceed one dollar this will not apply. Prounce 130¢ as one hundred thirty cents and $1.30 as one dollar and thirty cents.

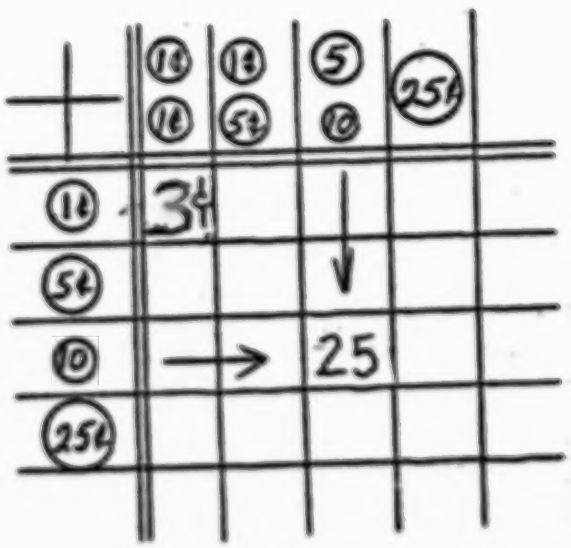


1. Have several envelopes at the game table that contain various amounts of play money.

2. After each child has finished his work, and during free time, he can come up and select an envelope.

3. He writes his name on the envelope and counts the money.

4. He then writes the amount on a line next to his name and returns it to the box. If he wishes, he may do several different envelopes.



Pennies, nickels, dimes, and quarters

1. Give each child a dittoed copy of the chart before the lesson.

2. Ask the child if he knows why the 3¢ is written in the square.

3. Talk about one penny and two pennies have the same value as three pennies or 3¢.

4. Point to the next two columns and write 7¢ in the square (5¢ and 1¢ and 1¢ are 7¢.)

5. Work a few more as a class and then have them do it on their own.

6. If needed let them use play money to clarify this concept.

5¢ can be made with two different combinations of coins. A nickel is worth 5¢. Five pennies are worth 5¢.

Tell the children that in your hand you have hidden 9¢ in coins. "Are you sure you know which coins?"

18:117

We can record in this way.

(1) 0 p, 1 n

(2) 5 p, 0 n

and . . . . . .

(six ways)

18 p 0 n 0 d

13 p 1 n 0 d

8 p 2 n 0 d

8 p 0 n 1 d

3 p 3 n 0 d

3 p 1 n 1 d

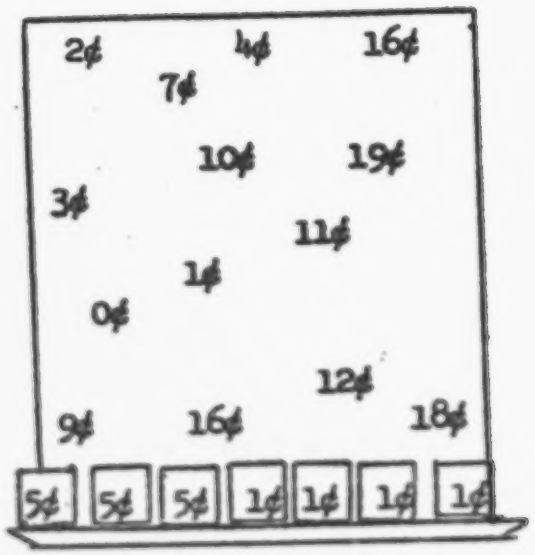
Dimes	Nickels	Pennies	Total No. of Coins
		13	13
	1	8	9
	2	3	5
1		3	4

No---they could be one of several combinations.

The words, "Can you say for sure," are important in playing this game. Ask questions like these: Can you say for sure whether or not I have any dimes? (yes) Can you say for sure that I have at least 5 coins? (yes) Can you say for sure that I do not have more than 9 coins? (yes) Can you say for sure that I have at least one nickle? (no) etc.

Change the coins in your hand and tell them you now have 13¢. After they make a few suggestions ask: "How many different combinations of coins are there that are worth 13¢?" Make a chart.

This type of chart can be made for any chosen amount of money.



Chalkrail

1. Place seven cards on the chalk tray. Each card bears an amount of many.

2. Write amounts of money from 0¢ to 19¢ on the chalk board.

3. Ask a child to come up, choose an amount of money, and circle it on the board. He then must remove cards from the chalk tray until the remaining cards "add up" to the amount he circled.

4. Continue the game until all the amounts are circled.

1. Make chart with money amounts across the top.

2. Choose two coins and list them to the left of the chart.

3. Have the children decide a way to reach the amount of money listed at the top of the chart with the fewest possible coins.

4. The coins can be changed as well as the amounts at the top of the chart.

AMOUNT	4¢	5¢	6¢	10¢	15¢	
Nickels	0	1				
Pennies	4	0				

1. Have children bring in different objects to be used in dramatic play for setting up a store (toys for a toy store, grocery cartons for a grocery store, etc.)

2. Place a monetary value on all objects and put them on shelves set up as a store.

3. Use a change box (cash register) containing play change.

4. Give each child $1.00 in change.

5. During dramatic play the children will be required to make change for their purchases.

6. The store clerk would receive a salary for working (the salary should also be set up by the class.) The children can sell their objects back to the store when they are through with their play.

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O.C.S.E.I.P. SYLLABUS

Grade 3

SE 006 069

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
OFFICE OF EDUCATION

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PREFACE

The Orange County Science Education Improvement Program (O.C.S.E.I.P.) is sponsored by the National Science Foundation and hosted by U.C. Irvine. It is a cooperative venture undertaken by the University of California, Irvine, California State College at Fullerton, the Orange County Schools Office and local school districts throughout Orange County. This syllabus was written by O.C.S.E.I.P. to help teachers teach the best aspects of recent mathematics programs. It is not meant to be another textbook for a new program. Instead, it is meant to be a sharing and synthesis of effective teaching methods. The outline of topics is a minimum coverage which is common to all schools in Orange County. Topics adequately covered in the majority of texts in use are given a minimum treatment in the syllabus.

The first draft of this syllabus was written during an 8 week session at University of California, Irvine during the summer of 1966 by:

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NUMBERS AND NUMERATION

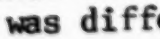
Grade 3

Grade 3

History of numbers and numerals.

One to one relationship

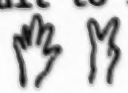
NUMBERS AND NUMERATION

In the Stone Age men knew nothing of numbers. They learned to tell by looking. Their eyes told them that was different from  . Thus the idea of numbers was discovered. As men became more proficient in obtaining food, growing grain, and raising sheep, they invented the tally.

The shepherd used to put down a pebble for each sheep in his flock as it went to pasture. In the evening, when the sheep came back, the shepherd had to check again on his flock. If he had a sheep for every pebble, he knew they had all come home.

Farmers tallyed the passing of time by cutting notches in wood. By tallying people could tell "How soon?" or "Are they all here?" but there was one important question tallying could not answer--"How many?"

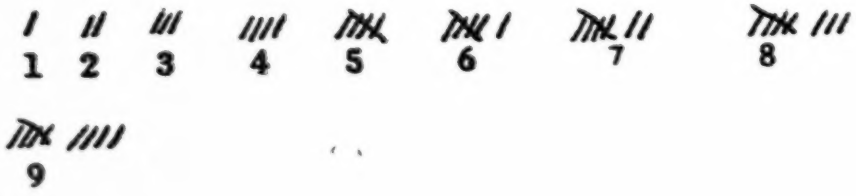
All the shepherds had was a big pile of stones. Was his flock bigger than his neighbors? The two shepherds had to sit down with their tallies and match them pebble for pebble.

In time, people found a better way. They began to tally on their fingers. For two reasons this was a good idea. Fingers are attached to the hands. Pebbles could easily be lost, the fingers could not be. It is difficult to remember •••••••• , but easy to remember 

Finger tallying soon gave numbers names such as "Hand (5) and two on the other hand." When the shepherd counted up to ten a second time, he knew he had twice used up all his fingers. The number of sheep was "Both hands twice." This put a pattern into counting. Every time he counted up to ten, he

began to count again. Some people then and some people now count only to five and then start over.

The next step forward was writing numbers. Some of the first written numbers were used in Egypt about 5,000 years ago. They learned to use tally marks and pictures for numerals.



Ten was noted as a heel bone.

One hundred was a coiled rope.

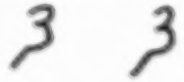
It took eleven pictures and tally marks to write our numeral 146.

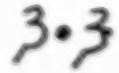
The Romans used letters for numbers. They wrote their smallest numbers with strokes, I, II, III. For 5 they used V, which represents the space between the thumb and forefinger when they were spread apart. Ten was X and 50 was L. For bigger numbers they used the first letter of the number word. Centum was their word for hundred. So they wrote C for 100. Mille meant thousand, so M stood for 1,000. We call these Roman Numerals. They are used in many places.

Long before the Romans used letters for numbers, the Hindus in India were writing on Palm leaves with just

strokes. They began to join the strokes.

If the Hindu wanted to write thirty three, he wrote

If he wanted to write three hundred and three, he wrote 

But this led to trouble, so they wrote it as 

Later this small dot became a circle. The circle is now called Zero.

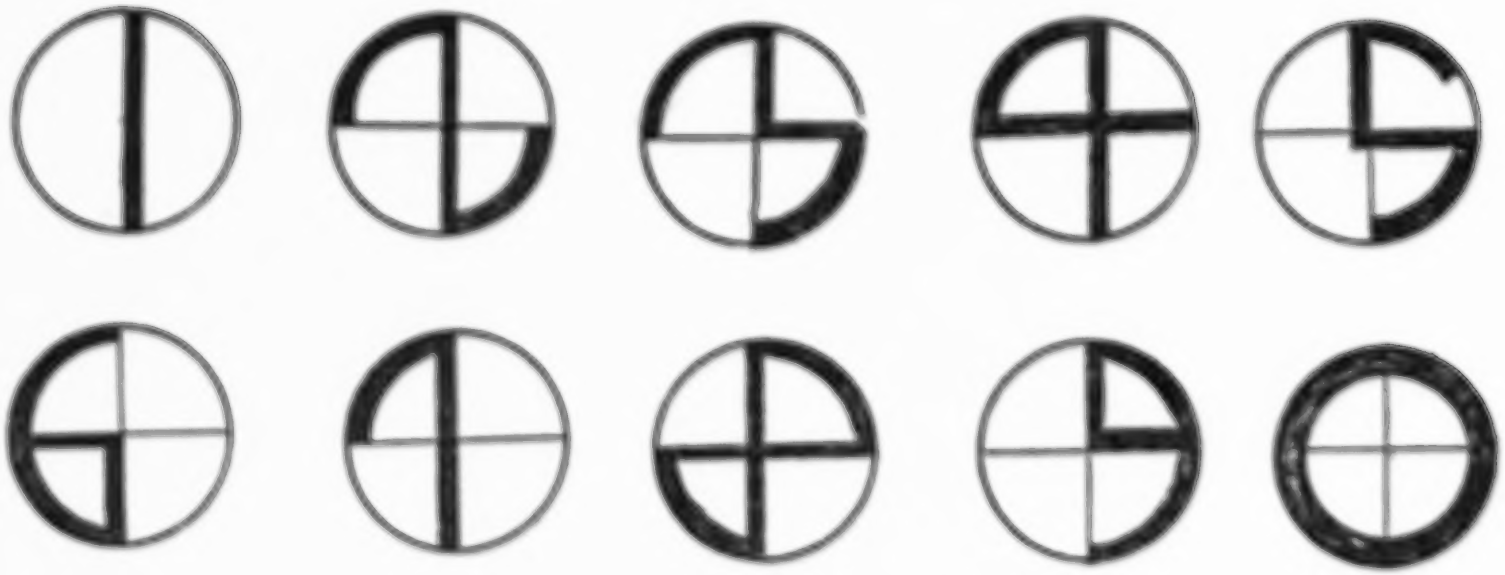
The Hindus were traders. They shared their number ideas with the Arabs. The Arabs carried these west to Spain and from there they spread throughout the rest of Europe. These are called Arabic Numerals.

The shape of numerals changed many times during these Travels. About 600 years ago, printing was invented. It was then that numbers took their present shapes and kept them.

A flannel board story could be made of the above history. It could be used to motivate the children to find out more about numbers and numerals.

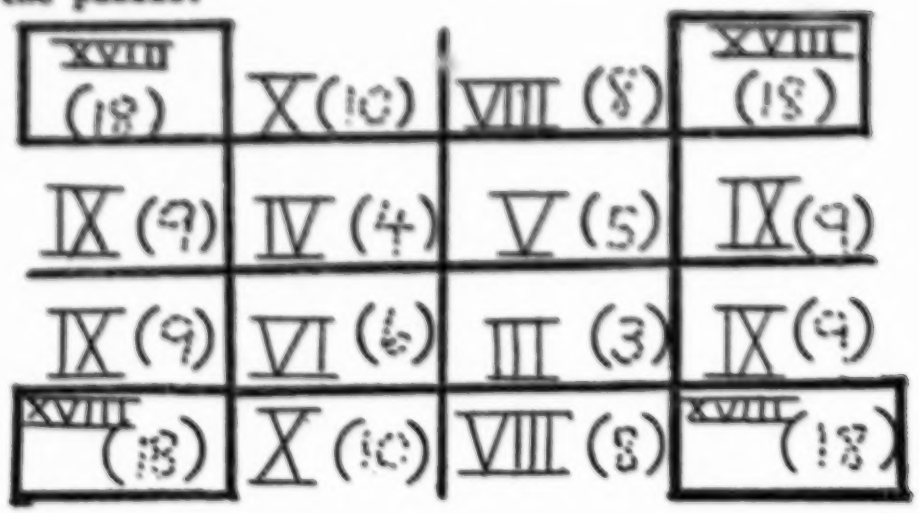
The Arabs first used the circle form for making their numerals.

Pupils might prepare a poster for the bulletin board display.



Roman Numerals I, V, X.

Cross number puzzles, using Roman Numerals, not only give practice in all operations, but keep interest at a high level. In the example below, the children transfer the Roman Numerals into Hindu Arabic and work the puzzle.



Simple addition and subtraction
without carrying and borrowing.

LCD
M

$$\begin{array}{r} XV \\ -XI \\ \hline IV \end{array} \qquad \begin{array}{r} IV \\ +\ V \\ \hline IX \end{array}$$

MDCCCCLXVIII
1968

Dictation of Numbers.
Place Value

	100	10	1		100	10	1	
					1	1	5	
							7	
						2	5	
					1	0	7	

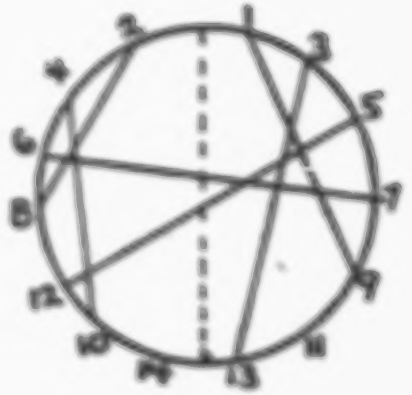
Children need to experience writing numbers that are dictated to them.

Give each child a sheet of lined paper. Direct him to place the paper so the lines are vertical. One space is to be designated ones, another tens, and another hundreds.

Dictate numbers to them and check to see that each is placed in it's proper place.

Review of Odd and Even Numbers.

Draw a circle on the chalkboard. Divide it in half. On one side write even numerals. On the other side, write odd numerals. Ask a child to come up and draw a line from one even number to another. Ask the class, "Is the sum of these two even numbers odd or even?" "Let's try it again before we come to a conclusion." Continue this by adding an odd number to an odd number and, finally, an even number to an odd number.



1 + 9 = 10
3 + 13 = 16

2 + 8 = 10
4 + 12 = 16

6 + 7 = 13
12 + 5 = 17

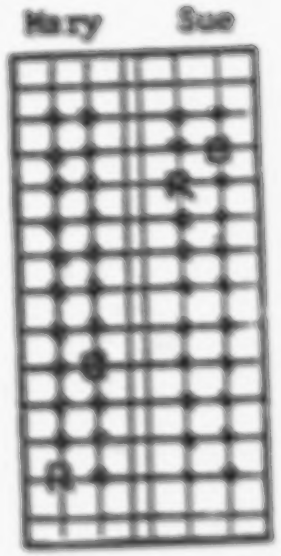
The following conclusions can be made:

even plus even = even
odd plus odd = even
even plus odd = odd

Continue by asking, "I wonder if we can form the same conclusions using subtraction?"

Addition and subtraction concepts and techniques redeveloped and extended.

Place Value.



Mary's score: ☐

Sue's score: ☐

"How many more points does Sue have?"

For keeping score in many game activities, a cribbage-type scoring board may be a pleasant way to use non-numerical arithmetic. 17:99

If the red markers (R) show tens, and the black markers (B) show ones, what is each girl's score?

This may be done on the chalkboard, printed on paper, or a regular cribbage board used.

"In these exercises, decide what fact that you know will help you. Then you can use the pattern to add."

Sets of examples can be arranged for pupils to make observations that will save them time. Start with the left hand example. It should be one that the pupils know well. The next example has one addend 10 greater, and the other addend the same as the first problem. Continue adding tens to take more examples. Skip to higher decades.

Place Value

	a	b	c	d
1)	8 + 6 = ☐	18 + 6 = ☐	28 + 6 = ☐	
2)	7 + 7 = ☐	17 + 7 = ☐	27 + 7 = ☐	
3)	9 + 7 = ☐	29 + 7 = ☐	59 + 7 = ☐	

Reading and writing whole numbers through ninety nine thousand nine hundred ninety nine. Understanding whole numbers through million.

It cannot be assumed that children have a true understanding of place value. In teaching the writing and reading of these numerals, the use of an individual place value chart as well as demonstration charts can be helpful.

10,000	1,000	100	10	1
Ten Thousands	Thousands	Hundreds	Tens	Ones

90,000	9,000	900	90	9
9	9	9	9	9

Dramatize a third grader being in a store with a gum machine in which the child can insert a penny for a ball of gum. He has two dimes and three pennies. He wants five pieces of gum. With two dimes and three pennies, how can he get five balls of gum from the machine?

He goes to the storekeeper and exchanges one dime (10) for ten pennies (ten ones) and now he has enough pennies (ones) to obtain his five balls of gum.

2 dimes + 3 cents becomes 1 dime + 13 cents
20 + 3 10 + 13

8:154-156

Understanding place value through expanded notation of 4 place numbers and column addition.

Example A.
4257 = 4 thousands + 2 hundreds + 5 tens + 7 ones.

Example B.
4257 = 4,000 + 200 + 50 + 7.

Example C.

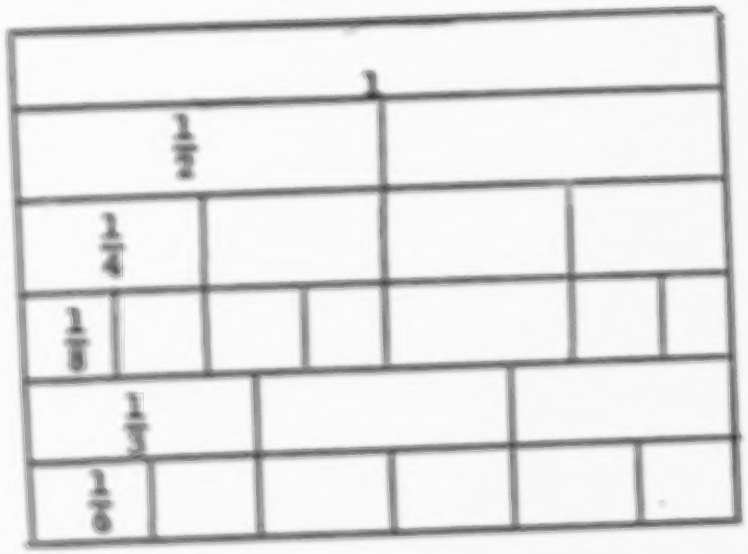
```
 4,000
   200
    50
+   7
-----
 4,257
```

Understanding basic structure in the addition operation should enable a pupil to move rapidly in addition of numbers named with two digits.

Expanded notation is an efficient method for introducing the addition of numbers that are named with two or more digits.

In expanded notation, a number is broken into component parts. In this way, children can better see the structure of numbers and are better able to rename the number in a form more convenient with which to operate.

Fractional Numerals.



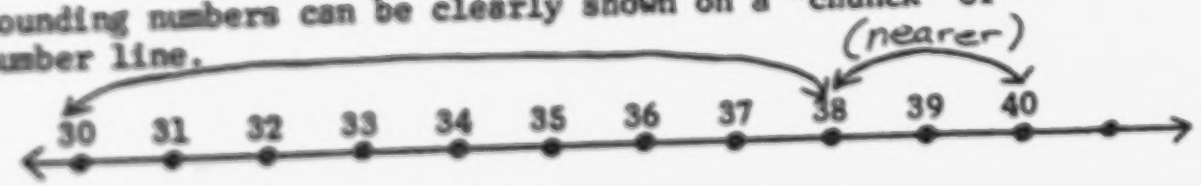
Use of the number line and other means such as acetate overlays using the same unit on each overlay--one with marking of halves, another for thirds, another for each of the following: fourths, sixths, and eighths. Laid one upon the other, it is possible to see other names for a same number.

On such a chart, equivalent fractions can be found and comparisons can be made between fractions of different values. For example, it can be seen that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.

In this way, rows of equivalent fractions can be determined. Furthermore, the relation between values such as 1/3 and 1/6 can readily be seen:
$\frac{1}{3} > \frac{1}{6}$ or $\frac{1}{6} < \frac{1}{3}$

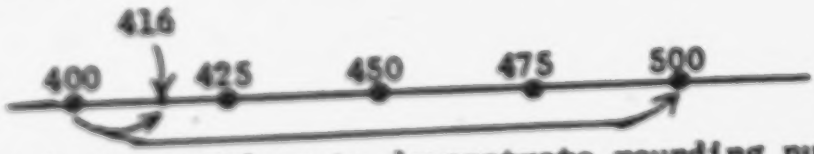
Rounding numbers (to the nearest 10 and nearest 100).

Rounding numbers can be clearly shown on a "chunck" of number line.



The number 38 rounded to the nearest ten is closer to 40 than to 30 on the number line. Therefore, 38 when rounded to the nearest 10, becomes 40.

On another "chunk" of a number line what would 416 be when rounded to the nearest hundred? It would be 400, because 416 is nearer to 400 than to 500.



Ask the children to demonstrate rounding numbers on a large blackboard number line.

ADDITION AND SUBTRACTION - GAMES AND DRILLS

Grade 3

ADDITION AND SUBTRACTION - GAMES AND DRILLS

Addition--Repeated addition

+	
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Mimeographed addition tables can be used as drill on combinations. A child or the teacher chooses the number to be used each time. The children add the numeral at the top of the page to the numerals on the left side.

Addition and Subtraction Drill.

Flash Card Relay.

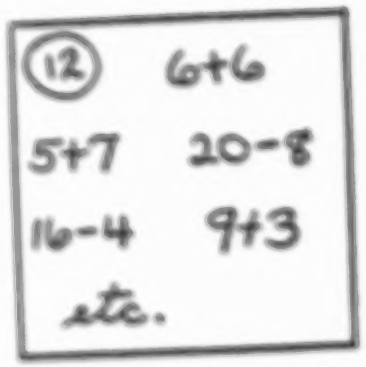
Arrange an uneven number of flash cards on the chalk tray. Two children play, one at each end. At a signal, they write answers on the chalkboard above the cards, working toward the middle. The one reaching the middle card first without mistakes wins a point. If played on a team basis, the point will add to the team score.

This game can also be used for multiplication and division drills.

Renaming Numbers.

Provide each student with a sheet of paper. Each child writes a numeral on the paper and circles it. Instruct them to write as many names for this number as they can think of.

This can also be made into a game by seeing who can write the most names for this number.



An addition grid provides for an interesting variation for drill. For ease in writing numbers, the squares should be 3/4" to 1" in size. To introduce the grid, use numbers in sequence as in figure 1. Then, use numbers not in sequence as in figure 2. Progress from this to the variation suggested on the next page.

Addition Grid.

+	1	2	3	4	5	6	7	8
1								
2								
3								
4								
5								
6								
7								
8								

fig 1

+	5	8	3	7	4	6
4						
7						
5						
8						
6						
3						

fig 2

Addition and Subtraction Concepts redeveloped and extended.

Let pupils fill in the missing signs, rather than finding a number.

Missing Signs.

$6 \triangle 3 = 5+4$ $\quad 17 \triangle 9 = 2+6$ $\quad 10 \triangle 7 = 12-9$ $\quad 7+7 = 17 \triangle 3$

$4 - 2 + (8-3) \underset{\cdot\cdot}{\overset{?}{\cdot}} 6 + 1$

$19 + 4 \underset{\cdot\cdot}{\overset{?}{\cdot}} 22 + 2$

$16 - 4 \underset{\cdot\cdot}{\overset{?}{\cdot}} (3x3) + 4$

Fill in equations with "greater than" "less than" or "equals" signs.

$> < =$

It is important for pupils to learn to recognize an incorrect mathematical statement. "True or False" is a game that can be played to develop this. A box containing a set of true and false number sentences on cards is used. A child picks a card, shows it to the class, and tells whether it is true or false. If false, he selects the correct answer from a set of numbered cards. 4

7+3 = 10	4+5 = 10
"True"	"False"

Chain Reactions

1+2 → 2 3 5 8 13 21 34 ← 2+3

__ __ __ __ 10 16 __

answer to example B.

2 3 4 6 10 16 26 42 68

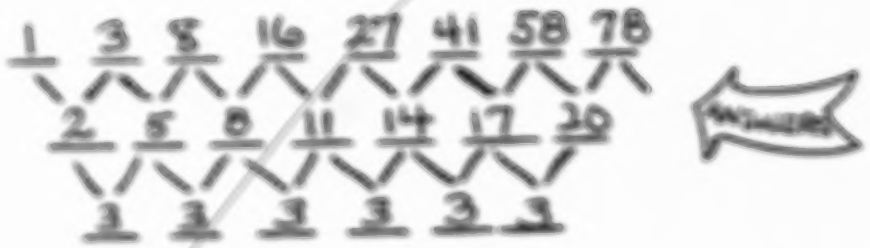
"Chain reactions" are forms of number puzzles that perform repeated operations on numbers. The answer to one "question" becomes the next "question." 22:ix

Many rules may be used. For example, a simple puzzle is given. The rule for this one is "each number is the sum of the two numbers before it." The first two numbers are 1 and 2. The next will be 3.

The one after 3 will be the sum of 2 and 3, etc. The reaction can go both directions. Keeping the same rule, but changing the numbers, a new chain reaction is begun. (In this example, the number left of 10 is (16 - 10) or 6. The next one is (10 - 6) or 4.

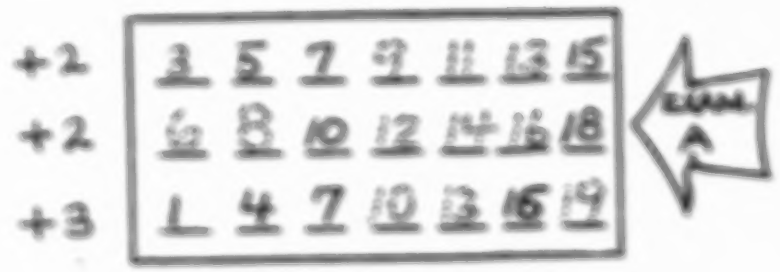
Puzzles may become more complex and use more elaborate rules.

In this example, the rule is, "Each number must be the difference of the two numbers above it; the bottom row is always the same."



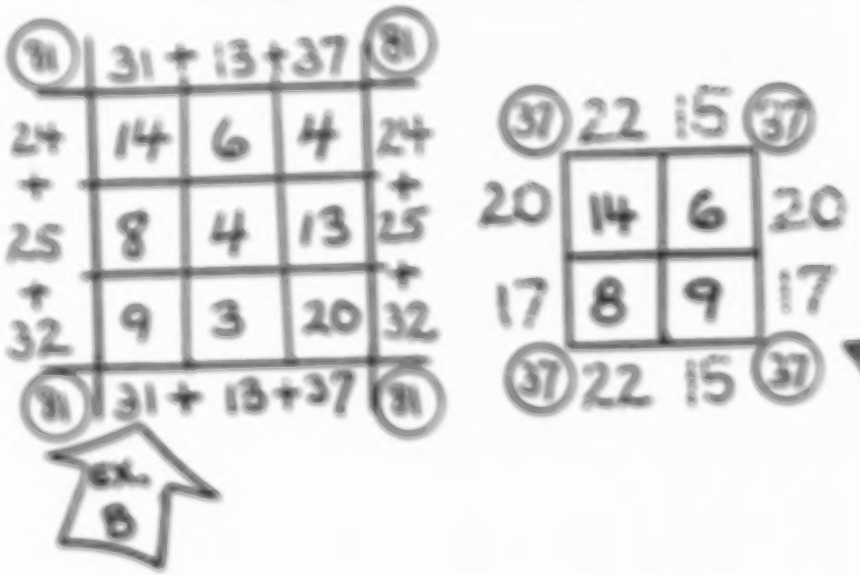
The value of such puzzles is that the child can go someplace and can "see" the pattern developing. Each answer depends on the one before, and a little slip puts him far off. An error "shows up" very quickly in the pattern.

Addition--Progressions.



Example A shows three "chain reactions." The first and second are built by adding "2," the third is built by adding "3." The directions to the pupil are "Find the pattern and finish."

Example B is a cross number puzzle which develops skill in the use of the associative property of addition. This may be varied by leaving blanks in the square and can be carried to having the pupil transpose the numerals from this math sentence to the squares. →14 + 6 + 4, ↓14 + 8 + 9, →8 + 4 + 13, ←20 + 3 + 9, ↑20 + 13 + 4, ←13 + 4 + 8, ←4 + 6 + 14.

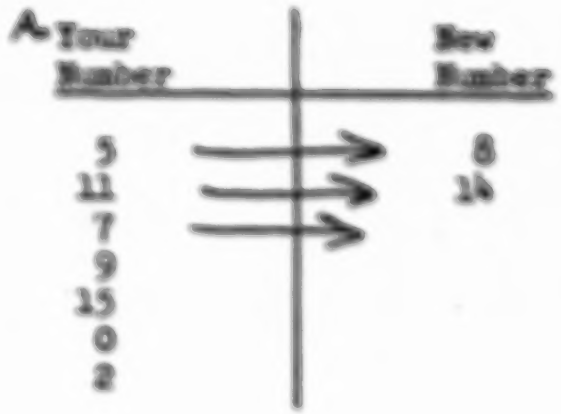


This puzzle develops skill in adding the following math sentences while applying the commutative law.

→14 + 6, ↑8 + 14, ↑9 + 6, →22 + 15, ↑+17 + 20, ←6 + 14, ←9 + 8, ↓14 + 8, ↓6 + 9, ←15 + 22, ↓20 + 17. The ears display an interesting pattern.

22:xui 82

A.



Many of these puzzles can be drawn on a slick surface with a felt tip pen. Pupils write their answers with grease pencil or crayon. This gives valuable, enjoyable work to the students who need a "little extra." Interest will drop off if puzzles aren't changed frequently. 11

Much can be learned by "silent teaching." Pupils who are normally unresponsive, or are "covered over" by the talkative pupils, begin to participate. Some pupils do not understand "math talk", but can see and make conclusions if they are confronted with a thought-producing silent lesson. 22

B.

Your Two Numbers		New Numbers
3,	5→	9
2,	1→	4
6,	8→	15
1,	1→	3

C.

6,	2→	4
1,	3→	2
2,	2→	
4,	5→	1
6,	7→	1

"What's my rule" can be played by all and may be as easy or as difficult as the class needs. In the simplest form, one operation is performed. For example, the rule in (A) is "add 3 to the number." Ask a child for a number, say, smaller than 15. Write his number and then your "new" number. (You might say "5 suggests 8". "11 suggests 14," etc. Any pupil who feels he knows what the rule is may not answer by telling the rule. (In fact, there are many "names" for one rule: e.g., "add 5 and subtract 2, and 1 and then add 2, etc.) Instead, he may say the number he thinks is "suggested" by a given number. He may not say "eleven suggests 14 because you add 3," but may say "11 suggests 14." He must then stay silent, and think up a rule of his own to try when its his turn.

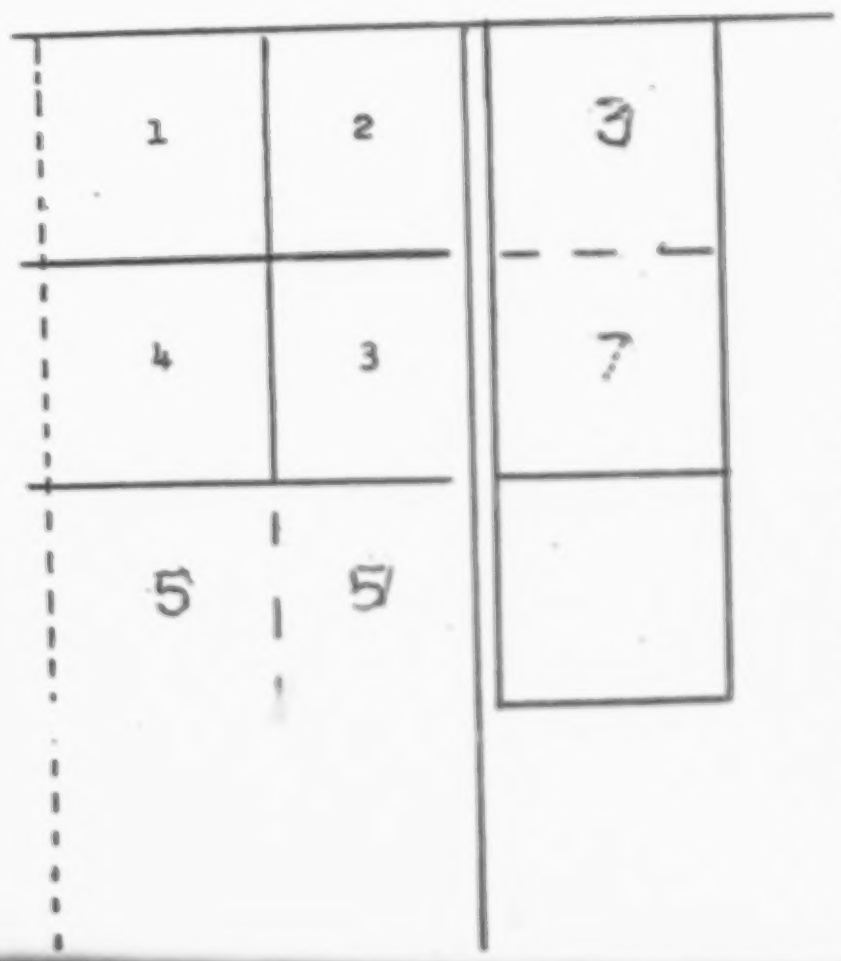
More difficult rules can be made, involving two or more given numbers. (See examples B & C). In B, the rule is, "Add the two numbers, and one more." In C, use subtraction. 11:262

Extension of addition and subtraction with 3 place and 4 place numbers with or without grouping sums.

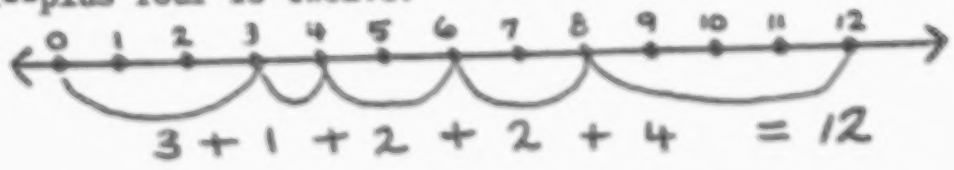
The math program in grades 1 and 2 has used aids in teaching basic facts and has employed games and techniques for enjoyment in learning addition, subtraction, expanded and place value.

The same techniques are a necessity in maintenance of quick computation and in development of extended activities. If the students have thoroughly mastered the concept, they may be challenged with addends up to 7 places.

Column addition defined through a mathematical sentence. $3 + 1 + 2 + 2 + 4 = \square$

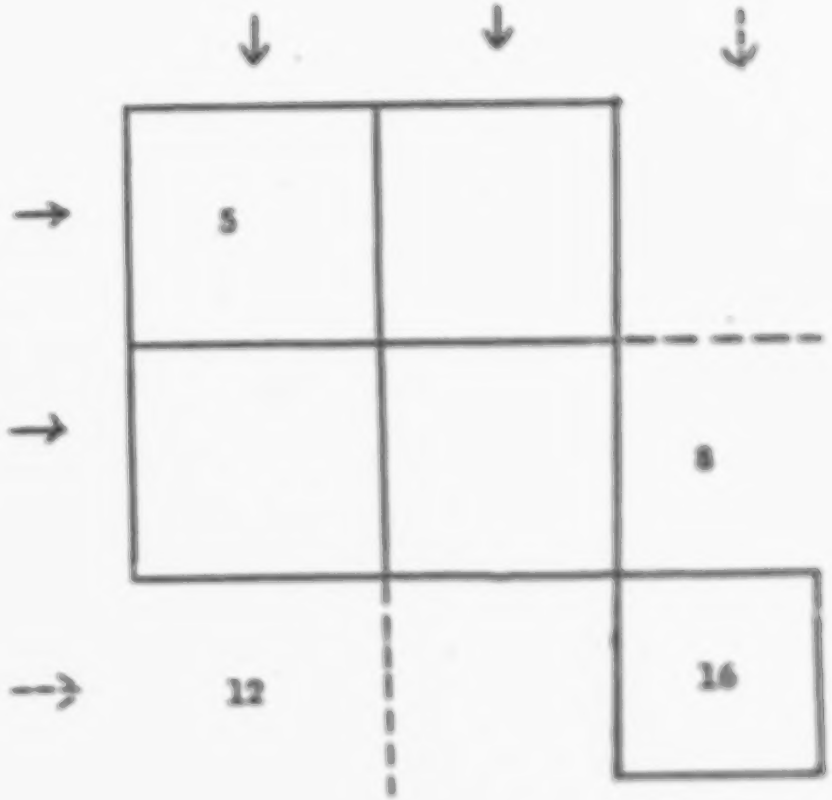


Have children verbalize as they add the number sentence. Three plus one is four--plus two is six--plus two is eight--plus four is twelve.



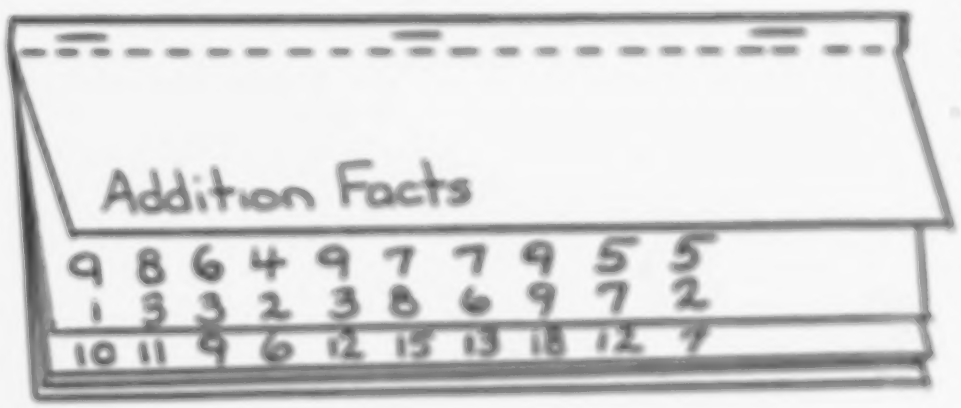
Pupils who have not used cross-number puzzles before may be amazed by doing their first such work. Explain that they are to add in the direction of the solid arrows, and put the sums outside the box.

The broken-line arrow tells you to add these sums. 3:12
Some pupils will be surprised that the two totals will be the same. After several puzzles, let pupils make up their own, to see if "special numbers" must be chosen, or if any will work. If pupils want to find out why the totals are always the same, set up some real objects or markers in the arrangement of a cross-number puzzles. "Sweep" them into groups at the end of the rows, and then "sweep" these piles into a joint group. "Do we get the same objects if we start out by "sweeping" down and then across? (Of course! - They're the same number of objects no matter how you group them!)



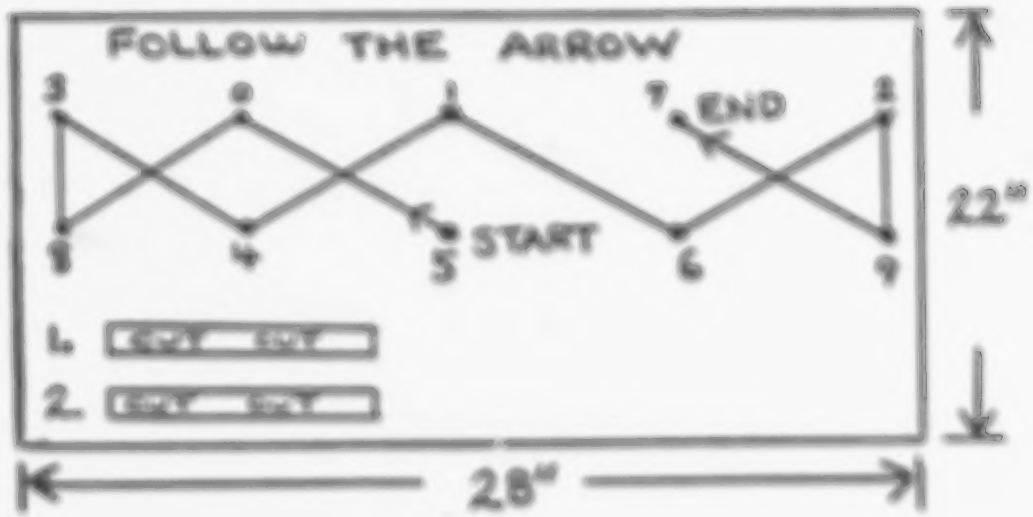
By leaving blanks in some of the four cells, the pupils will need to reconstruct by using subtraction.

Teaching Machine Drill of
Number Combinations.



Staple narrow strips of paper on one long side to make a slim book. Pages 1, 3, 5, and so on, contain number combination problems. Page 2, 4, and so on, have the answers. The child writes answers to the first page on a sheet of paper as quickly as possible, opens the second page, and corrects his own problems. Encourage the children to correct with a red pencil, then do again any page where they made mistakes. Have a series of these books graded from easy to more difficult.

The chart can be clipped over the chalkboard. Operations can be written with chalk in the cut-out areas of the chart. Other charts can be made with different starting points.



A commercially-made kit of cross number puzzles is available. One sample of their format is shown, giving a puzzle blank and the clues to match. Teachers can produce their own of a type similar to this, or purchase the set.

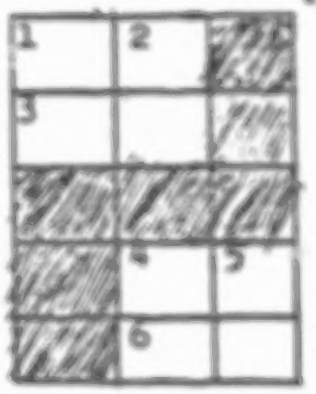
SRA Cross Number Puzzle Kit.

Across

1		12 + 15
3		12 + 22
4		42 + 43
6		13 + 23

Down

1		11 + 12
2		32 + 42
4		41 + 42
5		23 + 33



Addition and Subtraction Drill in Number facts.

+3	
9	12
5	8
10	13
2	5
6	9
4	7
8	11

+3	
9	12
5	8
10	13
2	5
6	9
4	7
8	11

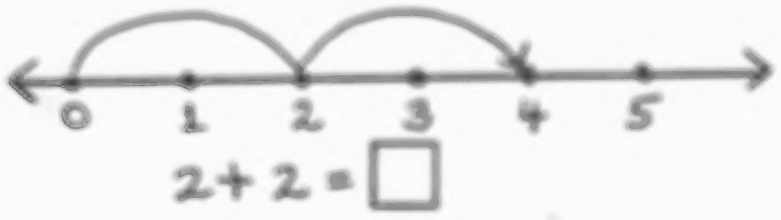
Choose some pupils for board work while the others work on paper at their desks.

Add the number at the top of each number on left hand side. Put the answers on the right hand side.

After the process is secure in the minds of the pupils, have those at the board erase the left-hand column, leaving the right hand column of answers. Then write in the answers. This is a good demonstration of the relationship between addition and subtraction.

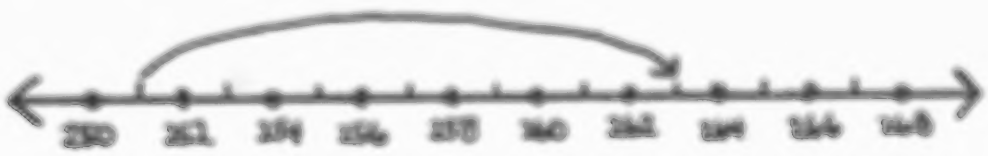
After multiplication and division work has been developed the T's can be adapted to use for learning their number facts.

Number Line to Illustrate Addition and Subtraction.



The use of a numberline can be extended to show operations in all levels of addition and subtraction. For some pupils, the most simple combinations can be shown to advantage on a numberline. The "in any order" grouping is also readily shown.

It is not always necessary to show the "zero" portion. A segment may be shown that includes just what is needed.



Number Line

$$\begin{array}{r} 175 \\ +\ 23 \end{array} = \text{2 tens} \cdot \text{3 ones}$$

$$\left(\begin{array}{l}\text{Down} \\ \text{2 rows}\end{array} \cdot \begin{array}{l}\text{Right} \\ \text{3 spaces}\end{array}\right)$$

$$\begin{array}{r} 53 \\ +\ 23 \\ \hline \end{array} = \text{2 tens} + \text{3 ones}$$

$$\left(\begin{array}{l}\text{DOWN} \\ \text{2 ROWS}\end{array} + \begin{array}{l}\text{RIGHT} \\ \text{3 SPACES}\end{array}\right)$$

The number line may be shown as a chart in the form of a grid as shown.

In this form, addition and subtraction can be shown by counting of single spaces (ones), and by jumping entire rows. (tens)

Using this method, pupils can check more difficult problems after doing the regular computation.

Below is an alternate chart that can be made. *Children work equally well with both.

Addition and Subtraction of Money.

Choose two players to stand before class and play "adding machine." One player chooses a set of five coins and calls off their names. His partner must call the total. This may continue until a wrong answer is given; then new players are selected.

Example:

First Player	Second Player
Quarter	Twenty-five cents
Dime	Thirty-five cents
Nickel	Forty cents
Penny	Forty-one cents
Penny	Forty-two cents

Give pupils practice with different coins.

In a cafeteria situation have pictures of food with prices attached. Items can be on table or pictures of food clipped on board. Each child has a sheet of paper money which he must cut out and label (total amount $3.50). A few children at a time may go to the "cafeteria" and decide on five items they wish to buy, keeping the total amount within their budget. When children return to their seats, they must paste coins on their papers to show what coins were used and what coins were left. The child is to also list the items bought and the cost per item.

Example:

$1.42	(doll)
.45	(dress)
$1.87	(total money spent)

Shopping List buying.

If you use a "store" setup to give pupils experience with money, let them make up their own. shopping list from a newspaper (or price list that they make up, written on the board.) Each child can supply a few empty cans or boxes, and the "shoppers" keep track of how much they think they're spending. An able student can be the "checker," and perhaps use one of the inexpensive pocket calculators for a "cash register." Pupils check to see if they were correct.

Catalog buying

For a "free-time" activity, supply several old catalogs. At a "math center" pupils can decide what to buy for Christmas presents, school, gifts for the family, etc., for, say, $15.28 total.

Toys Purchases

To work subtraction into "buying games," have a discount day. For example, every item under a dollar is discounted 3¢, over a dollar 11¢ off. Run off enough copies of a price list of toys for the group or class. Each pupil decides how much he can afford, and selects a group of toys within his price range.

Addition and subtraction concepts and techniques redeveloped and extended.

Blue	Dollars
Red	Dimes
Green	Nickels
Yellow	Pennies

The pupils may be interested in an Indian device for recording number values without using what they know as "numbers." Peruvian Indians used a set of knotted cords called a quipu (kē-pu). 17:101

Make several for pupils to figure out. Let colors of yarn represent, say, blue for dollars, red for dimes, green for nickels and yellow for pennies. Pupils who think they know the answer whisper it to the teacher without telling the others

Addition of halves, fourths, eighths, thirds, and sixths with sums not greater than one.

A numberline can be used to show fractions, and pupil's work pages can be made using the numberlines. It is recommended that the unit (0 → 1) be the same for each series of numberlines.

Example A:

Example A: $\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$

Example B:

Example B: $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$

Example C:

Example C: $\frac{3}{8} + \frac{3}{8} = \frac{6}{8} = \frac{3}{4}$

Comparing this numberline to Example B it can be shown that $\frac{6}{8} = \frac{3}{4}$

Example D:

Example D: $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

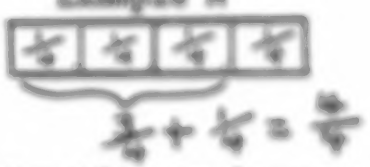
Example E:

Example E: $\frac{1}{6} + \frac{3}{6} = \frac{4}{6} = \frac{2}{3}$

Comparing this numberline to Example D it can be shown that $\frac{4}{6} = \frac{2}{3}$

Fractional numbers 3/4 + 1/4.

Example A



$\frac{3}{4} + \frac{1}{4} = \frac{4}{4}$

Subtraction as inverse operation of addition.

Example B

$\frac{2}{6} + \frac{1}{6} = \frac{3}{6}$ $\quad \frac{3}{6} - \frac{1}{6} = \frac{2}{6}$

The fractional pie has long been and continues to be used to show relationships and operations of fractional number. Not so often has the blocked unit been used. This method, however, can show either addition or subtraction of like fractions.

22: 100, 127-131

4: 308-320

This example shows both addition and subtraction of like fractions, since subtraction undoes what addition does. In subtraction of like fractions the sum (a minuend) should not exceed one.

Fractions--relationships

Make the following true sentences use $=$, $>$, or $<$.

$\frac{1}{3} \square \frac{1}{2}$ $\quad$ $\frac{1}{6} \square \frac{1}{8}$

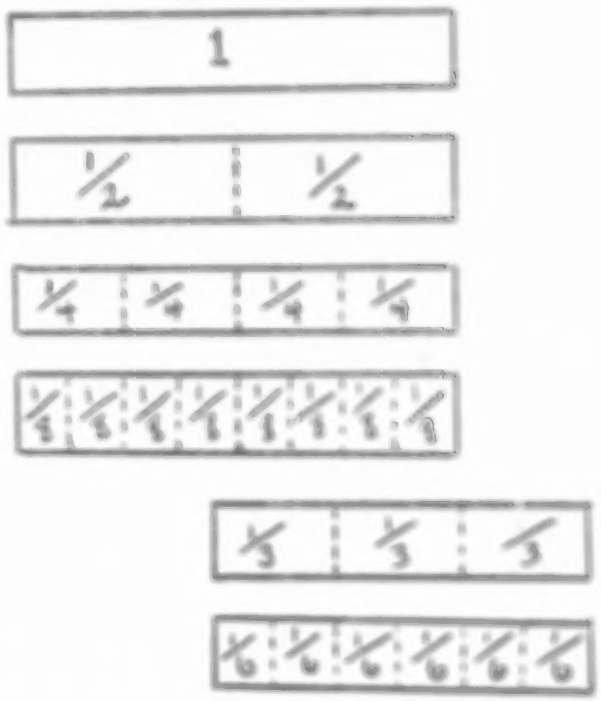
$\frac{1}{2} \square \frac{1}{4}$ $\quad$ $\frac{1}{2} \square \frac{2}{4} \square \frac{3}{6}$

$\frac{2}{4} \square \frac{2}{3} \square \frac{4}{6}$

Numberlines as per examples on the previous page will assist the pupil in the completion of mathematical sentences.

Class problems can include some open sentences that involve missing relationship signs.

Comparisons of fractional parts.



Manipulation of fractional pieces can help children gain better understanding of fractional relationships.

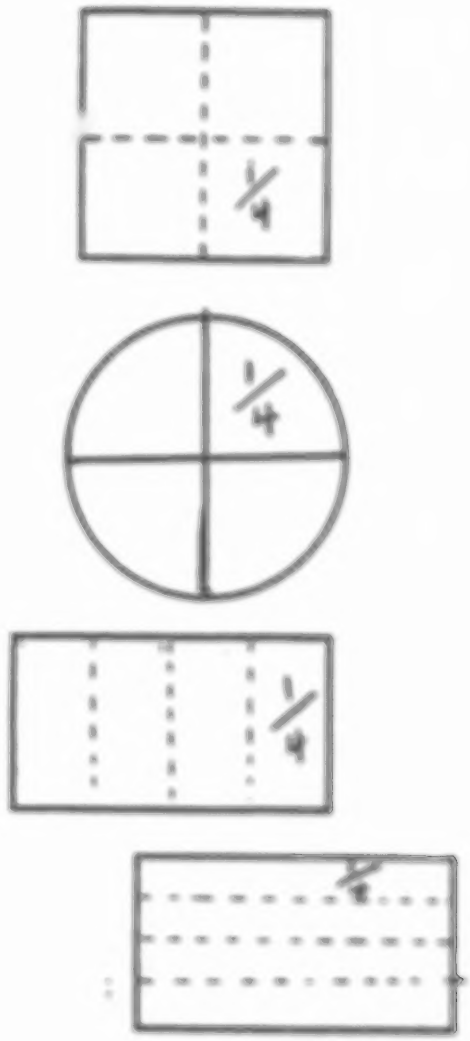
Each child will need six different colored strips of paper (2" x 8") which will be cut into fractional parts as directed.

1. Label one length as 1.
2. Fold one length to find the half of the whole. Label each as $\frac{1}{2}$, when cut.
3. Fold one length to find the fourth of the whole. Label each one as $\frac{1}{4}$ when cut.
4. Fold one length to find the eighth of the whole. Label each one as 1/8 when cut.

The fractional parts of 1/3 and 1/6 require use of a ruler to find the correct part of the whole. Measure, fold, and cut to find 1/3 and 1/6.

Each child now having his own fraction kit can use it to compare size relationships of fractional parts.

Addition of fractions.



Fraction kits can be made in several shapes. It is not wise to use only one shape when working with fractions. Students get the idea that "a fraction is a part of a circle." Circular, square, and rectangular shapes all work well.

An envelope should be provided to store the parts. Use the paper cutter on rectangular work.

MULTIPLICATION AND DIVISION

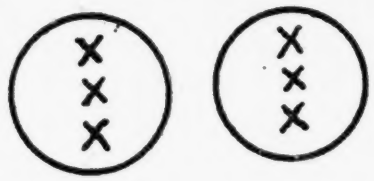
GRADE 3

GRADE 3

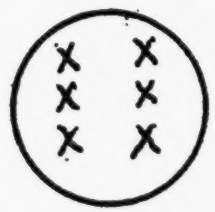
MULTIPLICATION AND DIVISION

Development of the concept of multiplication and division occupies a major percentage of the third grade mathematical program. The process was introduced in second grade. The third grade teacher cannot assume that the process is thoroughly understood by the pupils and should review, reintroduce, redevelop and extend experiences. They should carefully utilize concrete objects, semi-concrete experiences with pictures, and then the abstract algorism. Multiplication and division combinations, products through 81, are developed in the third grade. Division with one place divisors, two and three place dividends, and multiplication utilizing 2 place and 3 place multiplication are also developed.

Multiplication through set union.

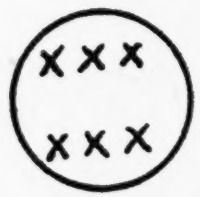


Join two sets of 3

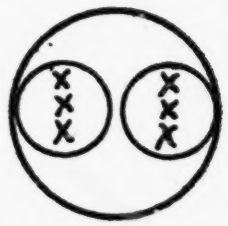


$2 \times 3 = 6$

Division through set partitioning



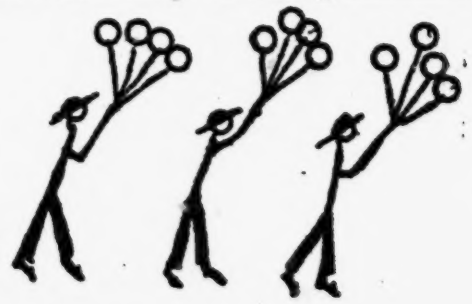
$6 \div 2 = 3$



$3 \times 2 = 6$
$6 \div 2 = 3$

Division as the inverse of multiplication can be shown effectively by using a game of jacks. This will help in understanding even-division (divisability) as well as division with remainders.

One-to-many correspondence.



Multiplication as repeated addition.

$4 + 4 + 4 = 12$

$3 \times 4 = 12$

Division as repeated subtraction.

$$\begin{array}{r} 12 \\ -\ 4 \\ \hline 8 \\ -\ 4 \\ \hline 4 \\ -\ 4 \\ \hline 0 \end{array}$$

$12 \div 4 = 3$

$12 \div 3 = 4$

Multiplication is repeated addition of like groups.

Since all children do not learn in the same way, the use of various materials and methods will help meet the needs of more children. Use school materials, such as hooks, erasers, pencils, and rulers for experimenting with the combinations in multiplication and division.

Multiplication is a process that resembles addition. However in multiplication the groups must be of like size.

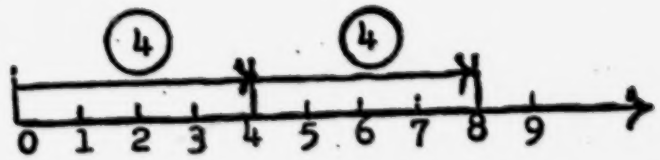
All the following groups can be added. Put a check after the ones that can be multiplied.

The wheels on three bicycles __x__
Six oranges and three apples _____
Three pencils and two pencils _____
The arms on six boys __x__

A worksheet with statements similar to the above may be given to the children as an activity.

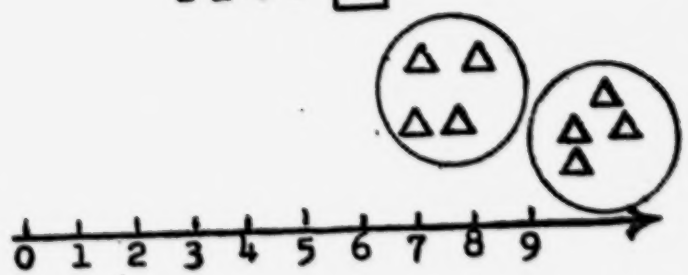
Multiplication and division concepts and techniques redeveloped and extended.

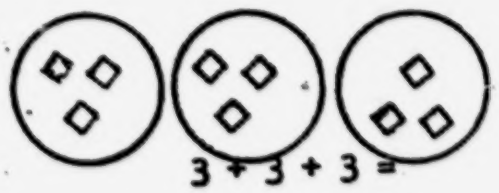
Number line



4 + 4 = 8

2 x 4 = □





3 + 3 + 3 =

3 x 3 = □

Have pupils show multiplication as repeated addition using number lines. Give them a few examples, and let them see what to do by example rather that instruction.

3:49

What patterns do you see?

A.

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30					
7	0	7	14	21	28	35					
8	0	8	16	24	32	40					
9	0	9	18	27	36	45					
10	0	10	20	30	40	50					

B.

X	0	1	2
0	0	0	0
1	0	1	2

A. Pupils can make their own multiplication grids to include 10.

B. (Use $\frac{1}{2}$" graph paper. Patterns are easier to see when the sections are equal.)

c. Patterns
In the 5's row the answers end in 0 or 5
In the 9's row the numerals in each answer add up to 9. e.g. 18 1 + 8 = 9

27 2 + 7 = 9

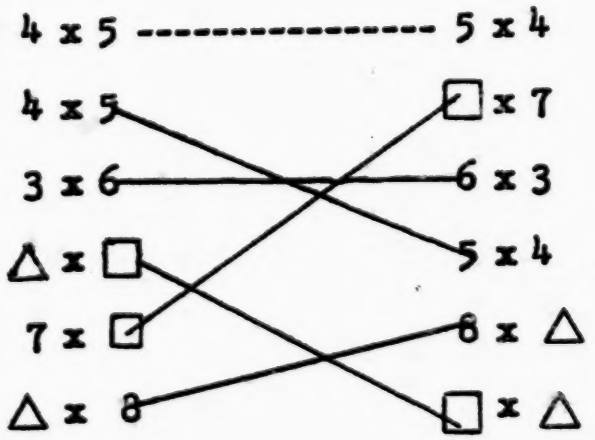
36 3 + 6 = 9

In the 1's and 2's rows look for fractions that are equal $\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$

(Other fraction patterns are found in the other rows. These fractions are in 4th and 5th grade work.)

The following may be used as an activity for a pupil worksheet.

Draw lines to connect two ways of naming the same number.



Multiplication

Divide the room into two baseball teams. Each team chooses a pitcher. The rest of the children on the team are batters. Designate four points in the room as first base, second base, third base, and home plate.

A child comes to bat. The pitcher calls out a multiplication fact or he may flash a card with the fact on it. If the child, who is batter, responds with the correct answer, he advances to first base and the second child comes to bat. If the second child responds correctly he advances to first and the child on first base goes to second base and so on.

If a child does not respond correctly, he strikes out (sits down). When a team has three strikes, it is the other team's time at bat. As many innings can be played as the time allows.

At the end of the last inning the team with the most children crossing homeplate wins the game.

These properties of multiplication can be discovered through various classroom experiences.

The properties of multiplication

The commutative principle $2 \times 3 = 3 \times 2$
$20 \times 3 = 3 \times 20$

The associative property $(4 \times 5) \times 3 = 60$
$4 \times (5 \times 3) = 60$

The distributive property $2 \times (3 + 4) = 14$
$(2 \times 3) + (2 \times 4) = 14$

Multiplication is commutative.

A.

0000 2 x 4 = 8
0000 4 x 2 = 8

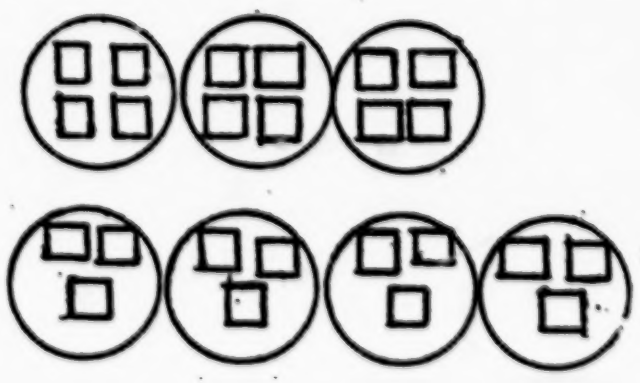
00000 3 x 5 = 15
00000 5 x 3 = 15
00000

B.

4 + 4 = 2 + 2 + 2 + 2
(2 x 4 = 4 x 2)

5 + 5 + 5 = 3 + 3 + 3 + 3 + 3
(3 x 5 = 5 x 3)

6 = 2 + 2 + 2
(1 x 6 = 3 x 2)



3 x 4 = □
4 x 3 = □

Children may discover that the principles that make possible the changing of the order of addends in addition apply to the factors in multiplication.

Worksheets may be made for Examples A and B as an activity.

Example A

"Write a pair of multiplication facts for each picture."

Example B

"After each addition sentence write the multiplication sentence."

To emphasize the commutative property, rearrange the same objects into, say, three groups of four, and then four groups of three. Pairs of examples should be written together.

Missing factors

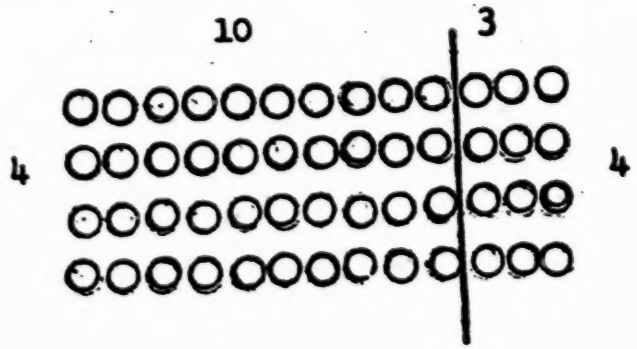
$4 \times 2 = \square$

$3 \times \square = 6$

$\square \times 4 = 12$

$2 \times \square = 0$

Distributive property of multiplication over addition



$$\begin{array}{r} 13 \\ \underline{\times 4} \end{array} \qquad \begin{array}{r} 10 + 3 \\ \underline{\times\ 4} \end{array}$$

$12 \rightarrow 4 \times 3$
$\underline{40} \rightarrow 4 \times 10$
$52 \rightarrow 4 \times 13$

Put examples on the chalkboard. After multiplying

$3 \times 5 = \boxed{15}$ $\quad 2 \times 4 = \boxed{8}$

$5 \times 3 = \boxed{15}$ $\quad 4 \times 2 = \boxed{8}$

erase one of the numerals in each example

$3 \times \square = 15$ $\quad 2 \times \square = 8$

$5 \times \square = 15$ $\quad 4 \times \square = 8$

This type of exercise is a valuable lead up to division.

Every two digit number can be renamed as a multiple of ten and a one digit number. For example, 17 can be renamed as 10 + 7; 23 can be renamed as 20 + 3. Multiplication of a two place number by "renaming" can be shown by means of an array, or with rows and columns of objects. 9:48,49

To multiply, say, 4 x 13, construct a 4 x 13 array. Draw a line segment that divides this array into 10 columns and 3 columns.

"How many objects are there in the 3 by 4 part? (12; write this down). How many in the 10 by 4 part? (40; write this down under the 12.) How many altogether?" If pupils have other ideas of breaking the array up, encourage them. "Why do you suppose I chose to separate into 10 and 3?"

"Rename the Factor"

A. $8 \times 4 = (7 + 1) \times 4$
$= (7 \times 4) + (1 \times 4)$
$= 28 + 4$
$= 32$

B. $8 \times 4 = (6 + 2) \times 4$
$= (\quad) + (\quad)$
$= \_\_\_\_ + \_\_\_\_$
$= \_\_\_\_$

C. $8 \times 4 = (5 + 3) \times 4$
$= (\quad) + (\quad)$
$= \_\_\_\_$
$= \_\_\_\_$

D. $8 \times 4 = (4 + 4) \times 4$
$= (\quad) + (\quad)$
$= \_\_\_\_ + \_\_\_\_$
$= \_\_\_\_$

Taking one multiplication fact, have pupils rename the first factor in as many ways as possible, to see if the products are the same for each renaming. For example, rename the 8 into (7 +1), (6 + 2), (5 + 3), (4 + 4).

This game may be repeated at a later time using the variation of letting the pupil rename the factor of his choice.

X	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

X	2	5	1	3	4	0
1	2	5				0
3			3	9	12	0
5	10	25				
2						
4				12		
0			0			

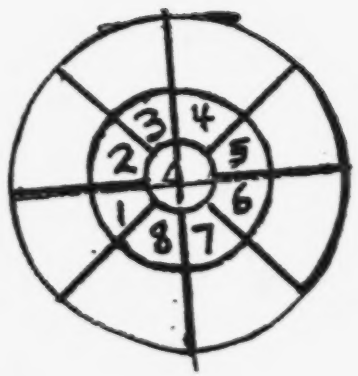
The multiplication grid can be used as usual, or may be "scrambled" to give greater variety. Use several "puzzles" of this type, leading up to those where there are many vacant squares. Many difficulty levels can be treated by puzzles of this sort.

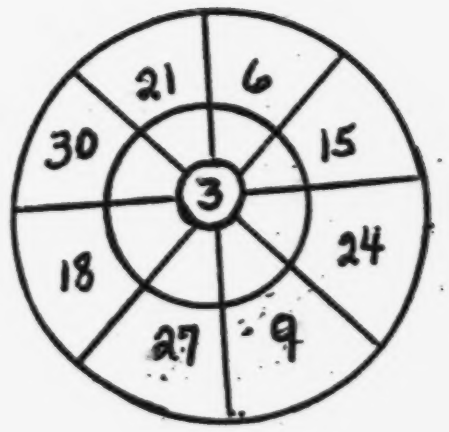
A number line may be used for pupils to "solve" unknown facts.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 →

Use repeated addition to find multiplication answers.

Number of stamps	1	2	3	4	5	6
Cost	8¢	16¢				





Mental arithmetic

Teacher dictates:

(1) "3 --- times
4 --- divided
by 2 --- times
3 ---.

Write your answer." (18)

Charts are another form of drill that can be used ocassionally as a type of cross number puzzle. 11:333

Circle puzzles offer another variation of number drill. Pupils fill in the blanks, using the number in the center as one factor, and the next ring out has second factors.

The outermost ring contains products.

Rapid calculation exercises help pupils develop strengths on the abstraction level.

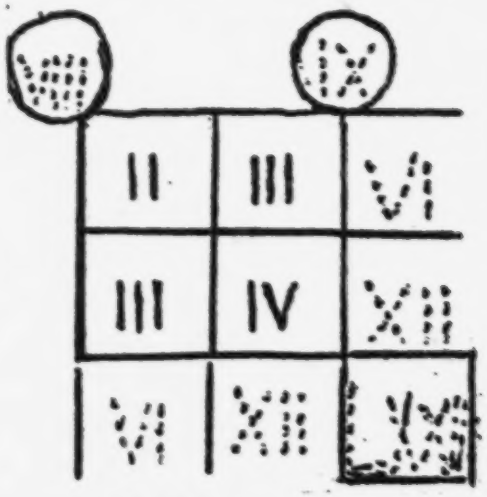
Have pupils number from 1-10 on a sheet of lined paper. Explain that they are to work each problem in their heads as the teacher dictates, and write only the answer on their answer sheets.

Mix the processes, if the pupils can handle such work, calling it "Quick Thinking."

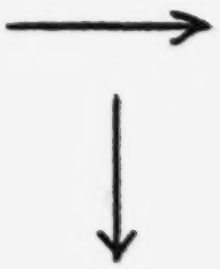
(2) 6--- plus 1 ---
times 2 ---
minus 2 ---
divided by 2 ---.
Write your answer. (6)

(3) 2 x 4 = (8)
(4) 2 x 14 = (28)
(5) 2 x 24 = (48)
(etc.)

Puzzles substituting Roman numerals



Multiply



1. Have children change Roman numerals to Hindu-Arabic numerals.
2. Then have them do the multiplication.
3. After work is done convert Hindu-Arabic numerals back to Roman numerals.

Relationship of multiplication and division

A.

0000
0000

$2 \times 4 = 8 \quad 8 \div 4 = 2$

$4 \times 2 = 8 \quad 8 \div 2 = 4$

00000
00000
00000

$3 \times 5 = 15 \quad 15 \div 5 = 3$

$5 \times 3 = 15 \quad 15 \div 3 = 5$

The relationship of multiplication and division may be shown by the use of arrays.

Reinforcing the inverse relationship of multiplication and division

The relationship between multiplication and division may be reviewed by giving the children a worksheet as follows:

Write these related number facts in three other ways.

$4 \times 5 = 20$ ($5 \times 4 = 20$)($20 \div 4 = 5$) ($20 \div 5 = 4$)
$27 \div 3 = 9$ ($27 \div 9 = 3$) ($3 \times 9 = 27$) ($9 \times 3 = 27$)

Write a multiplication fact on the board without using an operational sign; e.g., 2, 3, 6. The students then write 2 multiplication and 2 division facts from this:

$2 \times 3 = 6 \qquad 6 \div 3 = 2$
$3 \times 2 = 6 \qquad 6 \div 2 = 3$

From this progress to writing only 2 factors on the board -- 3, 4. The students have to supply the product and then write the four facts as above.

General Practice

"Guess Again"

Leader = "Something times something is 24"
1st guesser: "4 x 6 is 24"
Leader: "Guess again"
2nd guesser: "2 x 12 is 24"
Leader: "Guess again"
3rd guesser: "3 x 8 is 24"
Leader: "You guessed it!"

For the faster students, write a factor and a product -- 3, 27. They have to find the missing factor and then write the four facts.

"Guess again" involves supplying missing numbers 7:7 for division and multiplication facts. The leader says he is thinking of a multiplication fact having a certain product. Students try to guess which factors he used. For example, the leader is thinking of 3 x 8 = 24. He tells only the product, 24, to the class. One after another, pupils guess which factors he is thinking of. The correct guesser becomes the next leader. You and the class will need to decide whether to allow 8 x 3 and 3 x 8 both. Since you're teaching the commutative, "in any order" rule, you will probably want to accept either order.

"Lines Tell A Story"

Developing meaning of multiplication combinations including the identity element and the function of zero.

Rows →

Columns ↓

R = row
C = column
P = points
D = drawing of the lines
A.S - arithmetic sentences

By using Rows and Columns and counting the intersecting points, if there are any, children can graph the story of multiplication combinations

R	C	D	P	A.S
1	0	—	0	1×0=0, 0×1=0
2	0	=	0	2×0=0, 0×2=0
3	0	≡	0	3×0=0, 0×3=0

etc.

R	C	D	P	A.S
1	1	+	1	1×1=1
2	1	⫲	2	2×1=2, 1×2=2
1	3	⫼	3	1×3=3, 3×1=3
4	1	☰	4	4×1=4, 1×4=4

etc.

R	C	D	P	A.S.
3	5	▦	15	3×5=15, 5×3=15
4	7	▩	28	4×7=28, 7×4=28
6	9	▩	54	6×9=54, 9×6=54

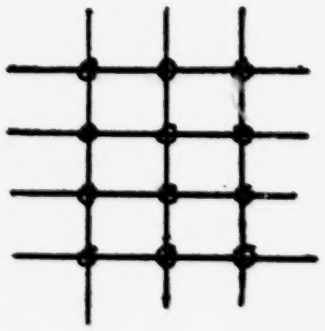
etc.

At this point children can be told to count intersections of lines.

Multiplication Arrays

Rows →

Columns ↓



3 x 4 = 12
3 by 4 array

There are many game like experiences for developing multiplication by utilizing the array. (Try the EHP Math Workshop for Children for ideas in this area) 8:170 22:7-15, 19,36,39, 48,52,58 74, 77

Identity element of multiplication

3 x 1 = 3
1 x 4 = 4
6 x 1 = 6
1 x 8 = 8

"One" is the identity element of multiplication. When we multiply 2 numbers if either of the factors is 1 the product is the other number. 8:174

The function of 0 (zero) in multiplication and division

0 x 8 = 0
8 x 0 = 0
0 x 4 = 0
4 x 0 = 0

"Nothing" multiplied 8 times is still "nothing."
8 multiplied by "nothing" is still "nothing." 4:125
etc.

Division using vertical notation and subtractive method.

A.

5 ⟌25	
-5	1 x 5
20	
-5	1 x 5
15	
-5	1 x 5
10	
-5	1 x 5
5	
-5	1 x 5
0	5 (answer)

B.

5 ⟌75	
-25	5
50	
-25	5
25	
-25	5
0	15(answer)

C.

5 ⟌80	
-50	10
30	
-25	5
5	
- 5	1
0	16 (answer)

The subtractive method of division (sometimes called the scaffold method) is demonstrated in examples A, B, and C. As multiplication is often referred to as repeated addition, and division is the inverse of multiplication and subtraction is the inverse of addition, then it is possible to divide simply by subtracting the divisor from the dividend as many times as is necessary to arrive at a remainder of zero (or a number less than the divisor). As the pupil improves in proficiency and skill he should begin to subtract larger multiples of the divisor.

Distributive property of division over addition

$30 \div 5 = (15 + 15) \div 5$
$= (15 \div 5) + (15 \div 5)$
$= 3 + 3$
$= 6$

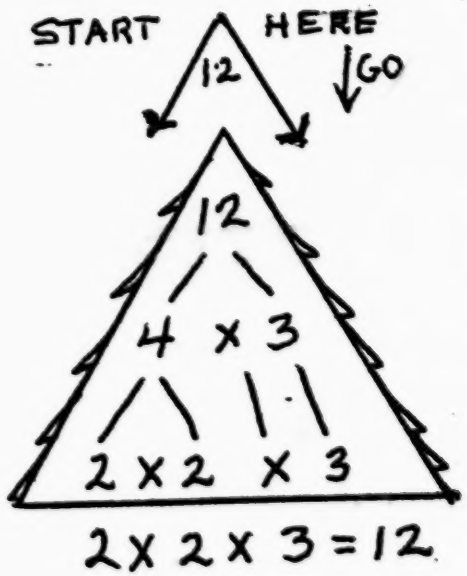
$24 \div 4 = (16 + 8) \div 4$
$= (16 \div 4) + (8 \div 4)$

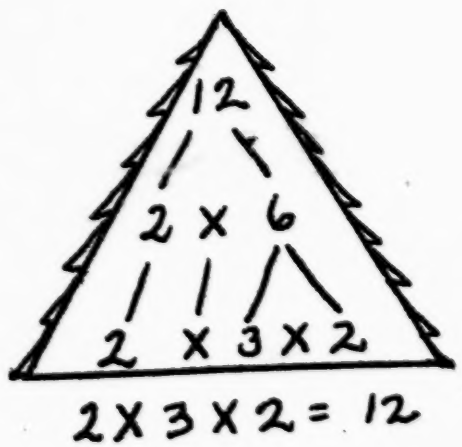
$20 \div 5 = (15 \div 5) + (5 \div 5)$
$= \_\_\_\_\_ + \_\_\_\_\_$, or $\_\_\_\_$

$166 \div 2 = (160 \div 2) + (6 \div 2)$
$= \_\_\_\_\_ + \_\_\_\_\_$, or $\_\_\_\_$

Pupils can find division facts that they have not yet learned by using the distributive method. This is not intended to become a substitute for memorizing the division facts, but is a meaningful extension of what the pupil knows.

Division: renaming into factors





Game "From the Tree Top Down." Children express each number as a product of 2 smaller numbers. They keep on until they can't express any of the terms in smaller factors.

Under each tree they write an arithmetic sentence which is true and whose product is the original number.

A.

$$\begin{aligned} 12 \div 2 &= (10 + 2) \div 2 \\ &= (10 \div 2) + (2 \div 2) \\ &= 5 + 1 \\ &= 6 \end{aligned}$$

B.

$$\begin{aligned} 99 \div 3 &= (90 + 9) \div 3 \\ &= (90 \div 3) + (9 \div 3) \\ &= 30 + 3 \\ &= 33 \end{aligned}$$

C.

$$\begin{aligned} 155 \div 5 &= (100 + 50 + 5) \div 5 \\ &= (100 \div 5) + (50 + 5) + (5 \div 5) \\ &= 20 + 10 + 1 \\ &= 30 + 1 \\ &= 31 \end{aligned}$$

D.

$$\begin{array}{r} 1 \\ 10 \\ 20 \\ \hline 5\,\overline{155} \\ \underline{100} \\ 55 \\ \underline{50} \\ 5 \\ \underline{5} \\ 0 \end{array} \qquad 20 + 10 + 1 = 31$$

Examples A, B, C and D demonstrate the development of the division algorism through the use of the distributive property of division over addition. It is to be remembered that in division the distributive property has the limitation that only the dividend may be renamed. The teacher will also do well to choose dividends that can be easily renamed with terms that can be evenly divided by the divisor.

Example D is an intermediate step between the distributive principle and the division algorism.

A.
$$646 \div 2 = (600 + 40 + 6) \div 2$$
$$= (600 \div 2) + (40 \div 2) + (6 \div 2)$$
$$= 300 + 20 + 3$$
$$= 323$$

B.
$$848 \div 4 = (800 + 40 + 8) \div 4$$
$$= (800 \div 4) + (40 \div 4) + (8 \div 4)$$
$$= \_\_\_\_ + \_\_\_ + \_\_\_\_$$
$$= \_\_\_\_$$

C.
$$333 \div 3 = (300 + 30 + 3) \div 3$$
$$= (\_\_\_ \div 3) + (30 \div \_\_) + (\_\_ \div \_\_)$$
$$= \_\_\_ + \_\_\_ + \_\_\_$$
$$= \_\_\_$$

The division algorism

$$\begin{array}{r} 213 \\ 4\overline{)852} \\ -8 \\ \hline 5 \\ -4 \\ \hline 12 \\ -12 \\ \hline 0 \end{array}$$

Expanded notation is one means of using the distributive property to make work more understandable for the child. Dividends should be selected so that the divisor will be a factor of each of its digits.

In example A: The numeral 646 was chosen because 2 is a factor of each digit - 6, 4, and 6.

At first, problems should be partially completed by the teacher. Later, the pupils can show the complete work themselves.

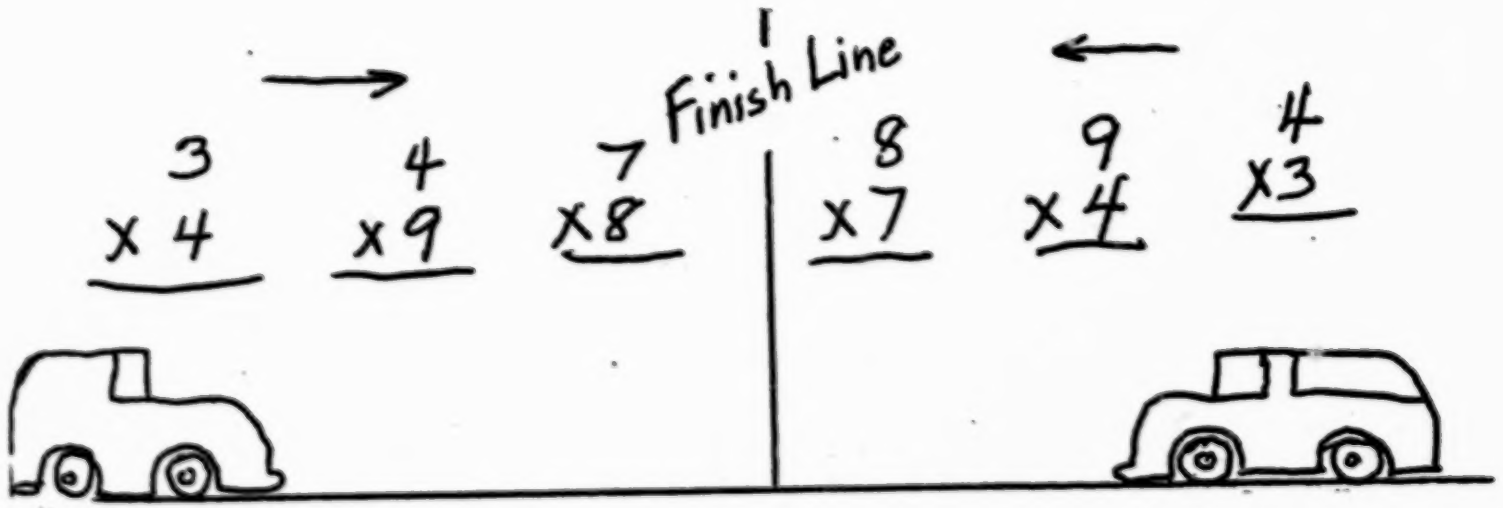
In the example $852 \div 4$ is done in the conventional algorism. Our first step is to ask ourselves "How many fours are there in 800?" The answer 200 is recorded in short form as just a 2, but placed in the hundred's place. In the same manner we record the product of 200 x 4 simply as 8 but it also is placed in hundreds place. We next determine that there are 10 fours in 52 and we record the one in the tens' place. The product of 10 x 4 is recorded as 4 but kept in the tens' place. Here we find we have some tens (one, to be exact) left over. Our next step is to think of the 12 as 12 ones and determine the number of fours in 12. Since we are finding "ones" the answer "three" is recorded in the ones' place. The simplified steps can be listed as follows:

1. Divide (by estimation)
2. Multiply
3. Subtract
4. Bring down the numeral in the next place.
5. Start over with step number 1.

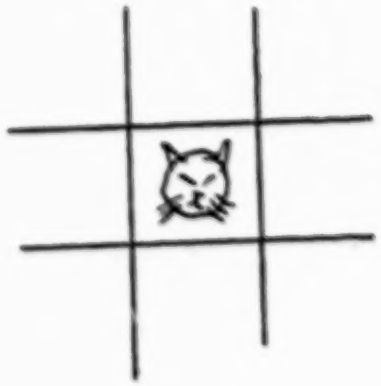
Drill - Blackboard Auto Races

Children draw and cut out their own cardboard cars (horses, turtles etc.) Two children are chosen to bring their cars up to the chalkboard. Cars are placed on opposite ends of chalkboard.

Teacher gives starting signal and the race to the center finish line is on as the children work the problems. The first car to reach the finish line is not always the winner. Classmates raise their hands when they spot a mistake. The child that reaches finish line should check his work before laying his chalk down.



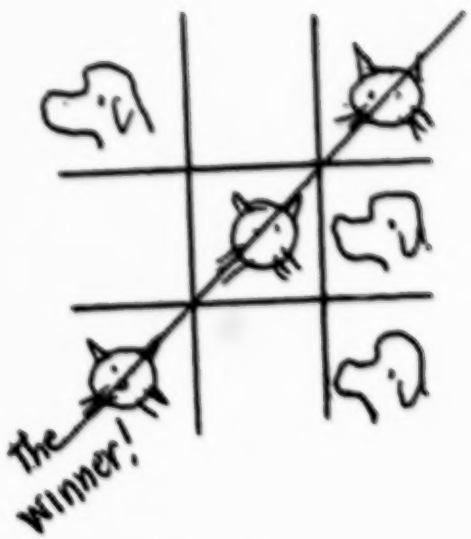
Multiplication or Division



Class is divided into 2 teams. Each team chooses its symbol. Teacher gives first children on cat's side a multiplication problem. If the answer is correct the child draws a cat's face otherwise he does not.

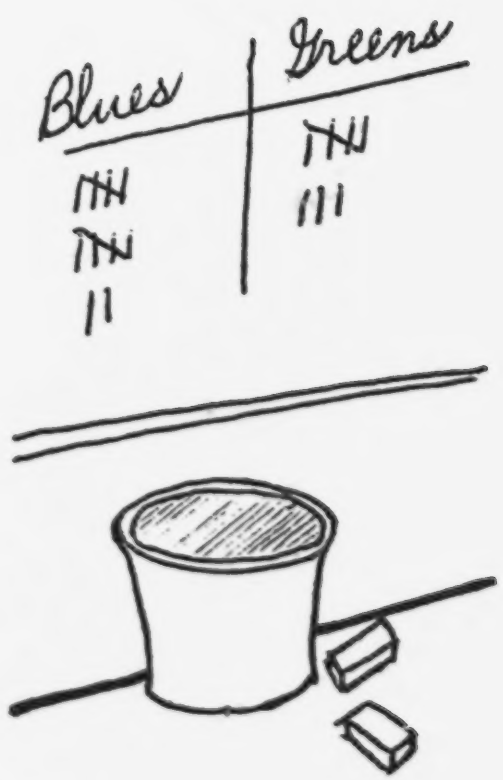
The dog team is then given a multiplication problem. The first team that has a straight row, either horizontal, vertical, or diagonal, wins.

Class can be divided into smaller units which operate on alternate days, etc.



Cats and Dogs

"Hole in One" Game



The class is divided into two teams, Blues and Greens. The first child on the Blues stands ten to twenty feet (varying with age) from a waste paper basket. Blue tries to throw an eraser into the basket. If the earser goes into the basket, Blue's own side will ask him a mental multiplication combination. (How much is 3 x 4?) If the eraser misses the basket Blue must let a Green ask him a question.

Children will try to get the eraser in the basket because their own side will give easier questions. Any child that asks a question, however, must know the answer himself. One point is scored for each correct answer.

"Blackboard Relay" - Multiplication or division

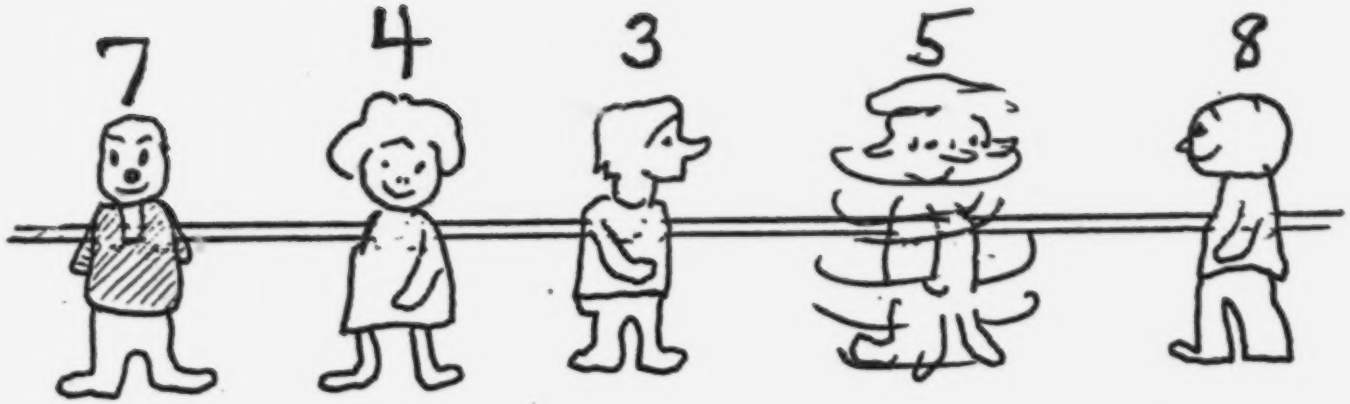
8	5	6	7
40	10	12	14
32	15	18	28
16	20	24	63
	45	54	
		48	

Have class divide into four relay lines of 5 children each. Vary size according to number of children playing. At a given signal the children in each line race to list multiples of numbers as shown in front of them. This is a timed game. Shift the teams to the left or right in order to give the children practice on all the numbers.

Multiplication - "Blackboard Spin the Bottle"

Five children ("bottles") are chosen to stand under five numbers on the chalkboard as shown in the diagram. Seated children, upon permission of the teacher, call out multiples of numbers on the board. Object is to "spin" the bottles. When a "bottle" hears a multiple of his number he quickly spins around once. (Example: if 25 is called out the "bottle" under the 5 turns around.)

Seated children watch for mistakes and take the place of "bottles" that did not spin around or who spun around when they should not have.



Properties of multiplication

Relationship Problems

(< , > , =)

a. 24
b. 30
c. 28

Examples that involve relationships other than equality allow pupils to reach many rather sophisticated mathematical conclusions on their own. Encourage pupils to make a predicition of the relationship before they do any computation. This can be done in several ways: Example A: Give the pupils a ditto sheet arranged as follows:

Write the correct symbol <, > or =

Predictions	Problems	Computations
Fold back	5 x 9 □ 9 x 5	40 = 40
	40 ÷ 5 □ (40 - 20) - 5	
	(3 + 3) x 4 □ 3 x 4	
	9 x 2 □ (9 x 2) + (9 x 0)	

Instruct the pupils to first make a prediction on each problem then fold the paper back on the dotted line. After that they are to do their computation and put the correct symbol in the frame. At the conclusion they can check their predicitions for accuracy. Another approach would be to put a problem on the chalkboard and have a class predicition and then a discussion regarding prediction. Continue with examples until the pupils understand that they make their prediction before computing.

Find three different ways to express the numbers as a product of 2 factors.

a. 12 (there are 6 ways)
b. 35 (there are 4 ways)
c. 42 (there are 8 ways)
d. 45 (there are 6 ways)

Express each of the numbers to the left as a product of 2 factors in every possible way.

Multiples of tens and hundreds.

$$\begin{array}{r} 12 \\ +\ 0 \\ \hline 12 \end{array}$$

It may be necessary to discourage the pupils from thinking that multiplying by 10 is the same as "just adding zero." Show them, say, 12, and "add a zero" using a conventional addition algorithm. "Twelve plus zero is not the same as 12 x 10." 14:613

This is more than simply a language problem. Pupils should know the structure of our decimal numeration system and see the reasoning back of the shortcut of affixing a zero to a numeral to show multiplication by 10.

Multiples of tens and hundreds leading to 2 and 3 place multiplication

Problems can be laid out in an order that allows pupils to make their own realizations about numbers. Give the instructions " Find the pattern and write the missing numeral."

Example:

	a.	b.	c.
1.	4 x 4 = ____	4 x 40 = ____	4 x 400 = ____
2.	3 x 3	30 x 3 = ____	300 x 3 = ____
3.	6 x 3 = ____	6 x 30 = ____	6 x 300 = ____

After this is clear, further number skills can be developed by using a 2 place factor with other than zero in the ones place.

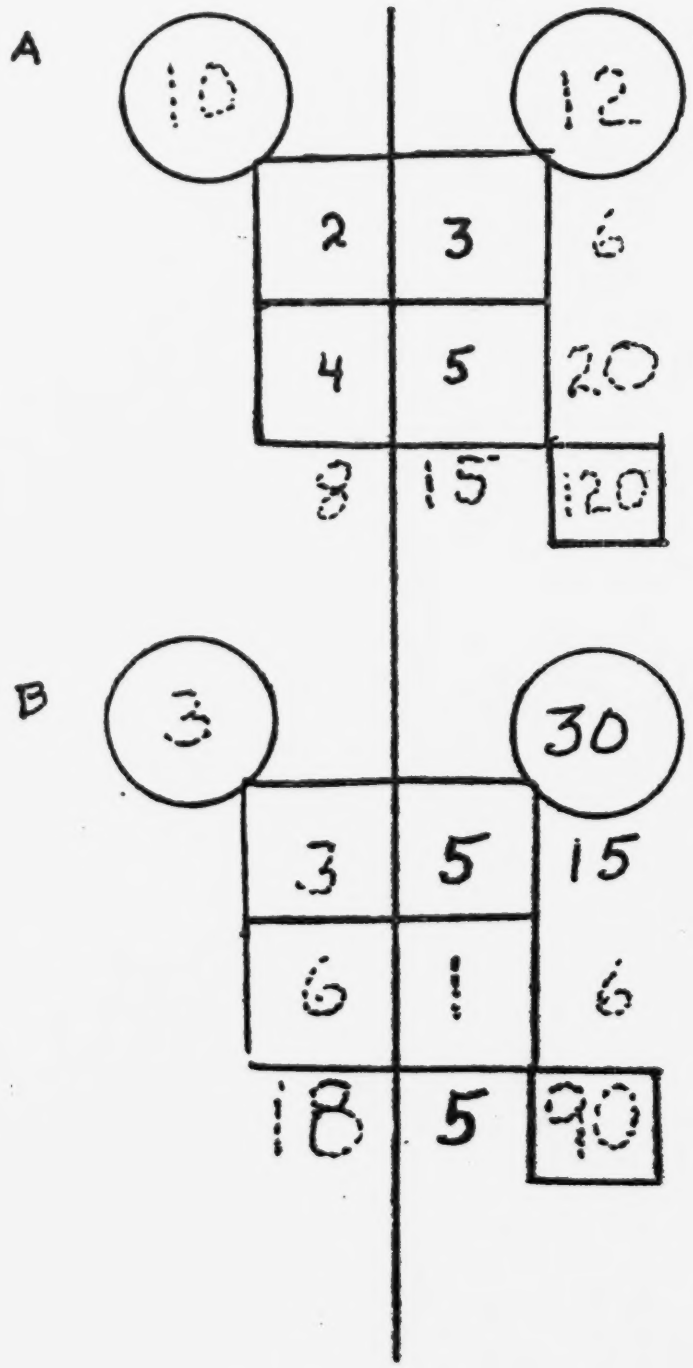
Example:

$$\begin{aligned} 2 \times 95 &= 2 \times (90 + 5) \\ &= (2 \times 90) + (2 \times 5) \\ &= 180 + 10 \text{ or } 190 \end{aligned}$$

$$\begin{aligned} 5 \times 56 &= 5 \times (50 + 6) \\ &= (5 \times 50) + (5 \times 6) \\ &= 250 + 30 \text{ or } 280 \end{aligned}$$

Cross number puzzles to illustrate multiplication and division

A



B

Cross number puzzles give practice and offer a method for pupils to discover checks for correct answers.

Help the pupils to discover these things:

2 x 3 x 4 x 5 = 120
Ears: 10 x 12 = 120
Sides: 6 x 20 = 120
Bottom: 8 x 15 = 120

After completing the puzzle erase all but four of the numerals and have the pupils fill in the missing ones again. Then give them a new puzzle as in Example B.

Encourage fast students to discover other patterns.

Have them make their own puzzles.

Remember that there need be enough clues in each puzzle so that can be worked.

Multiplication and division practice

	A	B	C	D	E
△÷□	2	4	5	5	0
△	6	8	20	10	0
□	3	2	4	2	3
△X□	18	16	80	20	0

"Study the table to see if you can discover how to arrive at the answers indicated by the broken line numerals. Work on <u>one column</u> at a time."

In the chart shown pupils follow the"directions" shown by the figures. 3:69

" △ ⋮ □ " tells pupils to divide the figure in the triangle row by the figure in the row marked by a square. For example, in column A, △ ⋮ □ would be 6 ⋮ 3; △ x □ would be 6 x 3.

Multiplication and division using dollar sign and decimal or separating point.

A. Solve problems on the chalkboard.

A.

$.72	$.63	$.92	$.51
x2	x3	x3	x4
$1.44	$1.89	$2.76	$2.04

B. Erase a part from each problem and put in a □ in its place, then solve the problems.

B.

$.7□	$.63	$.□2	$.5□
x2	x □	x3	x4
□1.44	$1.89	□2.76	$□.04

Test for divisibility by 2s or 5s

The pupils might be interested in seeing what the possible ones-place numerals are for 2s and 5s.

By looking at just the last figure in a numeral you can tell if 2 or 5 will go into it evenly.

Factor x Factor = Product

	tens	ones
2 x 1 =		2
2 =		4
3 =		6
4 =		8
5 =	1	0
6	1	2
		(etc.)
5 x 1 =		5
2 =	1	0
3 =	1	5
4 =	2	0
5 =	2	5
6 =	3	0
7 =	3	5
8 =	4	0
9 =	4	5
10 =	5	0
		(etc.)

PROBLEM SOLVING

Grade 3

Grade 3

Problem solving

Begin problem solving with simple dramatizations. Choose Mary and John to help. Discuss their dramatizations and write the problems on the board, having the class solve each problem jointly, and using number sentences. Use at least one dramatization each day.

Example:

1. Mary brought five books to Mrs. White's desk and John brought seven books. How many books did they put on the teacher's desk? 5 + 7 = 12

2. Mary had 10 pencils. She gave 4 of them to John. How many pencils did she have left? 10 - 4 = 6

3. Mary is 9 years old and John is 7. How many years older is Mary than John? 9 - 7 = 2

Ask the children to write three or four problems each. Give dittoed sheets to each child using the children's problems for use as a class assignment. Attach their names to them because they enjoy working them when they know who wrote them.

When multiplication is being studied, discuss both ways of writing the answers. Remember multiplication is a fast way of adding.

I baked 5 pies and cut each one into 6 pieces. How many pieces did I have:

6 + 6 + 6 + 6 + 6 = 30
5 x 6 = 30

Ask the pupils to write problems at home and bring them to school. (Suggest problems about games.) Do them orally in class.

Example: Donald's baseball team had 20 points and Joe's team had 10 points. How many more points did Donald's team get than Joe's team? 20 - 10 = 10

Suggest problems about food.

Examples: I made 14 pies and put them out to cool. When I came back there were only 10 pies on the table. How many pies were taken? 14 - 10 = 4

Mother made 36 cookies. The children ate 22 of them. How many cookies were left? 36 - 22 = 14.

Emphasizing step-by-step problem solving approach.

The following step-by-step approach to problem solving can be written as a chart and placed where the children can refer to it.

1. Find out what the problem asks.
2. Find out what the story tells.
3. Choose the correct operation
4. Solve the problem.
5. Make sure the answer is reasonable.
6. Check the accuracy of the answer.

Open End Problems

Example A is an example of a semantic puzzle, <u>not</u> a problem. Those who read well instantly get the answer.

Example A.

Johnny had 4 marbles.
He found 3 marbles.
How many marbles does
Johnny have now?

Example B.

Johnny has ____marbles.
He found _____ marbles.
Now he has 7 marbles.
How many marbles did
Johnny have. How many
did he find?

has	found
0	7
1	6
2	5
3	4
4	3
5	2
6	1
7	0

Check solution by translating into mathematical sentence.

Example B is an example of an open end problem. This situation requires some organized thinking. There can be eight possible correct answers. The possible answers can be set up in a column using the number pairs to make 7..

(Note: In the solution - "has 6 marbles, found 1 marble," - point out to the children the correct use of marble rather than marble(s). Children will sometimes be misled by this. Explain this one idea prior to solving.)

Open ended problems may be highly desirable and often yield creative results.

How many times a day does a clock strike if it only strikes the hour?

1,2,3,4,5,6,7,8,9,10,11,12 = 12 hr.
(1,2,3,4,5,6,7,8,9,10,11,12)2 = 24 hr.

1+1+1+1+1+1+1+1+1+1+1+1 = 12
1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1 = 24

Checking solutions by translating into mathematical sentences.

Ted found 2 rocks at the river for his collection. Then he found 8 more at the lake. How many did Ted find for his collection? 2 + 8 = 10

Aids needed for this activity:

1. 5-inch squares of oaktag with numerals 1 through 9 (several of each).
2. One each of all the operational signs, a place holder, and parenthesis.

Have the children read a problem from the text. Then have another student set up the equation on the chalkboard ledge using the proper numerals and signs to show the correct equation. Check and discuss. This could be used for any problem solving activity.

Aids needed for this activity:

1. 5" x 3" cards lettered A, B, C, etc.
   Write or paste one problem on each card.
   You may make up these problems or cut them from old textbooks. Be sure the problems are within the range of the processes taught thus far. Include cards for new problem types as they are taught.

Let each pupil pick out as many cards as he would like to do. On his paper, opposite the letter of the card, he is to write the equation for the problem with a placeholder for the answer. (If the children have been introduced to $\underline{n}$ or their maturity warrants, the letter $\underline{n}$ may be used as the placeholder.) If the problem requires computation, the child should show his computation, then write the equation as completed.

Example:

John had some apples. Tom gave him 3 apples. Then he had twelve apples. How many apples did John have to begin with?

"How Many to Begin With?"

Aids needed for this activity:

1. Box of objects such as shells, macaroni, stones, jacks, corks, etc.
2. One 12-inch square of cloth.

Children who have difficulty in finding how many there were to begin with when the number added and the resulting total are known (see example) may try the following activity.

Let them work in pairs (an able pupil may work with a slow learner). Each pair needs a box of aids. The first child takes a handful of the objects and, without counting, puts them under the cloth. The second child then takes a group of objects out of the box and counts them. He puts these on top of the cloth and pulls the cloth out so that all the objects fall into one group. The first child counts all the objects. Then they are to find how many objects were put under the cloth. This they can do, of course, by removing the numbers that were in the known group from the total group and counting the remainder. The children should write an equation to tell what happened, using a square, a line, or a question mark to hold the place for the answer (whichever they are familiar with).

If the children are mature enough and are ready, you may substitute a letter as the place holder.

Example: use n for the placeholder and compute

$(n + 3 = 12)$, $(3 + n = 12)$

$(12 - 3 = n)$, $(12 - n = 3)$

Checking reasonableness of solution to problems.

If a class has been making repeated errors of the same type, it is often helpful to have them discuss these problems orally.

Choose two or three problems with typical errors and write them on the chalkboard. Discuss orally the reasonableness of their answers.

Typical Student Errors:

Problem I:
$$\begin{array}{r} 19 \\ 19 \\ 42 \\ 68 \\ \hline 202 \end{array}$$

Challenge the children to give their reasons for doubting the answer to the column addition problem.

One child may offer the statement that in the ones place he sees only two tens. In the tens place he sees only twelve tens. Together, there couldn't be more than fourteen tens.

Problem II:
$$\begin{array}{r} 146 \\ -\ 23 \\ \hline 23 \end{array}$$

In the case of the second problem, a child may make the statement that 23 + 23 is only 46, whereas, we had 146.

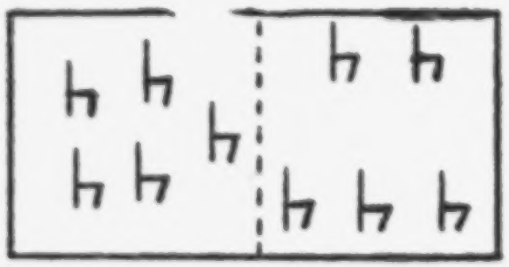
The discussion may be followed by a paper containing several problems. Each problem is followed by three possible answers. Direct the child to circle the most reasonable answer.

Estimating an answer in advance.

One approach to estimating is to have the pupils diagram the problem. Given the problem: Two rooms have the same number of chairs. If there are ten chairs in all, how many are there in each room?

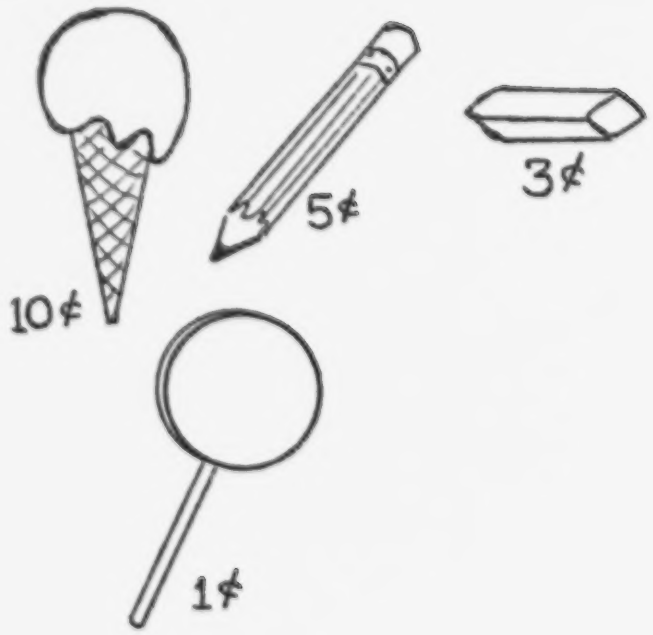
$10 \div 2 = \square$ or $\square \times 2 = 10$ (See Figure 1).

Figure 1



Another approach is to give the pupils a certain amount of money and the price of several items and ask them to determine how many different purchases they could possibly make.

Figure 2



Example: Given 50 cents to make some purchases. (pencils are 5 cents each, erasers are 3 cents each, and candy suckers are 1 cent each) Get as many of each item as you can. (See Figure 2). There are several possible answers. Each student's solution should be challenged by computation.

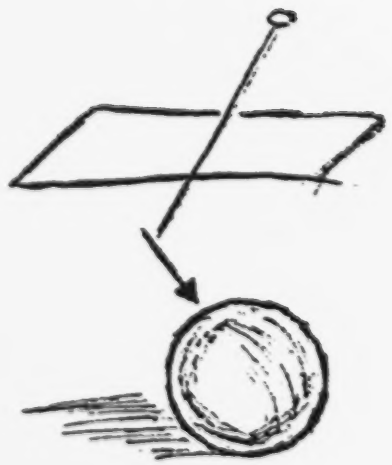
(___ x 10)+(___ x 5)+(___ x 3)+(___ x 1) = 50

GEOMETRY

GRADE 3

Formal definitions of point, line, and line segment.

Point Markers



GEOMETRY

Begin with a review of the informal, intuitive ideas of a point, a line, and a line segment that were explored in kindergarten, first and second grades. 11:189

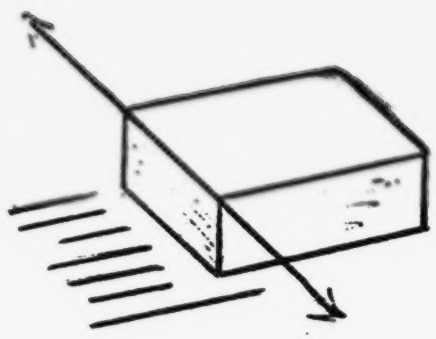
The point:

1. What can we use to mark a place on a table? (A spoon, checker, button, nail, hole, etc.)
2. What can we use to mark a place on the chalk board? (A cross, arrow, circle, dot, etc.) 4:2,12-14
3. What can we use to show the smallest mark of a place? (Dot made by sharp pencil point, or small hole made by pin.)
4. Can you <u>think</u> of a place in the air without a marker? Try it. 6:200-203
5. Tell me a point or place you can think about.
6. Tell where the point is.
7. Think of a point on a ball. What happens to the point when the ball is kicked? (It may move with the ball if the ball is our frame of reference. The point may remain fixed in space if the ground is our frame of reference. This is a sophisticated concept. Be happy if the students can think of a point as something that does not move or something that does move.)
8. Formal definitions to be reached: A point is an exact location in space (distinct from any physical representation!)

The line:

1. What do we see in the classroom that marks a line? (Table edges, intersections of walls and ceiling, lines on calendar, etc.) 12:190

Line Markers



2. Think of a line on the table. How could we mark it? (Erasers end-to-end, jump rope, pencil, yardstick, yarn) 14:4,5, 26-30

3. Think of a line on the chalkboard. How could we show the line? (Chalk mark, string, etc.)

4. What is the smallest mark we can use to show the location of a line? (A thin pencil mark along a straight edge will do.)

5. Can you think of a line in the air without a marker? Try it. 6:209

6. Tell about your line in space.

7. Tell the class where your imaginary line is.

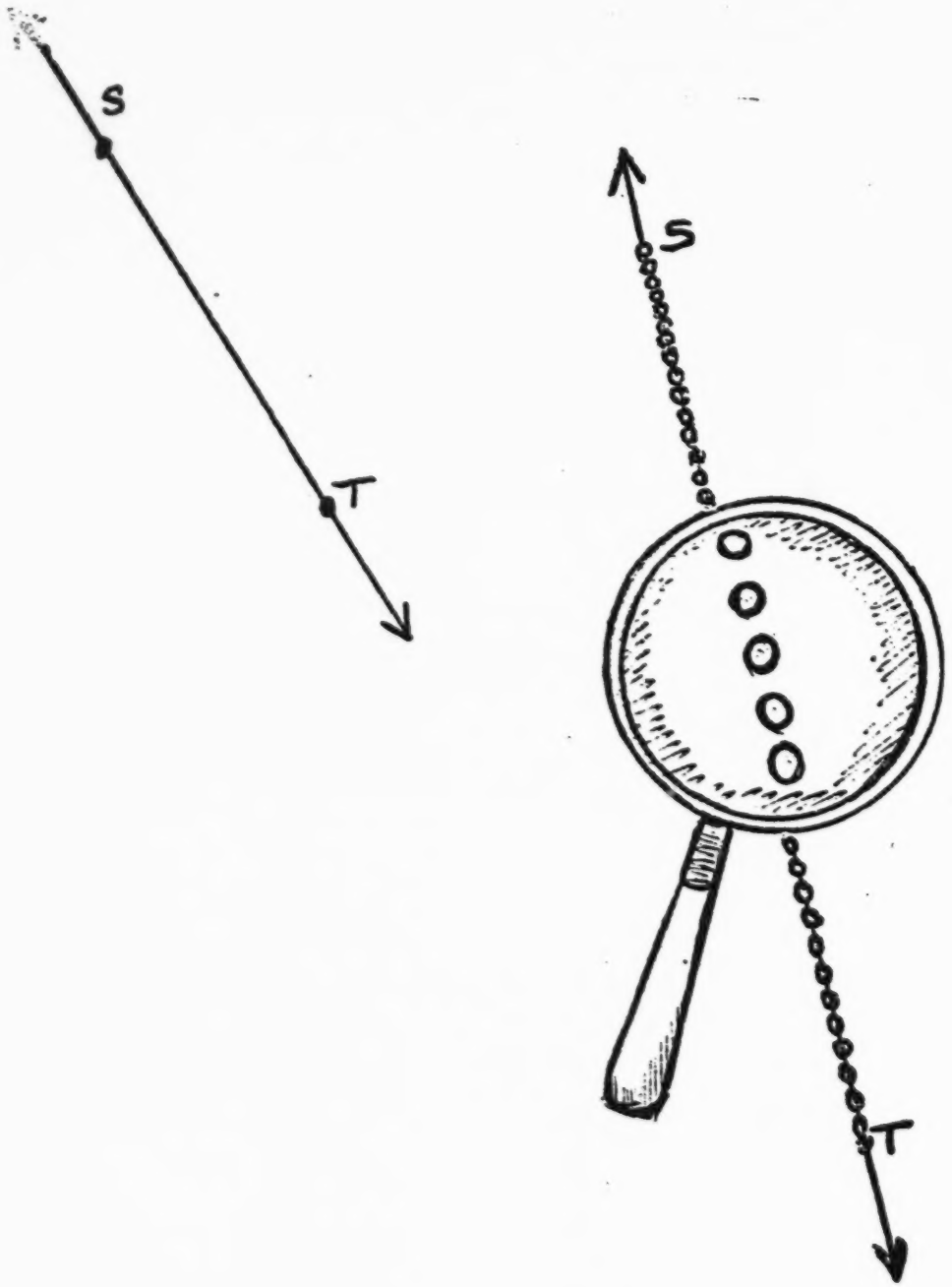
8. Think about a line represented by an edge of a box. What happens to the line when the box is moved? (The line moves or doesn't move depending on whether the box or ground is the frame of reference.)

9. Think of a point of a line. Think of ten points. How many points are in a line? (More than can be counted)

10. Draw a picture of a line on a piece of paper (or chalkboard). Does your picture of line end at the edges of the paper? (Yes) Does the line we are thinking of end at the edges of the paper? (No) Where does the line go? (On and on) Where does the line end? (It doesn't) When does the line stop going? (Never)

11. Formal definitions to be reached: A line is a set of points that continues on in both directions without end.

Line Segment



1. Draw a picture of a line on your paper. 12:191
   Use arrows to show that the set of points goes on in both directions. 14:4,15-23

2. Mark two points on the line and label them S and T.

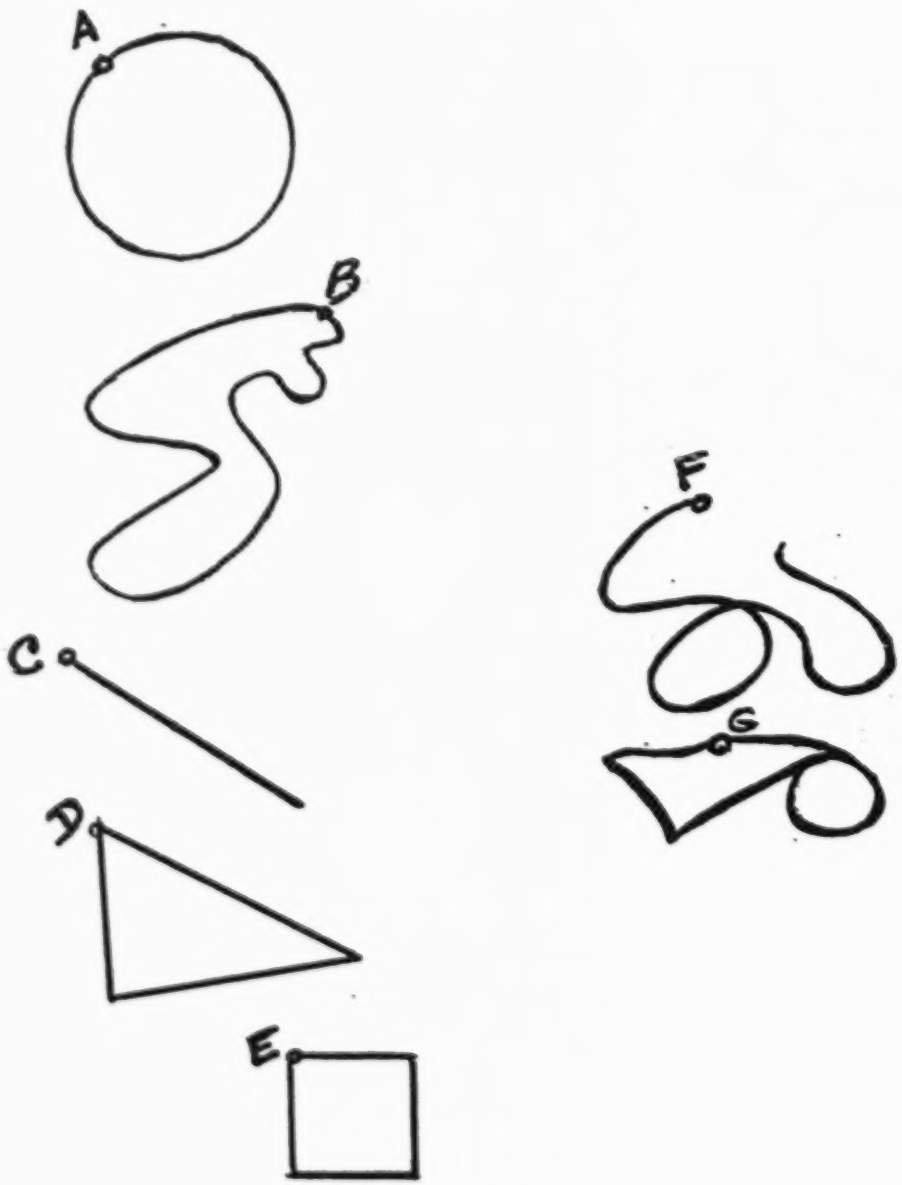
3. Think of a point between S and T. 6:204-208
   Think of ten points between S and T. Use your pencil to mark all the points you can on line segment S T. How many dots or marks could you draw? (Varies with length of line segment, sharpness of pencil lead and skill of students.) Are there more points between the dots you drew? (Yes) Are there always more points than we can mark? (Yes)

4. Are line segment S T and line S T the same length? (No, a line has no length)

5. Is line segment S T a part of a line? (Yes, line S T)

6. Definition to be reached: A line segment is any two points of a line and the set of countless points between the two points chosen.

Curves



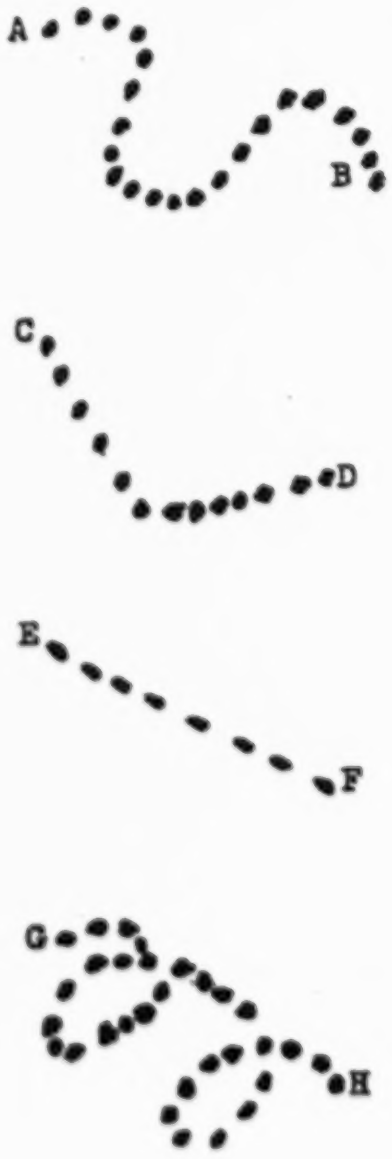
Give paper and pencil to students. "Think of a point on the paper, mark it, and label it "A". Place the point of your pencil on A and then move your pencil point over the surface of the paper in any path you want. Does the pencil lead mark a set of points? (Yes) A path of points is called a curve. Mark and label several more points on your paper. Then beginning at each of these points, draw curves. Draw curves that intersect (touch, end at, or meet) their beginning points and some curves that do not meet their beginning points." Use the chalkboard to show some student models of curves for class discussion. Show curves A, C, D, and E on the board. "What special name do we give these curves?" (Circle, line segment, triangle, and square respectively.

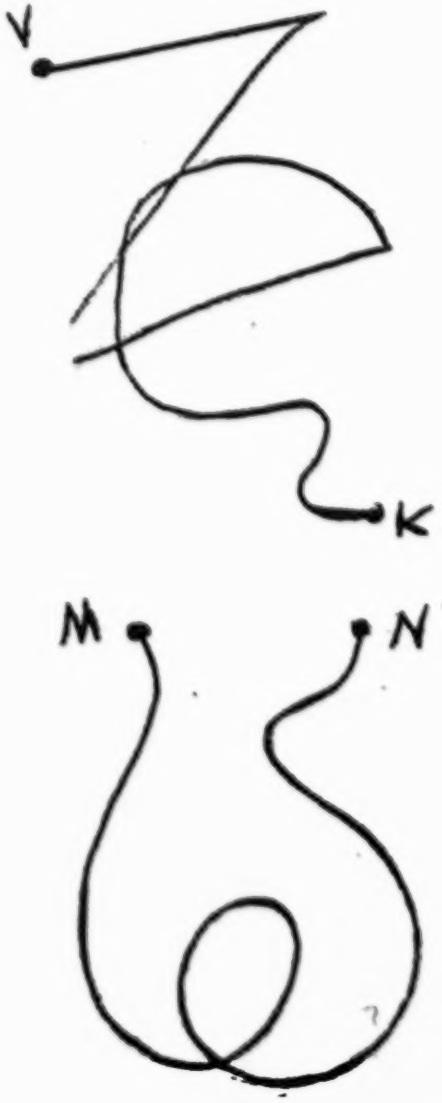
14:3,4

12:192,193

6:204,215-217

Open and closed curves





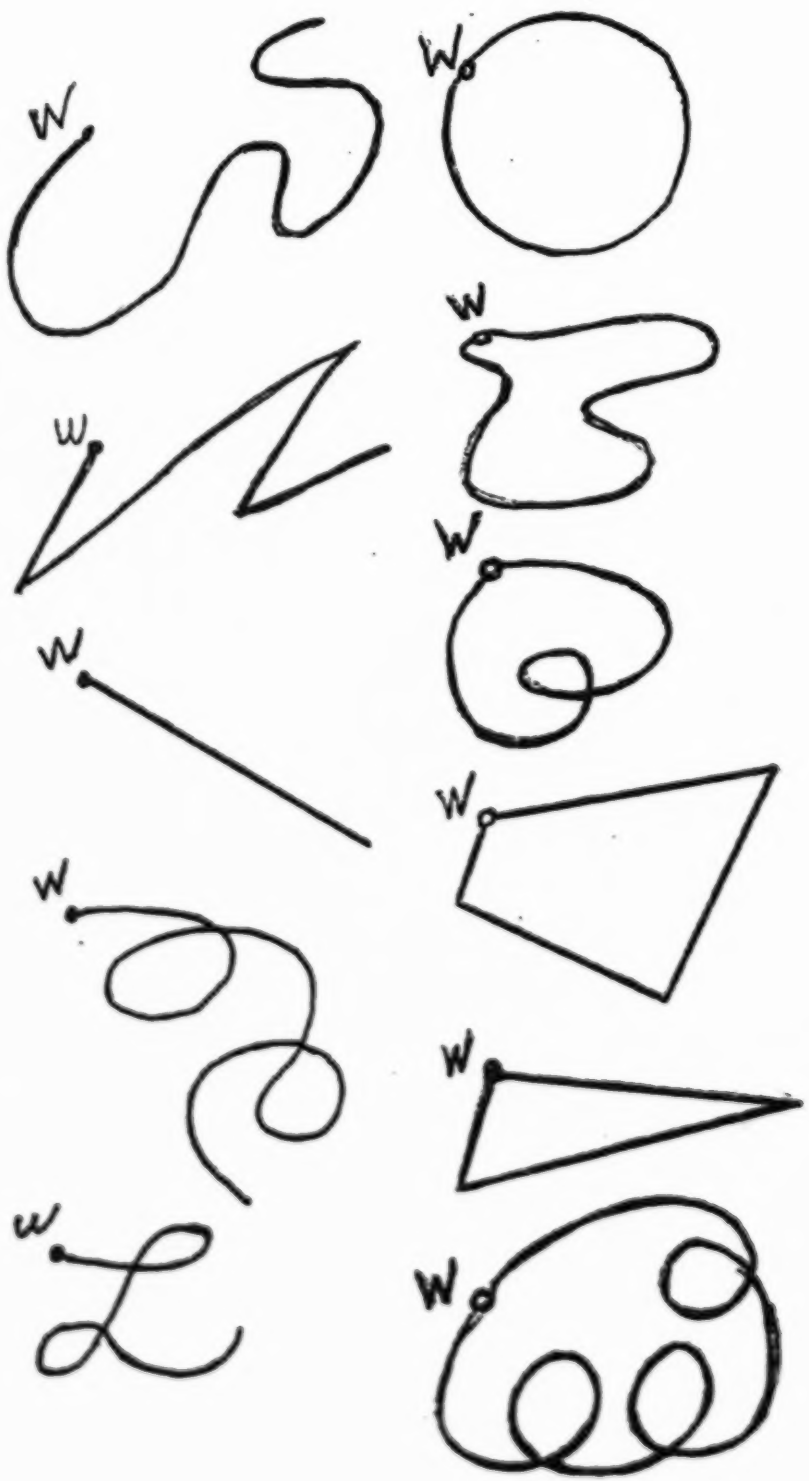
14:52-56

Choose any two points in the classroom or playground. Have students show various paths by walking between the two points.

Give students dittoed sheets with paths shown by dots. Ask "Can you show a path between A and B by tracing the dots with your finger? Use your pencil to connect the set of points between A and B, C and D, E and F, G and H. The curves you have drawn are open curves.

Choose several pairs of points on your paper and draw some more open curves. Remember to begin at one point and end at the other. Is a line segment on your curve? (Yes) Is a line an open curve? (Yes) A circle? (no).

"There are two kinds of curves, open and closed. Can you guess what a closed curve might look like? Find a point on your paper and label it W. Draw a curve that begins and ends at point W. You have just drawn a <u>closed curve</u>.



Draw three closed curves that begin and end at one point without crossing themselves. Closed curves that don't cross themselves are called simple closed curves.

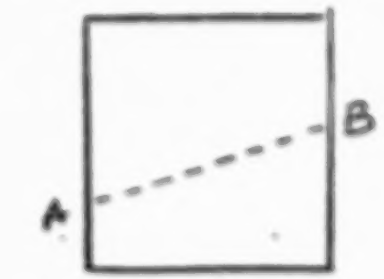
Place curves like the examples at left on the chalkboard. Discuss whether they are open curves or closed curves. "Are the closed curves simple closed curves? How can you decide?" (Some students may be curious to know that the not-simple closed curve is called a complex closed curve. Open curves are also divided into simple and complex categories but are not studied in the elementary school.)

Polygons

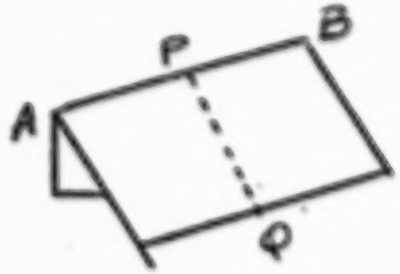
Show pairs of simple closed curves like those at left. "What is different about the two figures in each pair? How are the left-hand figures alike? (The left-hand figures are simple closed curves made up of line segments only.) Such curves are called <u>polygons</u> and their line segments are called <u>sides</u>. Is a square a polygon? (Yes) A rectangle? (Yes) A circle?" (no, not made of line segments.)

Properties of square, rectangle and triangle

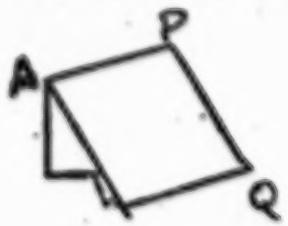
The "square corner"



1. FOLD AT AB



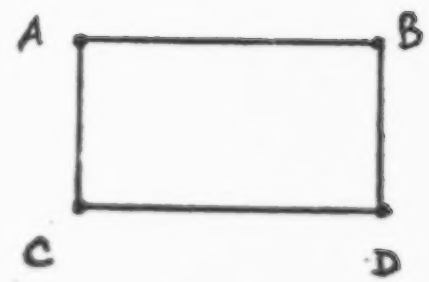
2. FOLD AT PQ
SO THAT A IS ON B

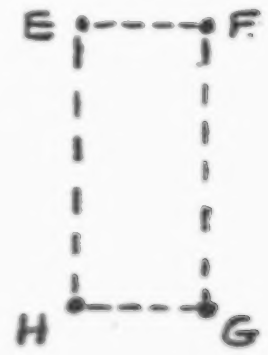


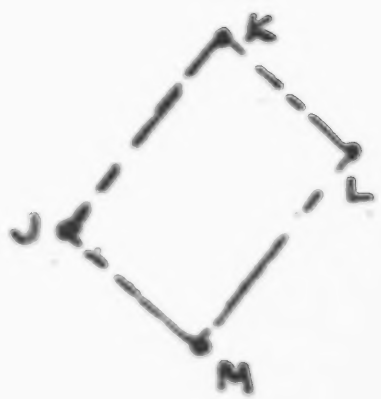
∠APQ IS A
"SQUARE CORNER"

Have students look for square corners in the room. (Book pages, walls, rectangular boxes, desk tops, etc.) Show students how to fold a square corner and have them use it to test corners in the room for "squareness." At this level let students see angles as corners and right angles as square corners.

The rectangle







Give students sheets of dittoed paper containing sets of dots which mark the vertices of various sized rectangles. To students: "Use your pencil and straightedge to draw line segments AB, CD, AC, and BD. What familiar polygon do you see? (Rectangle) Draw line segments EF, FG, GH, and EH. Continue until all figures are drawn." Ask: 12:195-196 18:4,5

1. How many line segments form a rectangle? (Four)

2. Are AC and BD the same length? (Yes) Measure to be sure.

3. Are AB and CD the same length? (Yes)

4. How many corners does the rectangle have? (Four)

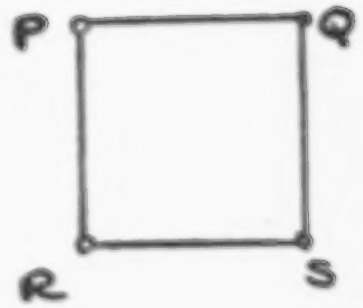
5. Are the corners the same size? (Looks like it)

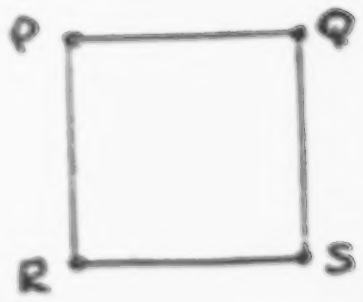
6. Are they square corners? (Yes) Test them to make sure.

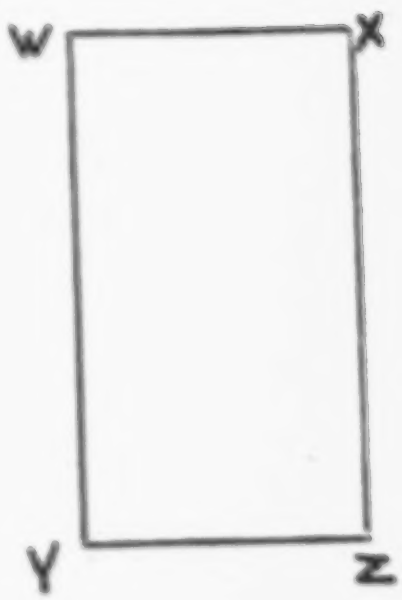
7. Definition to be reached: A rectangle is a four-sided polygon with four square corners.

Have students locate rectangular shapes in the classroom and test them to see if they meet the requirements given in the definition.

The square







Place square PQRS on the chalkboard. Ask students to test the polygon to see if it belongs to the rectangle family, i.e.

1. Four line segments? (Yes)

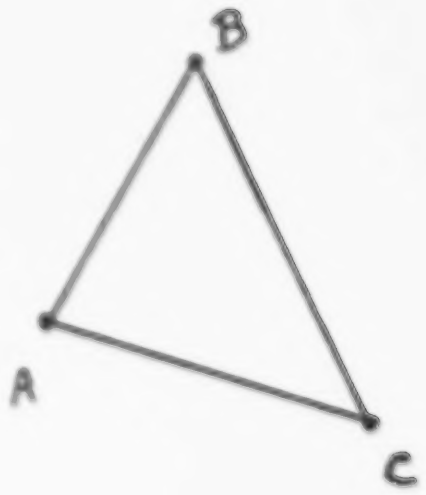
2. Four square corners? (Yes)

3. A rectangle? (Yes)

"What special name do we give this rectangle?" (Square) Draw a rectangle alongside the drawing of square PQRS. "Are both polygons rectangles? (Yes) Are both polygons squares? (No) How can we tell the difference?" Discuss how we could ask someone to draw a square who had never seen one. "What does a square have that other rectangles do not have?" (4 sides equal in length)

Definition to be reached: A square is a rectangle whose four sides have the <u>same</u> length (or are <u>equal</u> in length).

The triangle



Ask student to mark any three points on his paper, label them and connect the points with line segments. "What familiar polygon have you drawn? (Triangle) Draw more sets of three points and connect the three points with line segments. Do you always draw a triangle?" (Yes)

1. How many line segments form a triangle? (Three)

2. How many corners does a triangle have? (Three)

3. Are the three line segments the same in length? (Sometimes)

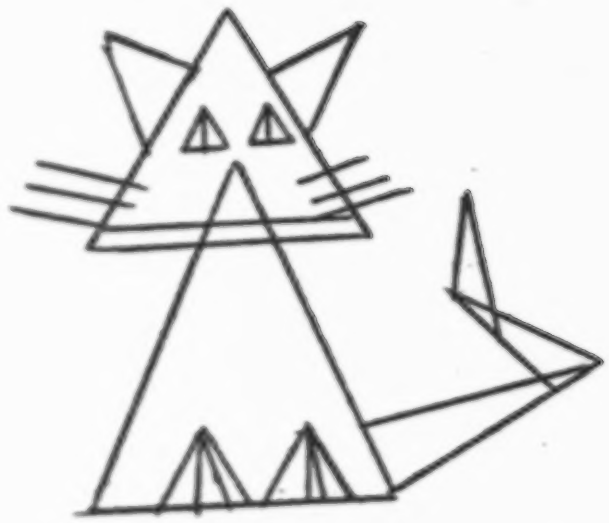
4. Are the three corners the same size? (Sometimes)

5. Is one of the corners a square corner? (Sometimes)

6. Definition to be reached: A triangle is a polygon with three sides and three "corners."

Find triangular shapes in the classroom and test them to see if they meet the requirements given in the definition.

Recognizing triangles



"How many triangles are there? "

Pupils can test their counting ability with the "Triangle cat" on a bulletin board. (There are 30 triangles) Pupils tell their answers and you say "right," or "count again." 17:96

Properties of the circle

Give students dittoed sheets of circles that are suggested by paths of dots. Ask them to label a point, start at the point with their pencil and trace the path of points. 11:197, 198

1. What familiar closed figure have you drawn? (Circle)

2. Does a circle have line segments? (No)

3. Can we think of a circle as a set of points? (Yes)

4. Name two points of the circle A and B. Think of two points between A and B.

5. Are there more points between A and B? (Yes)

6. Use your pencil to mark all the points you can on the circle between A and B.

7. How many marks or dots can you draw? (Varies)

8. Are there more points between A and B than you could mark? (Yes)

9. How many points are there between A and B? (More than we can count)

10. How many points are contained in the circle? (More than we can count)

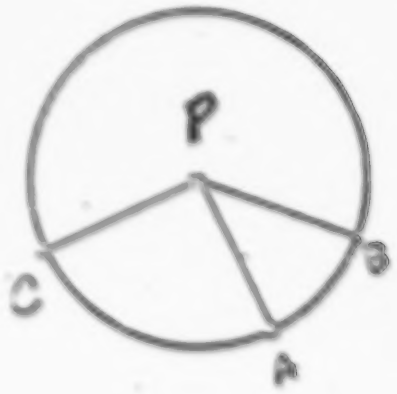
11. Generalizations to be reached:
    a. The circle is a set of countless points.
    b. The circle is a simple closed curve.
    c. The circle is <u>not</u> a polygon.

The center, the radius and diameter

Have students join hands and form a circle. Ask a student to stand exactly in the center of the circle. Discuss where the center is. "How can you tell if you're exactly on the center of our circle?" (Estimating distance to the circle of children around you.)

"How far is it from the center to the circle?"

Have students estimate and then measure with jump rope or string. "If we measure the distance from the center to many points of the circle does the distance remain the same?" Measure several line segments from center to various points of the circle. The lengths should be about equal in measure if the center has been located.



CP ≅ AP

"How far is it across the circle?" (Across meaning the distance measured along the diameter of the circle.) Measure the distance. "How does the distance across the middle of a circle compare with the distance between the center and a point of the circle?" (Distance across is twice the distance from point of circle to center.)

Give students dittoed sheets of various sized circles with centers marked. Have them think of three points of the circle and label them A, B and C. Draw line segments PA, PB, and PC. Measure the line segments. "Are the three line segments the same length? (Yes) Draw some more line segments between the center and points on the circle. What did you find out?"

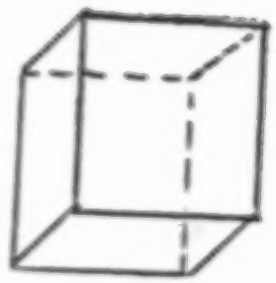
Generalization to be reached: All line segments drawn between the center and points of a given circle are equal in length. (Do not use term "radius" at this level.)

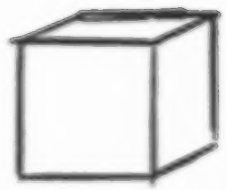
Ask students to draw a line segment from the center P of a circle to a point A of the circle. Have them measure line segment AP and then extend AP to point T. "Measure line segment AT. How does the length of line segment AP compare with the length of line segment AT? (It is half the length.) Measure line segment TP. How does the length of line segment TP compare with segment PA?" (Equal in length) Continue activity, measuring and comparing radii and diameters.

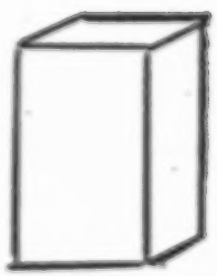
Generalizations to be reached: The measure of the diameter of a given circle equals the measure of two radii. (In the childrens' own words preferably!)

Properties of three dimensional shapes

Cubes







Provide models of cubes as nearly perfect as possible. Allow students to handle the cubes and to experiment by arranging and stacking them. Encourage them to use vocabulary terms, side (or face), edge, corner, and cube. 11:260-264

Ask:

1. How many faces does a cube have? (6)
2. What is the shape of a face? (Square)
3. Does a cube have corners? How many? (8)
4. Does a cube have edges? (Yes) How many? (12)
5. Measure the length of one edge of the cube. Can you tell me the lengths of all edges of the cube? (Yes - edges are all equal in length)

Give students pictures of cubes and right rectangular prisms. Ask them to distinguish the cubes from the other models. "How can you tell a cube from other models?" (Examine faces to see if they are squares)

The sphere

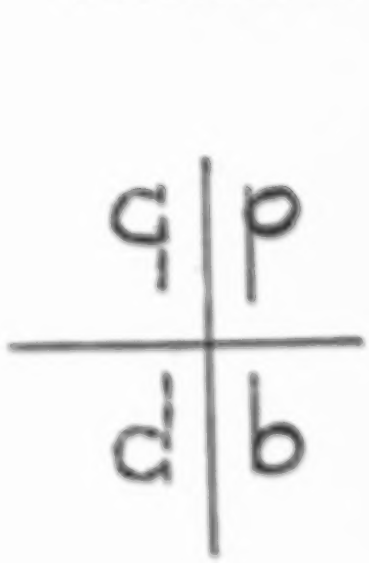
Show models of spheres to students, i.e., volleyball, basketball, globe. What familiar shapes does a ball make you think of? (Circle) Cut out a circle shape of oaktag whose diameter measures the same as the diameter of a volleyball. Glue a long wooden dowel along a diameter of the circle shape.

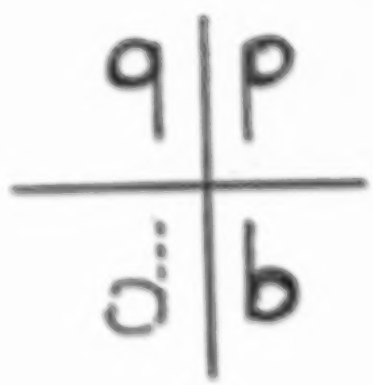
Rotate the dowel between the palms of the hands to cause the circle shape to rotate. The spinning will cause the illusion of a sphere comparable in size to a volleyball. Let students repeat the experiment.

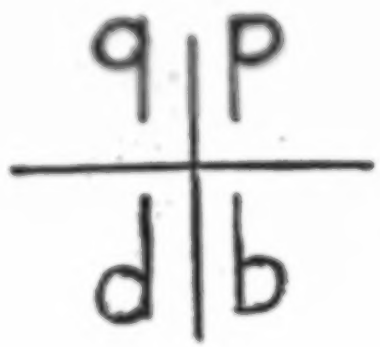
Examine the longitude and latitude lines on the globe. "Do these lines all have the same shape? (Yes) What shape do these lines form?" (Circle)

Discuss where we can see spheres in the world around us.

Reflections of curves and figures





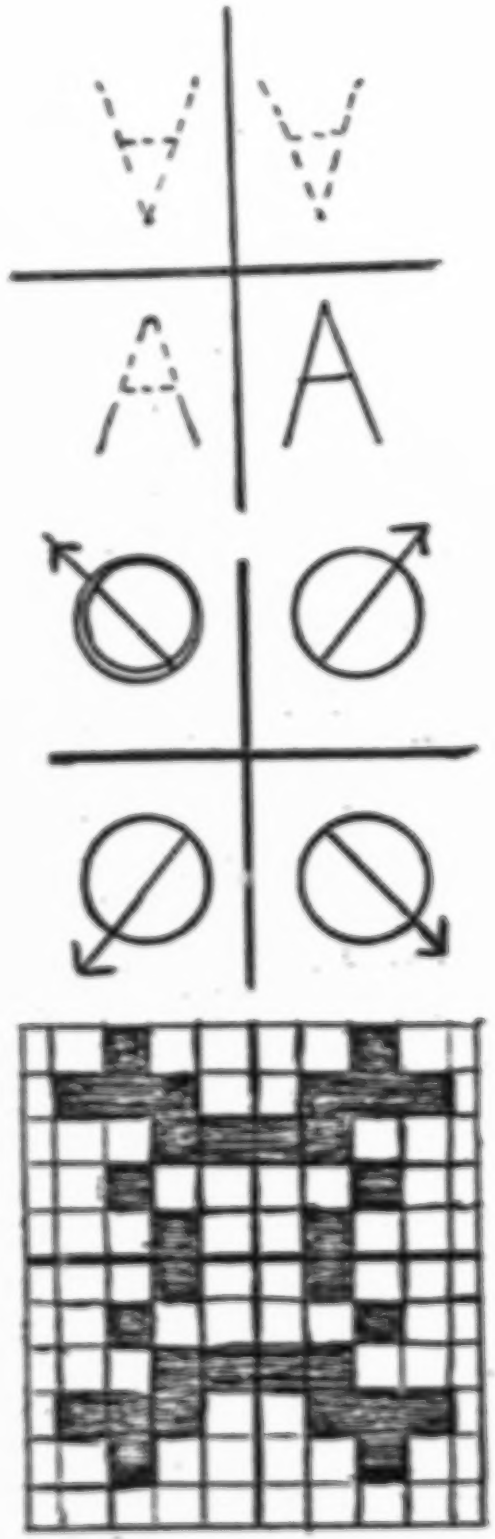


Have students fold paper in halves, draw simple figures on one half and then try to draw the reflections of the figures on the second half with pencil or crayon. It may be necessary to review extensively the concepts outlined in grade 2 for the slower students. Encourage students to use hand mirrors to judge what the reflected image should like before they begin drawing.

20:106,107, 112, 113

Draw a pair of intersecting lines (like the example at left) on the chalkboard. Write the letter "b" in the lower right quadran. "How will this letter look if we draw its reflection in the upper right quadrant?" (Point to the quadrant to which you are referring.) Discuss moving the line segment to the quadrant first. Then, "where will the circle be placed? What letter do you see now?" (Letter "p")

Continue reflecting in counterclockwise direction, i.e., reflect the "p" image to the left about the vertical line. The image in the upper left quadrant would resemble the letter "q". Reflect this image to the lower left quadrant, showing the letter "d".



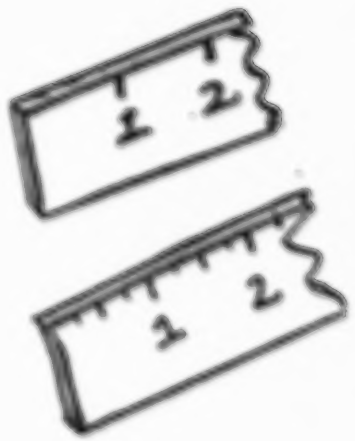
Give students sheets of 1" graph paper. Have them draw pairs of intersecting lines like those in the examples. Ask them to reflect the capital letter A in all four quadrants beginning with A in its proper writing position in the lower right hand quadrant. Vary exercise by using various letters, numerals, curves, or figures. Also, patterns can be used by coloring selected squares in a quadrant and reflecting these regions in the other three quadrants. Several colors of crayon may be used to make pleasing geometric designs.

MEASUREMENT

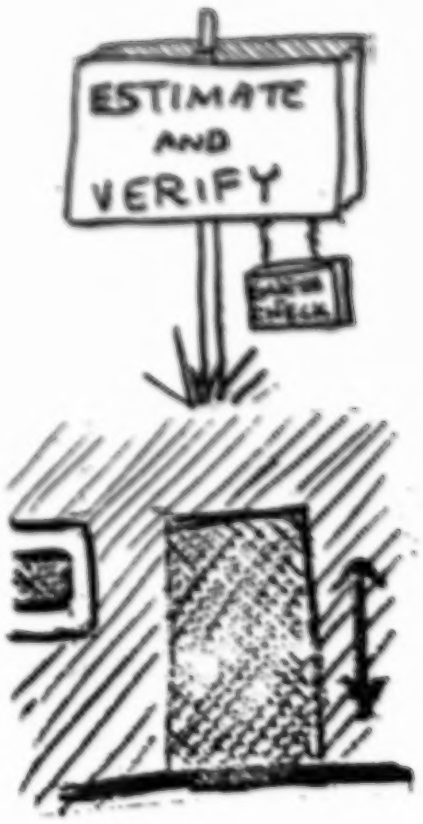
GRADE 3

GRADE 3

Length



Feet and yards



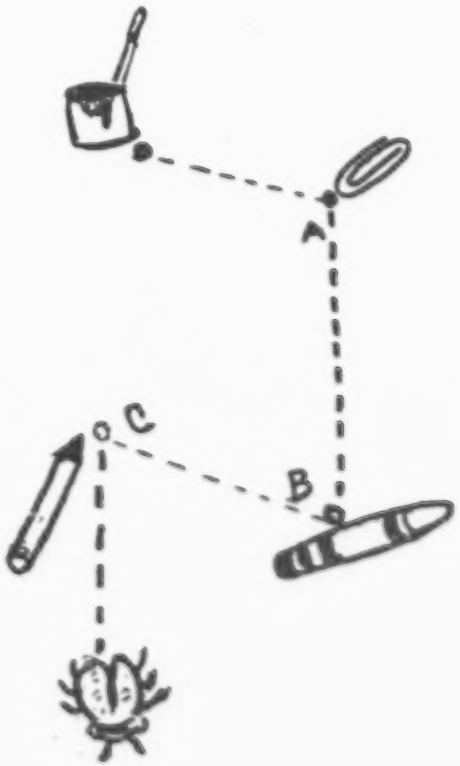
MEASUREMENT

Supply each child with two one foot rulers. One should have only inch markings and one quarter inch markings. The children should have many experiences early in the year in estimating lengths (using logical guesses) and then verifying these estimates by measuring.

Have the children guess the height of the door, then record their guesses. With a ruler let them verify its length. Give a child a yardstick to measure the door and discuss which is the better of the 2 measuring devices. The one most appropriate for the situation--a foot ruler if the door is shorter than a yard, and of course it isn't; a yardstick if the door is taller than a yard.)

Precision of measurement to nearer inch, half inch, quarter inch.

Map reading



On the blackboard draw a paint jar, paper clip, crayon and pencil. Show a ladybug taking a trip to all these objects. Use a straightedge to indicate (by dashed line) her journey. Howfar did the ladybug walk? Have one child come up to the blackboard and measure her trip with a ruler (or if you wish a larger picture use a yardstick). As the child measures and announces the distances, record them on the board as follows:

From paint jar to A	4 inches
From A to B	6 inches
From B to C	5 inches
From C to Bug	5 inches
Length of whole trip	20 inches

Have two or three dittoed worksheets for the children with similar drawings of the ladybug's journeys. Make all the distances she travels "even" inches (use no fractions here). Then have the children plan a trip in their imagination and draw the accompanying picture of their ladybug's journey, then measure it! You may use $\frac{1}{4}$ and $\frac{1}{2}$ inch measurements, if you ask the children only to find the distances between the objects and avoid discussing the whole trip.

Ask the children to estimate the distance from their classroom door to the door of the principal's office. Record their guesses, then use a tape measure to verify. If none is available use a jump rope as a unit of measure--then check its length.

The mile

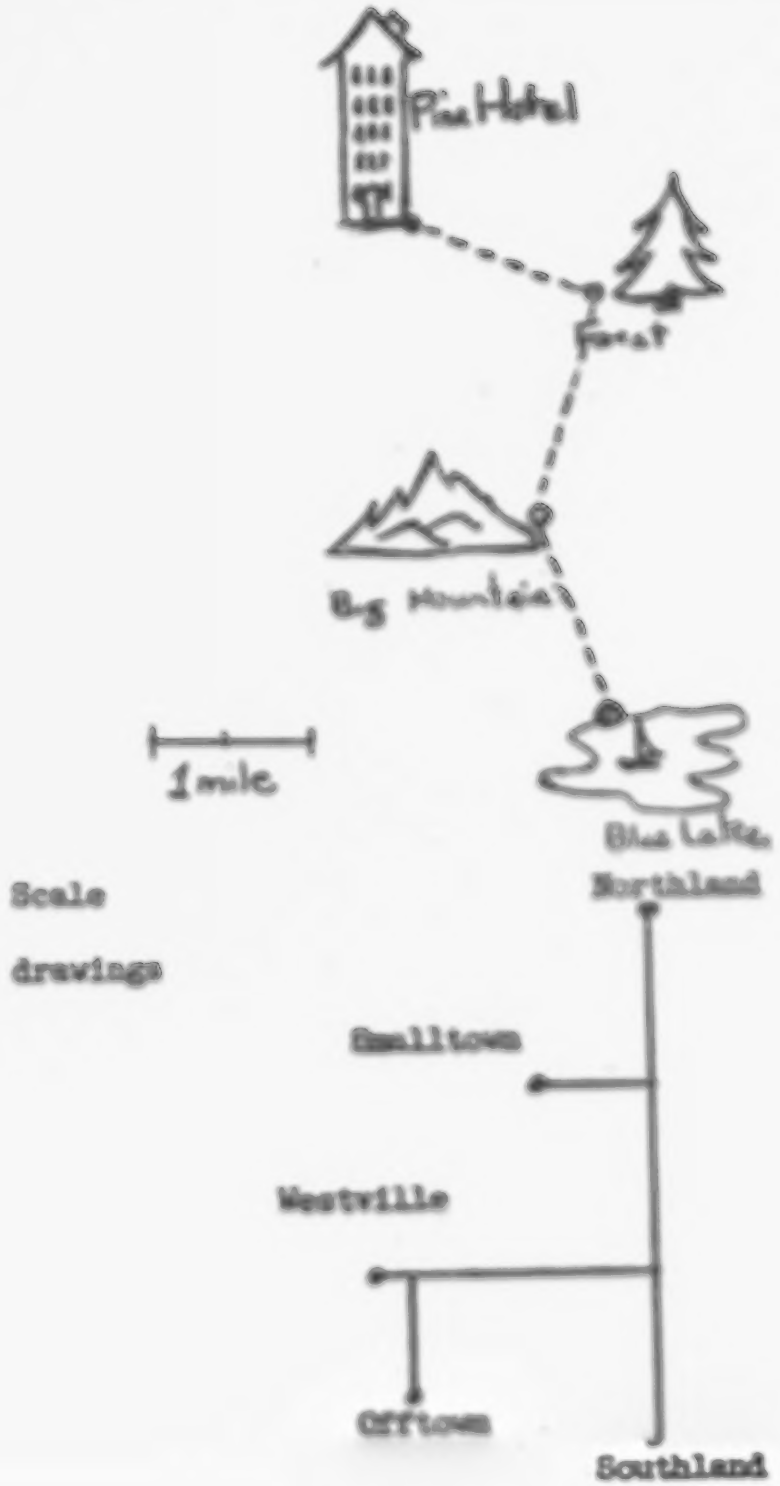
Allow this measuring experience to lead into a discussion of greater units of measure. How would we measure the distance from the school to our home? By blocks or miles. But how long is a mile? Take the class on a walk a measured mile from school in a fairly straight path of direction. (Use a pedometer, or drive the distance beforehand and measure on the car's odometer.)

Try to choose a familiar destination for your walk--some place they may have been before, like the local park, etc. Upon returning from the walking trip, draw on the blackboard a picture of the journey. Label the distance <u>one mile</u>.

Map reading

Have dittoed work papers showing an illustrated map. On this map, have 1 inch stand for 1 mile. The children should use their rulers to answer questions such as--How far apart is the hotel and the forest? (One and a quarter inch) On the map, what is the distance from the

11:264-265



Scale drawings

mountain to the lake? (1 inch) How far apart are these two places? (1 mile) If we walked from the hotel, through the forest, how far would it be? (-----), etc.

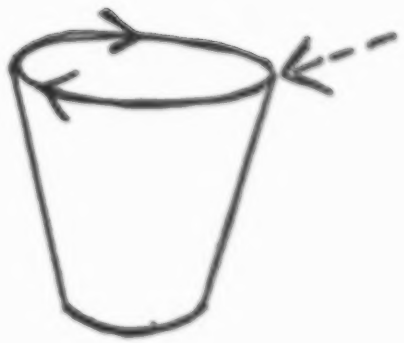
After several illustrated maps are worked out, try one with less clues. Let one inch equal 4 miles.

How many miles does $\frac{1}{2}$ inch show? (2)
What part of an inch shows 1 mile? ($\frac{1}{4}$ inch)
Along the roads what is the distance from:

1. Northland to Southland (13 miles)
2. Smalltown to Northland (3 miles)
3. Smalltown to Southland (11 miles)
4. Westville to Offtown (7 miles)
   etc.

Remind the children when they give answers to always say "7 miles," rather than just the number 7. A characteristic of measurement is that it is not exact. To record any measurement give both the number and the measuring unit used. It means nothing to say an object has the weight of 2, or that a distance is 7 long.

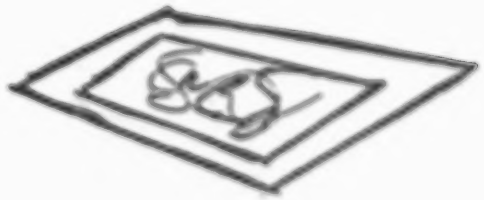
Measuring a curve

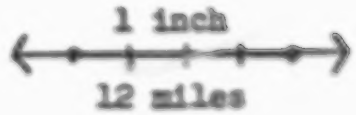


So far all the students have measured is line segments. Now we will briefly try a few measurements on another kind of line segment-- the curve. Our aim is to show the idea of length of a curve, not to measure it with accuracy. Ask the children to estimate the length of the equator on the globe, the distance around the rim of a desk or reading table, or the distance around the rim of the wastepaper basket. How can we verify our guesses? Try to get from the children the idea of laying a string along the curve being measured and then straightening out the string to be measured. Provide each child a string about 15" long and have available a ball of string or twine for team projects. (Make sure it does not stretch easily.) Look for answers to the nearest inch. Group the children into teams to measure various curves.

14:518
523

Curves and scale drawings





On a map, a child can lay a string on a road from one city to another--and then measure the string using the scale of miles to find the road distance between the cities. For added experiences in geometry please refer to the unit on Geometry (3rd Grade) contained in this syllabus. (Refer to 2nd Grade Geometry unit in the syllabus for explanation of the curve.

Provide each child with worksheets showing curves which they can measure using their string. The directions can read: Look at the picture of a curve. Place your string along the curve. Measure the part of the string you used. The length is _____ inches.

Measurement of perimeters

Cut different sized polygons from colored paper. Cut squares, rectangles, and triangles with perimeters that vary from 3 to 10 inches, then cut several strips of paper 1 inch wide and 11 inches long. On each strip draw a line that is the same length as the perimeter of one of the polygons. (Don't put any dimensions on the strips) Begin each line at a different distance from the left end of the strip. Do several strips with lines that do not match the perimeters of any of the polygons. 1:16

Working in small teams, put sets of polygons and strips on a table top. Each child then chooses a polygon and decides, from eyeing the strips, which of the lines is the same as the perimeter of the polygon he is holding. When he has chosen a line, he verifies his guess by placing one side of the polygon on the line, puts a finger on the line to indicate the lengths of the side, turns the polygon and places the next side at that point on the line, etc.

Some polygons with perimeters and matching lines might be as follows:

1 inch square; perimeter 4 inches
2 inch square; perimeter 8 inches
Rectangle 1 inch x 4 inches; perimeter 10 inches
Rectangle 1 inch x 2 inches; perimeter 6 inches
Equilateral triangle with one inch sides; perimeter 3 inches
Equilateral triangle with 3 inch sides; perimeter 9 inches
Right triangle 1½" x 2" x 2½"; perimeter 6 inches
Scalene 2" x 1 ¾ x 2 ¼"; perimeter 4½ inches

Area

Since measurement of area requires a standard unit as in any kind of measurement, we will try to lead up to the need for this standard through the use of beans and gummed labels. We will then go into the use of square inches, etc., using a formula. Many children, even in junior high 15:42-48

Limitations of using beans as a standard

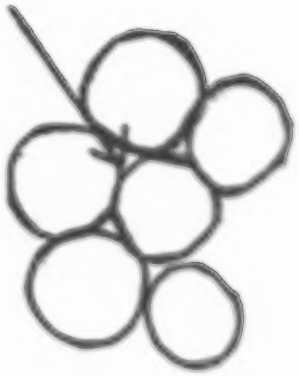
school, still do not understand the concept of measuring area because they weren't given the opportunity to discover the concept of area and the need for a standard unit of measure.

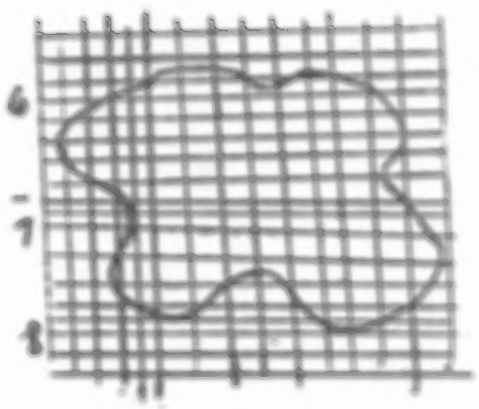
Draw two lakes on the board. Ask which is bigger--has more surface, or room for more boats. List on the board the ideas they have for measuring the two so they can be compared.

Spill a can of beans on a table. Have the lakes mimeographed on paper for each team. Ask them to estimate, by looking, how many beans will fit on each lake. Then let them check and verify by placing the beans on the lake and counting them. We choose beans to use first since they vary so in size. The use of such a standard will point out the value of a clear-cut equally proportioned standard such as squares of paper.

Ask each team to announce their results. Note their answers will be different even though the pictures of the lakes are the same.

Ask what might be a standard to use. Answers might be gummed labels, pennies, bingo markers, buttons, poker chips,etc. Whatever the standard decided on, the children should then measure the lakes' surfaces using the new standard and record the results. If





labels are not mentioned, point out the fact that the coins, etc., leave small space between which are not measured. Someone will, ideally come up with cutting squared or rectangles of paper and using them as a measuring unit. Parts of these pieces of paper are very important too. The children must estimate the number of whole squares to which the parts are equal.

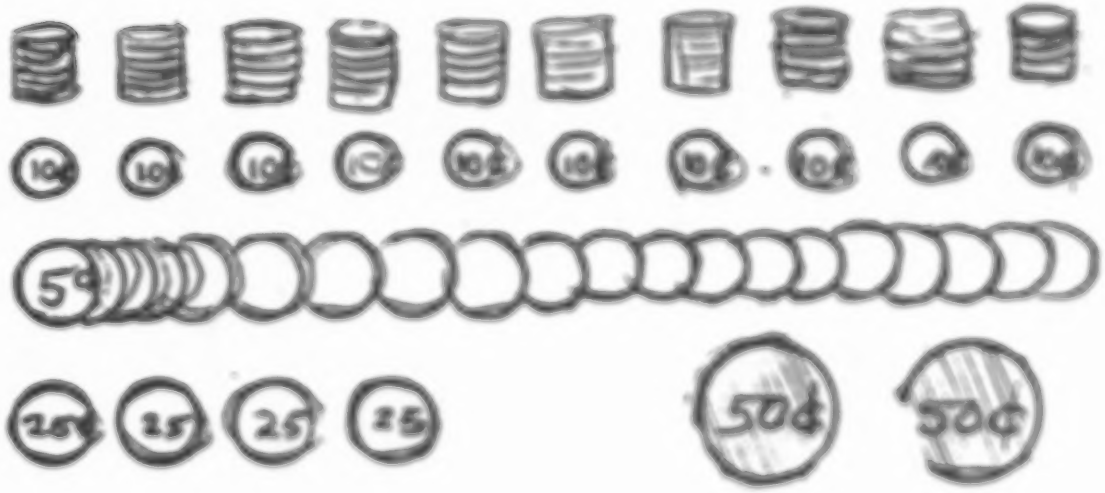
What might be better than many paper squares? Make a grid or use graph paper with 1" squares. Give each team this grid and pictures of the lakes reproduced on tracing or other transparent paper. 5:34-37
Have them place the transparent paper over the graph paper and count the squares enclosed by the boundaries of the lakes. They will later appreciate better the short cut algorithm for computing areas.

*Also ditto a grid on onion skin or other transparent paper to be placed over pictures or other objects whose area is to be measured.

Money

Draw several number lines on the board and discuss the dollar and its equivalent amounts of coins. 12:80-85

Have teams of children, using their toy money, make sets of coins that equal one dollar. Make a large chart and glue real money to it as shown to the left. 4:200-214



One dollar has the same value as any set of coins with the value of one hundred cents.

$1.00 = 100¢

35¢
25¢
10¢
70¢

Stress that when we add money we add numbers, not dollars or cents. However, we must always label the answer with the unit of measure. We don't say "It cost 70" -- 70 what? So we don't write 70 -- we write 70¢. (Or $.70).

Dollar sign and separating point

	Dollars	Dimes	Pennies
$	6	2	1
	6	2	1

(Use dotted lines)

Show the comparison of place value and money symbols on a large chart on the blackboard. Start with (1¢) $.01 and work down having children fill in the amount--write it again in a list on the board.

	Dollars	Dimes	Pennies
1¢	$	0	1
2¢	$	0	2
3¢	$	0	3
4¢	$	0	4

$.01	$.02	$.03	$.04
$.05	$.06	$.07	$.08
$.09	$.10	--------->	
etc.			

Making change

Cost	Amount Given	Change
12¢	15¢	3¢
14¢	25¢	11¢
20¢	25¢	5¢
45¢	50¢	5¢
32¢	40¢	8¢
6¢	10¢	4¢
48¢	50¢	2¢
70¢	$1.00	$.30

Dramatize customer and clerk situations, using play money and objects for sale. Give the customer varying amounts of money as follows:

"I would like a chocolate bar."
"It costs 12¢."
"Here is a quarter."
"Here is your candy. 12¢ and 3 pennies make 15¢ and a dime makes 25¢.

Have charts available on dittoed paper -- for instance:

Tom has $1.00. He bought a book for $.50. How much change did he get back? 21:71

$.50

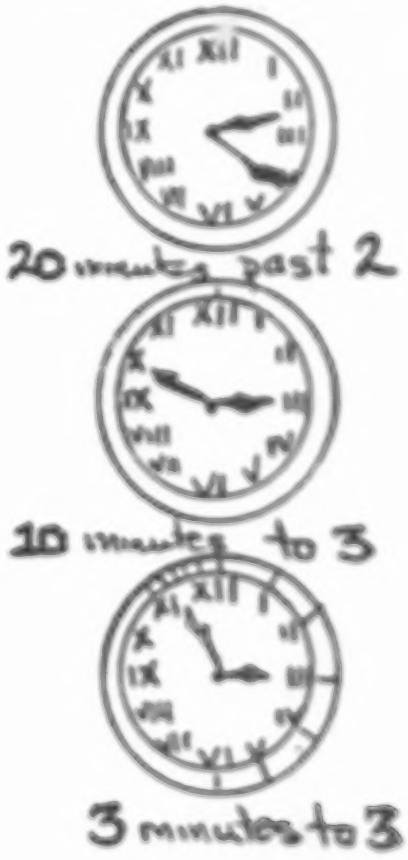
Time

Drew a clock on the board, with the minute hand pointing to 12. Also display a large demonstration clock. 22:98,99

Ask where the minute hand will be in a half-hour. Have a child move the hand on the play clock and drew an indication of the hand's new position on the board.

Shade in the area the minute hand covered as it traveled from first to second position. How much of the clock face is colored? (Use colored chalk).

Clocks with Roman numerals



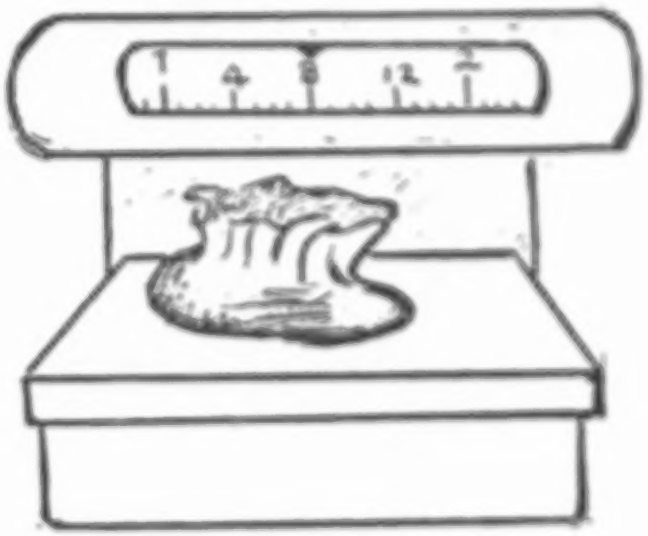
Do similar experiences with quarter hours and 5 minute, 10 minute, 20 minute, 30 minute, etc., sections of the clock face.

Use clock faces with Roman numerals. If the children haven't been introduced to Roman numerals as yet provide them the experience of games with these numbers. Have them fill in clock faces using them on dittoed paper.

Weight -- there are 16 oz. in a pound

Have the children name things that come in pounds. (Butter, margarine, candy, beans, nuts, etc.) Provide a scale clearly marked off in pounds and ounces.

Bring in a pound of margarine and various objects of different weights. Have children estimate by lifting objects and recording their guesses. (Man first balanced objects in his hands to guess their



What does the candy weight?

a. 1½ lbs.
b. 1 lb. 8 oz.
c. 24 oz.

Fish weights (sinkers) can be purchased which come in ounces.

relative weights.) Have teams decide relative weight of objects and put them on a table in the order of lightest to heaviest. Then weigh them on the scale to verify. Note that a pound is made up of smaller units of measure called ounces. There are 16 oz. in 1 pound. Discuss fractions of a pound and make a chart with all teams working together.

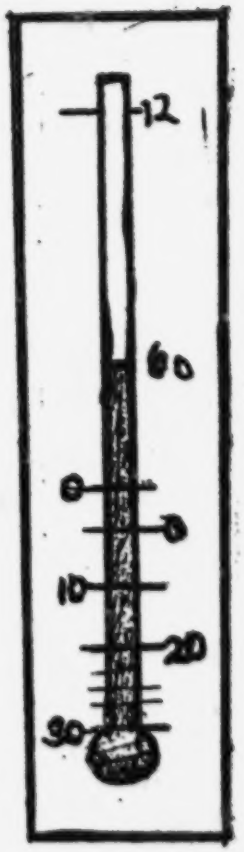
1 pound = 16 oz.
½ pound = 8 oz.
¼ pound = 4 oz.
etc.

Have more than one kind of scale available. 10:112
A team of children could try making a simple balance scale themselves. Explain that a balance scale can not tell us how much something weighs -- it is just a device to compare the weights of objects.

If a teeter-totter is available have the children use it as a balance scale to compare their relative weights.

Temperature

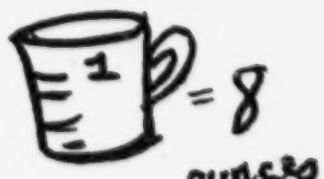
32 ° freezing point



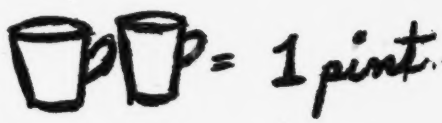
Have the children make their own thermometers. Use strips of heavy tagboard, cut slits in top and bottom and insert elastic which is colored $\frac{1}{2}$ red and $\frac{1}{2}$ white. Use a ruler to evenly space the marks for degrees. Note that the thermometer scale usually has a division for each 2 degrees and only the 10 degree marks have numerals.

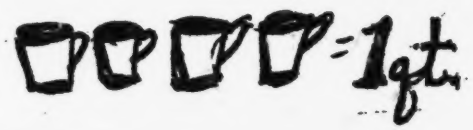
Keep a daily temperature chart and if the children have not had the experience of discovering that 32 degrees is the freezing point for water, refer to the study of temperature in second grade.

Liquid



Provide the math interest center with containers holding cups, pints, $\frac{1}{2}$ pints, quarts, half gallons, gallons. Remember that we discovered in 2nd grade that one measuring cup = 8 liquid ounces. By doing pouring experiences discover how man 8 oz. cups are in a pint, quart, etc. Make a chart for this information.





8 oz.
x 4
32 oz.

By multiplying find how many liquid ounces there are in some of the larger containers of measure.

32 oz. = 1 quart

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O.C.S.E.I.P. SYLLABUS

Grade 4

SE006 065

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
OFFICE OF EDUCATION

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PREFACE

The Orange County Science Education Improvement Program (O.C.S.E.I.P.) is sponsored by the National Science Foundation and hosted by U.C. Irvine. It is a cooperative venture undertaken by the University of California, Irvine, California State College at Fullerton, the Orange County Schools Office and local school districts throughout Orange County. This syllabus was written by O.C.S.E.I.P. to help teachers teach the best aspects of recent mathematics programs. It is not meant to be another textbook for a new program. Instead, it is meant to be a sharing and synthesis of effective teaching methods. The outline of topics is a minimum coverage which is common to all schools in Orange County. Topics adequately covered in the majority of texts in use are given a minimum treatment in the syllabus.

The first draft of this syllabus was written during an 8 week session at University of California, Irvine during the summer of 1966 by:

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We wish to thank all the participants in this program for their hard work and fine cooperation.

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NUMBERS AND NUMERATION

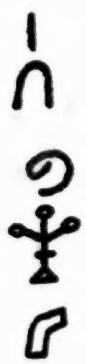
GRADE 4

Grade 4

NUMBERS AND NUMERATION

Place Value.

1) and 2)



staff = 1

heel bone = 10

scroll = 100

lotus flower = 1,000

pointed finger = 10,000 etc.

1) For motivation, prepare a bulletin board displaying Egyptian numerals and their values in our number system.

2) Have children practice writing Egyptian numerals.
   a. Lead children to discover that addition was used in writing Egyptian numeral. No place value was used.

3) Roman Numerals:

CCCLIII = 353 (Addition only)

CMXLIV = 944 (Subtraction only)

XCVII = 97 (Addition and Subtraction combined)

3) Through oral discussion and worksheets, introduce and work with Roman numerals.
   a. Lead children to discover that the Roman system used both addition and subtraction in writing numerals.

4) <u>569</u> (Hindu-Arabic)

Egyptian	Roman
	DLXIX

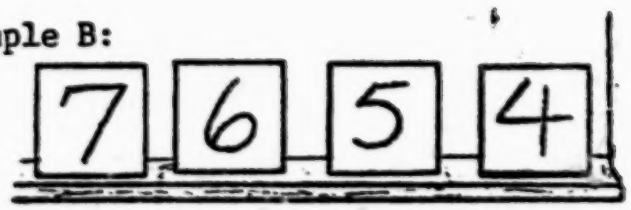
4) Review our place value system.
   a. Lead children to discover the efficiency of our system as compared to the two above.

Reading and writing numerals through 99,999,999.

Example A:

hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones

Example B:



Review place value with the class. Introduce millions.

The next number after 99 is
The next number after 699 is
The next number after 999,999 is

Have the class practice reading and writing numerals:

1) Put place value grid on the chalkboard with place value names and period names indicated. Give students numerals orally which they must write in the grid. (Example A)

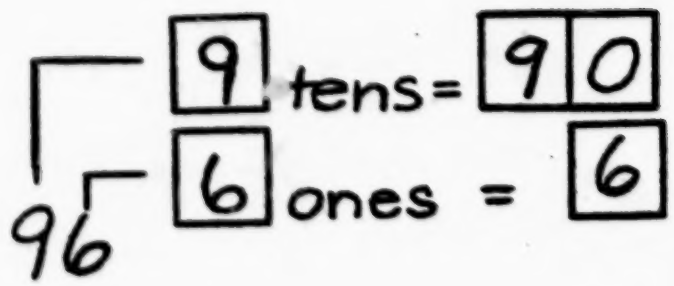
2) Give each student a 4 x 6 index card with a large numeral written on it. Call students one by one to place their card in the chalk tray. As each numeral is placed, have the class say its name aloud. Begin by placing cards only right to left. Then allow each student to place his numeral in any place he chooses, i.e., at the left, at the right, next to the third card, etc. (See Example B)

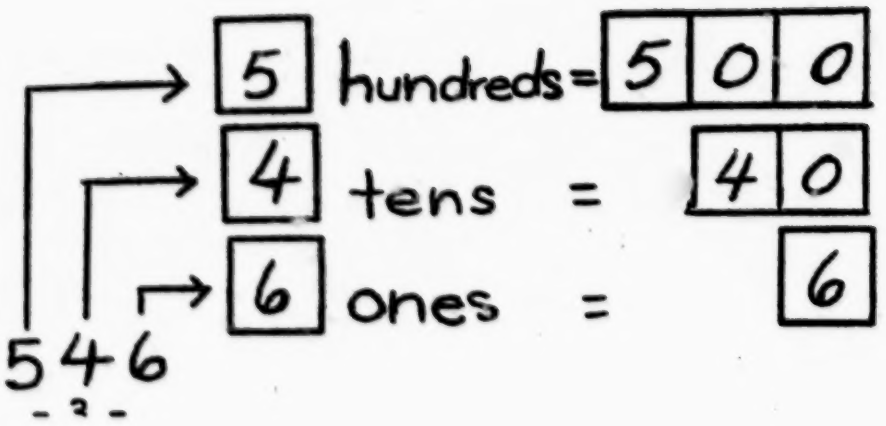
3) Using the same index cards, call five students to the front and let them each choose a numeral to hold before the class. Using only the five numerals, let them rearrange themselves before the class to show as many different number names as they can. After each numeral is shown, have class call its name and the place value names of some or all of its digits. A record can be kept of the different names made from only five numerals by each team. Thus, a continuing contest can be carried on during the year which is basically a review of place value.

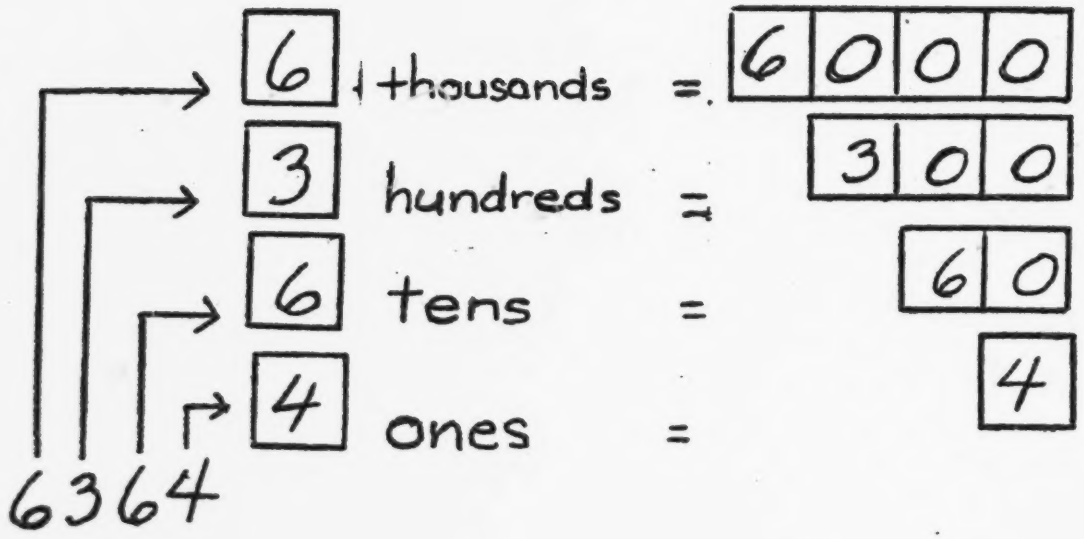
Review other number names for numbers smaller than 100. Then build on this concept.

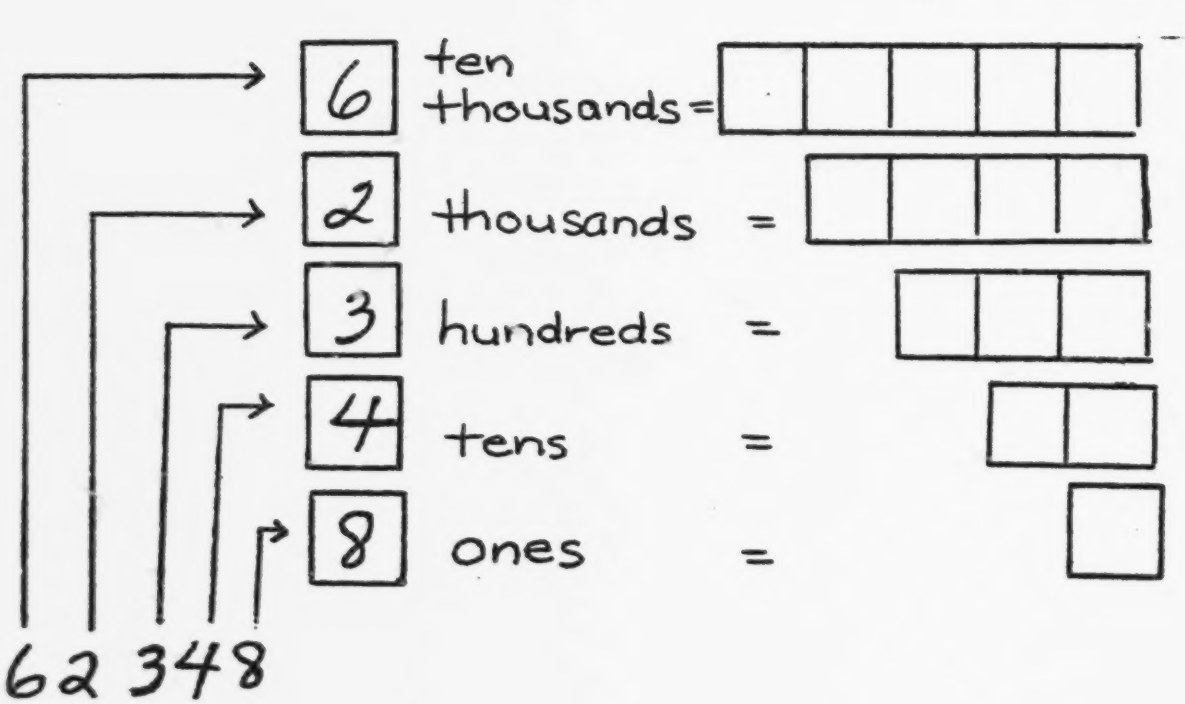
Expanded notation through five-place numerals.

62,348
↓
60,000 + 2,000 + 300 + 40 + 8






6 thousands = 6000
3 hundreds = 300
6 tens = 60
4 ones = 4
6364


6 ten thousands =
2 thousands =
3 hundreds =
4 tens =
8 ones =
62348

Give students many expanded numerals to name orally. "Name the number that has four thousands, three hundreds, seven tens, and five ones." (4,375

Written exercises should include opportunities to name the number that is shown in expanded form as:

a. 5 hundred thousands, 7 ten thousands, 3 hundreds, 9 tens, and 3 ones.

b. 9 x (100,000) + 1 x (10,000) + 3 x (1,000) + 9 x (10) + 7 x (1) = _____.

Ordinal numbers beyond 31st.

32nd, 33rd, 34th, 35th, 36th, 37th, 38th, 39th, 40th, etc.

The ordinal numbers from 1st through 31st have been covered by reference to our calendar.

Show the children orally and by board examples that the numbers between 30 and 40, 40 and 50, 50 and 60, etc., can all be read in the same manner as were the numbers between 20 and 30. (22nd, 23rd, 24th, etc.)

Practical application suggestions can be drawn from the class.

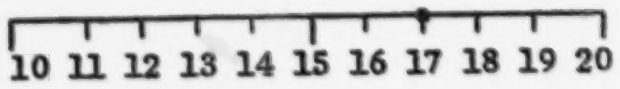
Teacher examples:

a. Placement in class = 34th
b. Placement of P.E. achievement or order (40th at bat this week).

c. Contest - 1000th customer in 1966.

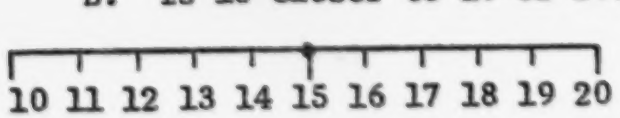
Rounding Numerals.

A. Is 17 closer to 10 or 20?



In teaching students to round off numbers, it is best to use a visual aid such as the number line. In using the number line, the youngsters are better able to see the relationship between 10 and 17 and 17 and 20. (Example A)

B. Is 15 closer to 10 or 20?



In a case such as this (Example B), it is best to let the class decide whether the numeral 15 is to be considered closer to 10 or 20. It is not important whether they decide that 15 is closer to 10 rather than 20. The important thing here is that the whole class agrees. It would also be a good idea to let the youngster know that in most situations when one comes to 5 or a numeral is considered closer to the highest numeral mentioned. But when a numeral is lower than 5, the numeral is then said to be closer to the lower of the numerals mentioned.

ADDITION AND SUBTRACTION

Grade 4

Grade 4

ADDITION AND SUBTRACTION

Addition of whole numbers with four-place addends

Regular	Irregular
3,673	2,765
2,364	329
6,037	7,654
	10,758

Review the meanings of addition and subtraction. Review the types of addition and subtraction examples learned in the third grade and extend the addition to include four-place numbers, both regular and irregular columns.

A)
$$\begin{array}{r} {\scriptstyle 1\ \ \ } \\ 39 \\ +\ 21 \\ \hline 60 \end{array}$$

B)
$$\begin{array}{r} {\scriptstyle 11\ \ \ } \\ 999 \\ +\ 1 \\ \hline 1{,}000 \end{array}$$

C)
$$\begin{array}{r} {\scriptstyle 11\ \ \ } \\ 826 \\ +\ 795 \\ \hline 1{,}621 \end{array}$$

D)
$$\begin{array}{r} {\scriptstyle 11\ \ \ } \\ 832 \\ 326 \\ 427 \\ +\ 620 \\ \hline 2{,}205 \end{array}$$

The children should be familiar with place value of ones, tens, and hundreds from the third grade. They now must add to their mathematical language the next (4th) place of "thousands"; this can be achieved by asking them what becomes the next step after we reach 999, as when 99 became 100? The children have previously learned that 9 + 1 = 10 and that we have had to regroup or carry when we are using two or more addends whose sum is ten or more (Example A).

Point out that when we add 1 to 999, the mechanics of working it out (carrying, regrouping) is the same technique, but just expanded one more column. This column is then named the thousand's column (Example B). Have them work several examples in which the hundred's column total will become a sum in the thousand's column (Example C). More addends can be added as proficiency increases (Example D). Give the children sufficient practice for them to feel secure enough to go on to 9,999 + 1 = 10,000.

Referring to the transformation from hundreds to thousands by adding 1 to 999, the same procedure can be used to direct the children into discovering

D) $\overset{1}{3},\overset{1}{6}42 + 9,838 = 14,480$ etc.

E) $99,999 + 1 = 100,000$

that 9,999 + 1 = 10,000. Again add to their vocabulary and their understanding that this five-digit numeral is now called ten-thousands; the same procedure can be used to take them to the six-place column of 100,000(Example D and E).

$$\begin{array}{r} {\scriptstyle 1\ 11} \\ 86,476 \\ +\ 54,936 \\ \hline 141,412 \end{array}$$

Stress the fact that basically the mechanics are the same. You can make the point more meaningful by explaining that the comma acts as a divider progressing to the left. That, as in the one's, ten's, and hundred's columns to the right of the comma, we merely repeat the same procedure in naming our columns progressively to the left of the comma: ONE thousands, TEN thousands, ONE HUNDRED thousands, etc.

$$\begin{array}{rl} 5,235 = & 5000 + 200 + 30 + 5 \\ +\ 6,804 = & 6000 + 800 + 00 + 4 \\ \hline & 11000 + 1000 + 30 + 9 \end{array}$$

Expanded notation can be used as a solidifying factor towards strengthening the concept of regrouping.

Column addition utilizing convenient grouping

$$\begin{array}{rl} 5\left(\begin{smallmatrix}2\\3\end{smallmatrix}\right. & 5 + 5 = 10 \\ 6 & 10 + 6 = 16 \\ 5\left(\begin{smallmatrix}1\\4\end{smallmatrix}\right. & \end{array}$$

or: 6 + 4 = 10

10 + (5 + 1) = 16

The decimal system provides ease in computation. Children need to discover this ease by applying the associative and commutative principle so that the ten or five may be taken advantage of.

$$\begin{array}{r} 23 \\ 14 \\ 16 \\ +\ 22 \\ \hline \end{array}$$

(a) $20 + 10 + 10 + 20 = 60$

(b) $4 + 6 = 10$
$10 + (3 + 2) = 15$
$60 + 15 = 75$

Whenever children have a thorough understanding of place value, it is often times as easy or easier to add the tens, then the ones, and finally regrouping for the final sum.

Subtraction with three-place subtrahends and four-place minuends

Before students are able to do a subtraction problem with a three-place subtrahends and four-place minuends, they must be able to understand place value and be able to analyze numeral regrouping up to four places.

First experiences should include problems with no regrouping or carrying as in example A. These simple problems should be expanded occasionally to refresh student understanding of place value and to smooth the way for computation involving regrouping.

Regrouping should begin in ten's place, then hundred's place, and slowly moving to more complex work involving zeros in the minuend and "bridging" (regrouping in two places not adjoining such as tens and thousands). Students should be allowed to work within place value grids or on squared off dittos or graph paper.

A. No Regrouping:

$$\begin{array}{r} 5764 \\ -\ 132 \\ \hline 5632 \end{array}$$

$$\begin{array}{r} 5000 + 700 + 60 + 4 \\ -\quad (100 + 30 + 2) \\ \hline 5000 + 600 + 30 + 2 \end{array}$$

B. Regrouping Tens

$$\begin{array}{r} 2,632 \\ -\ 215 \\ \hline 2,417 \end{array}$$

$$\begin{array}{r} 2000 + 600 + 30 + 2 \\ -\quad (200 + 10 + 5) \\ \hline \end{array}$$

$$\begin{array}{r} 2000 + 600 + 20 + 12 \\ -\quad (200 + 10 + 5) \\ \hline 2000 + 400 + 10 + 7 \end{array}$$

C. Regrouping Tens and Thousands

$$\begin{array}{r} 5{,}078 \\ -\ 329 \\ \hline 4{,}749 \end{array}$$

$$\begin{array}{l} 5000 + 0 + 70 + 8 \\ -\quad (300 + 20 + 9) \end{array}$$

$$\begin{array}{l} 4000 + 1000 + 60 + 18 \\ -\quad (300 + 20 + 9) \\ \hline 4000 + 700 + 40 + 9 \end{array}$$

Expanded notation should precede the presentation of the conventional form to build understanding into the subtraction operation and the mechanics of regrouping.

Fractions

The word "fraction" is derived from "frangere" (Latin word meaning to break).

Fractions are equal parts of a whole object or a set of objects. It is important for children to know that every fractional number has many names. The lowest term is merely one of those names.

Uses of Fractions

1) Through discussion, guide students to suggest ways in which they use fractions.

   a. clock ($\frac{1}{2}$ hour, $\frac{1}{4}$ hour)
   b. sharing ($\frac{1}{2}$ an orange, etc.)

2) Teacher-motivated experiences:

   a. stress parts of a group (set) begin with small groups.
   b. a number line can be used to show parts of a whole.

c. dividing geometric shapes into fractional parts.
d. showing relationships of fractional parts.

Addition and subtraction of like fractional numbers, including proper and mixed fractional numerals

Use of the number line and diagrams will facilitate understanding of fraction work. Fractional numbers having the same denominator are less difficult for children at this level. When they need to think in terms of equivalent fractions to perform operations, the situation becomes more complex.

Let the students find out if the properties of whole numbers apply to fractional numbers. Let them discuss their results and justify their reasoning.

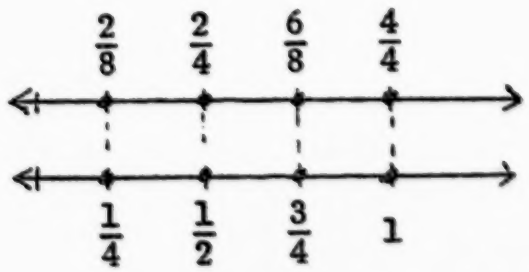
At this grade level, the important concept is the ability to add and subtract. Reducing to lowest terms need not be the focal point of this concept.

<u>Commutativity</u>: (True or False)

$\frac{1}{4} + \frac{2}{4} = \frac{2}{4} + \frac{1}{4}$

<u>Associativity</u>: (True or False)

$(\frac{1}{5} + \frac{2}{5}) + \frac{2}{5} = \frac{1}{5} + (\frac{2}{5} + \frac{2}{5})$



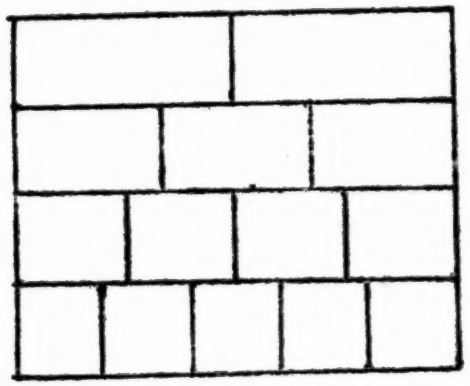
On a number line label the points to show halves, third, fourths, sixths, and twelfths. Ask the children to make up some examples of their own that can be used with the same number line.

The number line can also be used to show how equivalent fractions are related. Making comparisons of fractions by measuring physical objects with the eye provides a good foundation for abstract comparisons later on.

$\frac{1}{2} + \frac{1}{2} =$ △

$\frac{0}{4} + \frac{2}{4} =$ △

Comparison of fractional parts



Color 1/2 red.
Color 3/4 green.
Color 4/8 blue
Color 1/3 orange, etc.

Once the concept of a fractional number is understood, the student needs to see relationships between them. One method of comparison is a rectangle marked off into equal parallel lines. Each line is then divided into different fractional parts.

The students are to use different colors to show specified fractional parts of each line.

This exercise can also re-inforce the concept of equal parts of a whole.

$\_\_\_ + \frac{2}{4} = \frac{3}{4}$

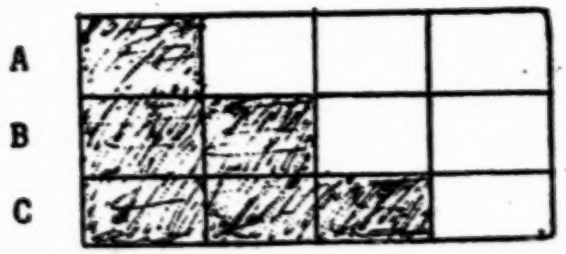
$\frac{2}{5} + \_\_\_ = \frac{5}{5}$

$\frac{3}{4} - \frac{2}{4} = \_\_\_$

Find the missing addends of the examples at the left. Answers should be based on examination of visual aids such as fraction rods, fraction circles, number lines, sets of counters, etc. Missing addends is an important bridge to subtraction.

$\frac{2}{8} \triangle \frac{3}{8} = \frac{5}{8}$ $\qquad$ $\frac{11}{11} = \frac{11}{11} \triangle \frac{0}{11}$

Write the correct sign of operation (+ or -) in each $\triangle$.



A _____

B _____

C _____

A + B + C = △

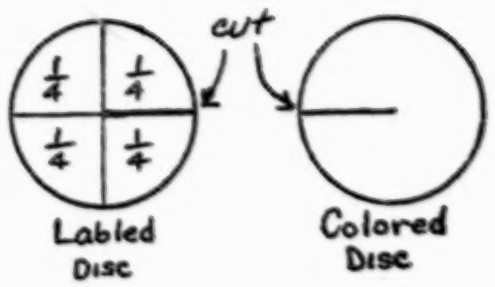
Write the fraction that represents the shaded area.

a. Write an addition sentence expressing the sum of A and B.
b. Write a subtraction sentence using A and B.
c. Write a subtraction sentence using C and A.

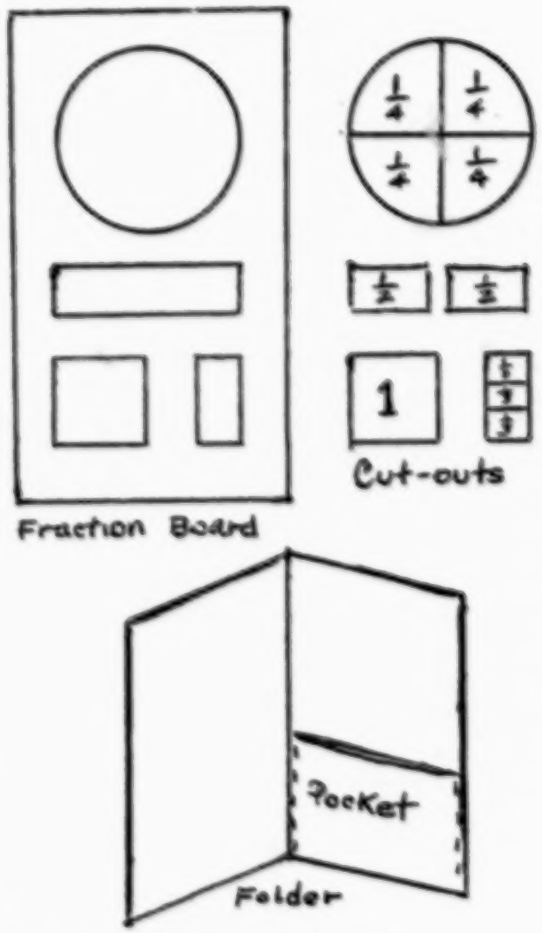
$\frac{1}{2} + \frac{1}{2} = \frac{1+1}{2} = \frac{2}{2} = 1$

$1 - \frac{1}{2} = \frac{2}{2} - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$

Subtraction as the inverse of addition can be shown by manipulating fractional parts and use of the number line.



Split two disks to their centers. Fit split sections together. Turn discs to discover addition and subtraction facts. Write numerals for facts as they are discovered. Use as class project. Later students may make up their own problems.



Individual fraction boards are made of tagboard or construction paper. Cutouts match fraction board. Place cutouts on board to add. Remove parts to subtract. A similar presentation can be performed using a flannel board.

Students may keep pocket folders of fraction cutouts for use in solving problems.

If $\frac{1}{2} + \frac{1}{2} = 1$, then $1\frac{1}{2} + \frac{1}{2} =$ ____

If $\frac{1}{4}$ and $\frac{3}{4} = 1$, then $2\ \frac{1}{4} + \frac{3}{4} =$ ____

If $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$, then $3\frac{1}{4} + 1\frac{1}{4} =$ ____

Using what the children have learned about whole numbers and fractional numbers, have them work with the following mixed fractions.

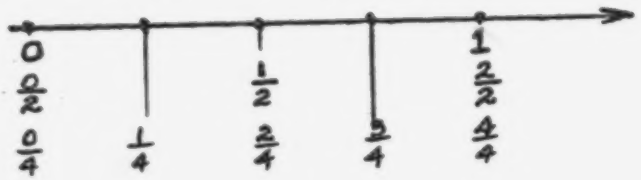
Other names for fractional numbers

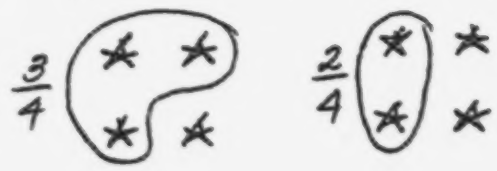
The students should be lead to the understanding that fractions have different names.

Equivalent Fractions

$\frac{1}{2} = \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}$

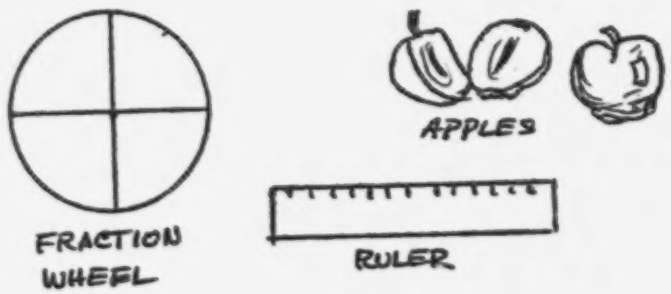
Relationship of fractional numbers--
halves, thirds, fourths, sixths





$\frac{3}{4}$ numerator

denominator



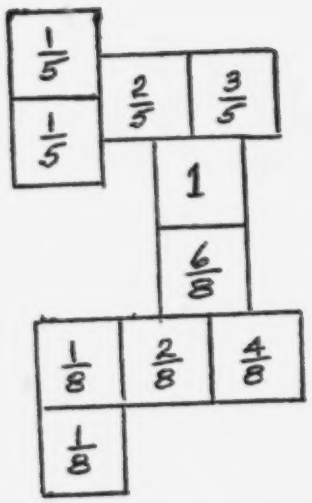
With visual aids (ie., Flannel parts, construction paper, and drawings on the board) have the children discover different names for the same fractional number.

1) Use the number line to identify different fractional names for the same number.

2) Use set concept. Which is greater: 3/4 or 2/4? Use sets having a like number of objects.

3) Use names for fractional parts. Denominator tells into how many parts the whole or set is divided. Numerator tells how many parts are given.

4) Use objects to identify and compare fractional parts.



5. Learn to read and identify fractional numbers named by equivalent, improper, proper, and mixed fractional numerals.

Fraction Dominoes

Purpose: To give practice in addition and subtraction of fractions

Players: Two to four

Materials: $1\frac{1}{2}$" x 3" oaktag dominoes with different fractions on each one.

Directions: Four dominoes are dealt to each player; the rest are put into the "bone pile." The game is started by the person with the highest double, such as 1/5 and 1/5, who puts it in the middle of the table. From then on, if the sum of the two fractions on the domino on the table equals one of the fractions on a domino in the player's hand, the domino is put down as shown in the example. A domino can also be put down if the sum of its two fractions equals a fraction on a domino on the table. If the player can't find a domino in his hand to play, he draws from the bone pile.

Cautions: Be sure the pupils thoroughly understand the directions for playing this game. Should be used as enrichment.

Familiarity of single division must precede this lesson.

To find $\frac{1}{2}$ of a number, divide the number by 2.

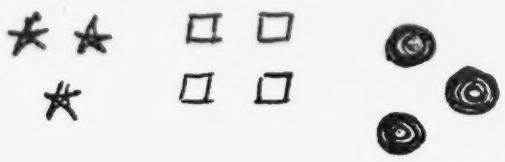
Bob took $\frac{1}{2}$ of 8. $\frac{1}{2}$ of 8 = 4 ($2\overline{)8}^{4}$).

How do you find 1/3 of a number? (Students should respond with "divide by 3.") 1/3 of 6; 1/3 of 15; 1/3 of 12.

Under various examples, write the fractions that show 1/2, 1/3, etc.

a) Draw a ring around apples that show halves, thirds, etc.

b) Draw a ring around pies that show quarters.



c) Draw a ring around each group that shows third.

d) Draw a ring around each group that shows fourths.

Properties of the operations of fractional numbers

Lead students to extend their generalizations about whole numbers to the fractional numbers. Place sentences showing associativity and commutativity of addition with fractions on the chalkboard. Ask students whether the sentences are true or false. Use manipulative aids or number lines to prove answers. Encourage students to use the terms "commutative" and "associative" at this level.

Commutative Property

$2 + 4 = 4 + 2$

$\frac{1}{2} + \frac{1}{4} = \frac{1}{4} + \frac{1}{2}$

Associative Property

$(1 + 2) + 3 = 6$

$1 + (2 + 3) = 6$

$(\frac{1}{4} + \frac{2}{4}) + \frac{3}{4} = \frac{6}{4}$

$\frac{1}{4} + (\frac{2}{4} + \frac{3}{4}) = \frac{6}{4}$

MULTIPLICATION AND DIVISION

Grade 4

MULTIPLICATION AND DIVISION

Multiplication and division concepts and techniques redeveloped and extended

Commutative Property

$8 \times 1 = 8$
$1 \times 8 = 8$

Expanded Notation

$$2 \times 42 = 2 \times (40 + 2)$$
$$= (2 \times 40) + (2 \times 2)$$
$$= (80 + 4)$$
$$= 84$$

Algorism

$$\begin{array}{r} 42 \\ \times\ 2 \\ \hline 4 \\ 80 \\ \hline 84 \end{array} \qquad \begin{array}{r} 42 \\ \times\ 2 \\ \hline 84 \end{array}$$

Just as zero has a special property, so does the number one. The number 1 is known as the identity element for multiplication. Any number multiplied by one or one multiplied by any number results in that same number. Again examples will clarify this for children.

This property of one has frequently been used by teachers to simplify learning of multiplication facts, just as the addition property of zero simplified the learning of addition facts.

Our numeration system is a place-value system; thus, the shorter algorism represents a brief, accurate method of performing the operation. In other words, multiplying 2 ones by 2 ones gives 4 ones. Then, multiplying 4 tens by 2 ones gives 8 tens. The 8 is put into the ten's column to the left of the 4 in the one's column. The alignment of the numerals in the vertical algorism is correct for two reasons: 1) place value of our numeration system and 2) the distributive principle of multiplication over addition.

With children, examples are more meaningful if they are explicit and simple. Several examples with numerals instead of letters may be tested by children for their accuracy.

Multiplication is interpreted as repeated addition. Most pupils will benefit by working several examples of number lines even though they may have reasonable rote memory mastery.

Commutative

$3 \times 2 = 2 \times 3$
$a \times b = b \times a$

Associative

$(9 \times 5) \times 4 = 9 \times (5 \times 4)$
$(a \times b) \times c = a \times (b \times c)$

Distributive

$3 \times (20 + 3) = (3 \times 20) + (3 \times 3)$
$= 60 + 9$
$= 69$

When children have seen the freedom of approach permitted in multiplication because of the commutative, associative, and distributive properties, they can incorporate these ideas in looking for the simplest way to solve any problem. This is one of the chief aims of any good mathematics program--to lead children to discover and understand the easy way to solutions, not the difficult way.

However, in seeing that multiplication is distributive over addition, they may discover that, in some cases, division is also distributive over addition.

$405 \div 5 =$
$(40 + 5) \div 5 =$
$(400 \div 5) + (5 \div 5) =$
$80 + 1 = 81$

Multiplication with two-place multipliers and 3-place multiplicands

a.)

37	
x 40	
0	(ones x ones)
0	(ones x tens)
280	(tens x ones)
1,200	(tens x tens)
1,480	

Multiplication with two-place multipliers can begin with problems where the multiplier is some multiple of ten. This underscores the concept of place value and brings later understanding to the convention of indenting second partial products.

Several approaches can be used before the final algorithm is presented. Some are shown at the left and on the next page

b)
$$\begin{array}{r} 37 \\ \times\ 40 \\ \hline 0 \\ 0 \\ 280 \\ 1{,}200 \\ \hline 1{,}480 \end{array} \quad \begin{array}{l} (\ 0 \times 7 = 0 \\ (0 \times 30 = 0) \\ (40 \times 7 = 280) \\ (40 \times 30 = 1200) \end{array}$$

c)
$$\begin{aligned} 40 \times 37 &= 40 \times (30 + 7) \\ &= (40 \times 30) + (40 \times 7) \\ &= 1200 + 280 \\ &= 1480 \end{aligned}$$

d)
$$\begin{array}{r} 37 = 30 + 7 \\ \times\ 40 \quad \times\ \ 40 \\ \hline 1200 + 280 = 1480 \end{array}$$

e)
$$\begin{array}{r} 37 \\ \times\ 40 \\ \hline 00 \\ 1480 \\ \hline 1{,}480 \end{array}$$

f)
$$\begin{array}{r} 37 \\ \times\ 40 \\ \hline 00 \\ 148 \\ \hline 1{,}480 \end{array}$$

g)
$$\begin{array}{r} 37 \\ \times\ 40 \\ \hline 1480 \end{array}$$
← Conventional Algorithm

These approaches can be made even more meaningful if students work problems on graph paper and label the place value columns "ones, tens, hundreds, and thousands."

Work can now be extended to include two-place multipliers not ending in zero. Avoid complex regrouping at first, using simple examples such as:

a)
$$\begin{array}{r} 23 \\ \times\ 12 \\ \hline 6 \\ 40 \\ 30 \\ 200 \\ \hline 276 \end{array} \quad \begin{array}{l} \text{(ones x ones)} \\ \text{(ones x tens)} \\ \text{(tens x ones)} \\ \text{(tens x tens)} \end{array}$$

b)
$$\begin{array}{r} 23 \\ \times\ 12 \\ \hline 6 \\ 40 \\ 30 \\ 200 \\ \hline 276 \end{array} \quad \begin{array}{l} (2 \times 3 = 6) \\ (2 \times 20 = 40) \\ (10 \times 3 = 30\ ) \\ (10 \times 20 = 200) \end{array}$$

c)
$$\begin{aligned} 12 \times 23 &= 12 \times (20 + 3) \\ &= (12 \times 20) + (12 \times 3) \\ &= 240 + 36 \\ &= 276 \end{aligned}$$

d.)
$$\begin{array}{r} 23 = 20 + 3 \\ \times 12 \quad \times \quad 12 \\ \hline 240 + 36 = 276 \end{array}$$

e.)
$$\begin{array}{r} 23 \\ \times\ 12 \\ \hline 46 \\ 230 \\ \hline 276 \end{array}$$

f.)
$$\begin{array}{r} 23 \\ \times\ 12 \\ \hline 46 \\ 23 \\ \hline 276 \end{array}$$

Conventional Algorithm

More complex work involving three-place multiplicands and regrouping (carrying) should be accompanied by the expanded notation form to insure real insight into the computation procedure.

a.) Multiplicand Expanded:

$$\begin{aligned} 36 \times 589 &= 36 \times (500 + 80 + 9) \\ &= (36 \times 500) + (36 \times 80) + (36 \times 9) \\ &= (18{,}000) + (2{,}880) + (324) \\ &= 21{,}204 \end{aligned}$$

b.) Multiplicand and Multiplier Expanded:

$$\begin{aligned} 36 \times 589 &= (30 + 6) \times (500 + 80 + 9) \\ &= (30 \times 500) + (30 \times 80) + (30 \times 9) + (6 \times 500) + \\ &\quad (6 \times 80) + (6 \times 9) \\ &= 15{,}000 + 2{,}400 + 270 + 3{,}000 + 480 + 54 \\ &= 21{,}204 \end{aligned}$$

c.)
$$\begin{array}{r} 589 \\ \times\ 36 \\ \hline 54 \\ 480 \\ 3000 \\ 270 \\ 2400 \\ 15000 \\ \hline 21{,}204 \end{array}$$

d.)
$$\begin{array}{r} 589 \\ \times\ 36 \\ \hline 3534 \\ 17670 \\ \hline 21{,}204 \end{array}$$

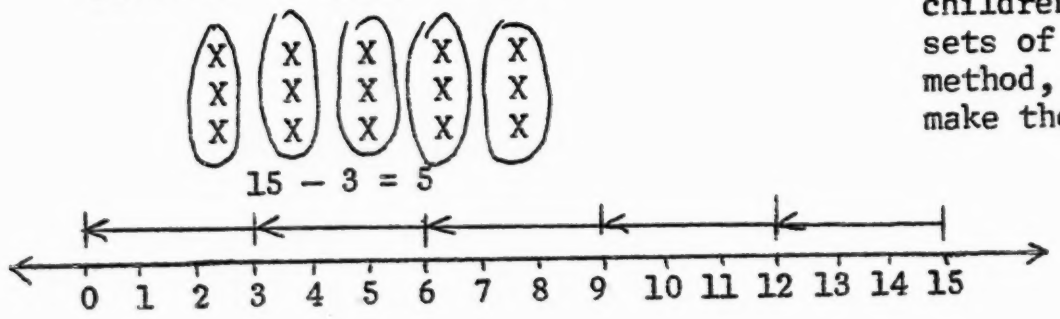
e.)
$$\begin{array}{r} 589 \\ 36 \\ \hline 3534 \\ 1767 \\ \hline 21{,}204 \end{array}$$

Conventional Algorithm

When students have trouble understanding why the second partial dividend is indented but do not seem to profit from the expanded notation analogies, try this variation:

589	589	589
x 6	x 30	x 36
3,534	17,670	
		????

Division of whole numbers



15	12	9	6	3
- 3	- 3	-3	-3	-3
12	9	6	3	0

3 was subtracted from 15 _____ times.

Division, one of the most difficult concepts for children, needs to be carefully developed using sets of objects, the number line, the subtractive method, and renaming the dividend in any way to make the solution easier to find.

15 ÷ 3 =
(12 + 3) ÷ 3 =
(12 ÷ 3) + (3 ÷ 3) =
4 + 1 = 5

15 ÷ 3 =
(9 + 6) ÷ 3 =
(9 ÷ 3) + (6 ÷ 3) =
3 + 2 = 5

Guide students to see whether or not the properties of multiplication also apply to the operation of division.

A. $\begin{array}{l} 2 \times 8 = 8 \times 2 \\ 2 \div 8 = 8 \div 2 \\ 2/8 \neq 4 \end{array}$

B. $\begin{array}{r} (8 \times 4) \times 2 = 8 \times (4 \times 2) \\ (8 \div 4) \div 2 = 8 \div (4 \div 2) \\ 2 \div 2 = 8 \div 2 \\ 1 \neq 4 \end{array}$

C. $\begin{array}{r} 12 \div 6 = (6+6) \div 6 \\ 2 = (6 \div 6) + (6 \div 6) \\ 2 = 1 + 1 \end{array}$

D. $\begin{array}{r} 12 \div 6 = 12 \div (4+2) \\ 2 = 12 \div 4) + (12 \div 2) \\ 2 = 3 + 6 \\ 2 \neq 9 \end{array}$

E. $\begin{array}{l} 1 \div 4 \neq 4 \div 1 \\ \frac{1}{4} \neq 4 \end{array}$

Guide students to see whether or not the properties of multiplication also apply to the operation of division. Review the commutative principle of multiplication on the chalkboard. Erase the multiplication signs and insert the division signs. Ask whether the sentence is now true (Ex. A). Repeat this procedure with the associative and distributive principles (Ex. B and C). Example C shows that division is distributive over addition when the dividend is renamed. Example D points out that division cannot be distributed over the divisor. In example E it is shown that division does not have an identity element.

Patterns in division

$3 \times 1 =$	$3 \div 1 =$
$3 \times 10 =$	$3 \div 10 =$
$3 \times 100 =$	$3 \div 100 =$
$3 \times 1000 =$	$3 \div 1000 =$

A crucial prerequisite to division is experience in estimation built on division patterns that reinforce the child's understanding of multiples. Thus, division by a one-digit numeral depends on the multiples of the set of counting numbers or simply the products found in the multiplication table. To divide a number by 6, the student must know the multiples of 6 (6, 12, 18, 24, 30, 36,...). To divide efficiently by two-digit numerals, he must know the multiples of ten to estimate the quotient. Division by three-digit numerals demands knowledge of

multiples of one hundred, etc.

$3 \times 31 < 100$
$200 \times 31 < 7000$
$1000 \times 31 < 42{,}000$

Inequalities such as these at the left are a further aid to estimating quotients in division. The student seeks the largest multiple of 1, 10, 100, or 1000 that makes the sentence true.

Experience in estimation, multiples of numbers, and relationships of numbers all help to strengthen division skills and concepts. An analogy between any two of the experiences must be closely related in order to be effective instead of forcing the student into thinking about the experiences as separate work orders. The immature workers will need ample practice in estimating trial quotients. The following activity points out how multiples and inequalities work together to bring efficiency to estimation of trial quotients when dividing by a one-digit numeral. When this stage is mastered, similar exercise work should precede division with two digit divisors.

1:71

7:223-227

1:71

1) Circle the numeral naming the greatest multiple of 10 that will make each sentence a true statement:

$3 \times n < 157$	20	30	40	(50)	60
$3 \times n < 218$	40	50	60	(70)	80
$9 \times n < 567$	50	(60)	70	80	90

2) Circle the numeral naming the greatest multiple of 100 that will make each sentence a true statement.

4 x n < 899	100	(200)	300	400	500
3 x n < 754	100	(200)	300	400	500
6 x n < 927	(100)	200	300	400	500

3) Find the quotient for each division problem below:

a. $3\overline{)157}$ d. $4\overline{)899}$

b. $3\overline{)218}$ e. $3\overline{)754}$

c. $9\overline{)567}$ f. $6\overline{)927}$

Division with two-place divisors ending in 0, 1, 2, 3

Much practice should be given in division by two-place divisors ending in zero first. Encourage students to think of the inequality sentence that would go with each division example.

```
      22 r 4
30 ) 674
   -600 | 20
     74 |
    -60 |  2
     14 | 22
```

30 x ? < 674
30 x 20 < 674
30 x ? < 74
30 x 2 < 74

```
      21 r4
15 ) 349
   - 300 | 20
     49  |
    -45  |  3
      4  | 23
```

```
      31
23 ) 713
   -69 | 30
    23 |
   -23 |  1
       | 31
```

Consider this example: How many multiples of fifteen can be subtracted from 349? Can 100 fifteens be subtracted? (15 x 100 = 1500) 1500 > 349. Can 10 fifteens be subtracted? (15 x 10 = 150) 150 < 349. Subtract 20 fifteens from 349. Now, how many fifteens can be subtracted from 49?

Division with three-place quotients with or without remainders, using the subtractive method.

```
    608 r 7
9 ) 5479 |
    5400 |  600        9 x ? < 5479
    ---------          9 x 600 < 5479
      79 |             9 x ? < 79
      72 |    8        9 x 8 < 79
    ---------
       7 |  608
```

Division by three-place quotients requires knowledge of multiples of 100. Again encourage students to think of, or write out if necessary, the corresponding inequality sentences.

Place value in division

Often children have trouble in estimating "how many" places a division problem will have in the quotient. If a chart such as the one below is made available, it eases the learning process.

HOW MANY PLACES

tens ÷ tens = ones	$12\overline{)36}$ = 3
hundreds ÷ tens = tens	$12\overline{)372}$ = 31
thousands ÷ tens = hundreds	$12\overline{)3720}$ = 310
hundreds ÷ hundreds = ones	$224\overline{)672}$ = 3
thousands ÷ hundreds = tens	$224\overline{)7168}$ = 3
thousands ÷ thousands = ones	$2244\overline{)4488}$ = 2

Place value in multiplication

Likewise, a chart can be used to determine the number of places in a multiplication algorism.

HOW MANY PLACES	
tens x ones = tens	12 x 4 48
tens x tens = hundreds	12 x 24 48 24 288
tens x hundreds = thousands	248 x 12 496 248 2976
hundreds x ones = hundreds	120 x 4 480
hundreds x tens = thousands	124 x 36 744 472 5464
thousands x ones = thousands	3426 x 5 17,130

Number relationships to estimate quotient

```
      547
9/ 4907 |
   4500 | 500
    407 |
    360 |  40
     47 |
     45 |   5
      2 |
```

Thousands ÷ tens = hundreds! Answer will be hundreds.

9 x ? hundreds	<	4907
9 x 500	<	4907
9 x 500	=	4500
9 x ? tens	<	407
9 x 40	<	407
9 x 40	=	360
9 x ? ones	<	47
9 x 5	<	47
9 x 5	=	45

The computation may also be explained by the example 4699 ÷ 12.

According to the chart, thousands divided by tens will have an answer in the hundreds.

```
12/ 4699
   3600   300
   1099
   1080    90
     19
     12     1
      7
```

Think:

12 x ? hundreds < 4699

12 x 300 < 4699

12 x 300 = 3600

12 x ? tens < 1099

12 x 90 < 1099

12 x 90 = 1080

12 x ? ones < 19

12 x 1 < 19

12 x 1 = 12

ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION GAMES AND DRILL

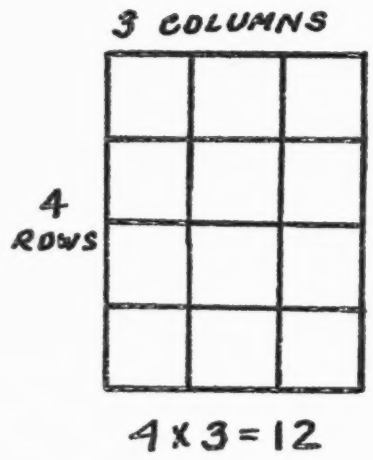
Grade 4

Multiplication and Division Games and Drill

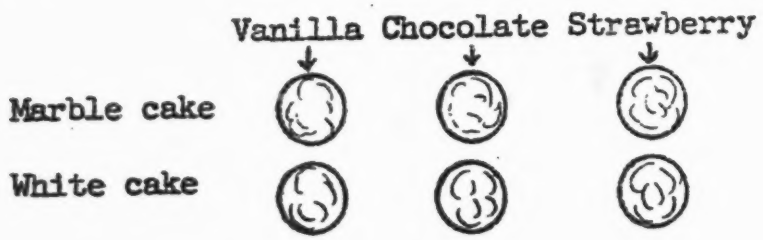
Arrays used in multiplying

a. eggs in a carton
b. panes of glass
c. seats in an auditorium
d. pieces of candy in a box
e. crayons in a box
f. linoleum blocks, tile, etc.

"Can you think of some objects arranged in arrays to add to the list of examples?"



(a)



Arrays can be used to show matchings of one set with another or as a model to interpret multiplication. An array is an orderly arrangement of rows and columns. 11:229-250

Some examples of sets of objects that form arrays are shown at the left.

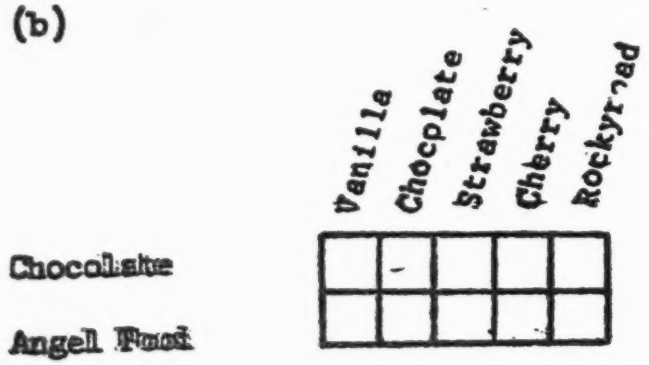
Draw an array of dots which represents a possible arrangement of 15 panes in a window.

How many panes are there in a window whose panes form a 4 by3 array? (12) Note that the first numeral (4) refers to the number of rows and the second numeral (3) refers to the number of columns.

Arrays are used to demonstrate the operation of multiplication. Their use does not imply that pupils are not to learn the multiplication facts for immediate recall. Facility with multiplication facts is essential before division and more difficult multiplication are attempted.

John went to the ice cream parlor to buy cake and ice cream and found that the shop sells two kinds of cake and three flavors of ice cream. How could he show how many choices of the combinations of cake and ice cream there are? We can make an array (a) to show the choices. (6)

(b)



Array (b) shows two kinds of cake and five kinds of ice cream combinations. How many choices are there? (10)

Use the dot arrays at the left to help you complete the table below.

Example:

a.

b.

c.

d.

Exercise	Rows	Columns	Elements
Example:	3	7	21
a	4	5	20
b	3	4	12
c	4	2	8
d	4	1	4

Addition, subtraction, multiplication and division of whole numbers

A.

15	1	11	27
5	9	13	27
7	17	3	27
27	27	27	27

B. (A + 26)

41	27	37	105
31	35	39	105
33	43	29	105
105	105	105	105

C. (B-13)

28	14	24	66
18	22	26	66
20	30	16	66
66	66	66	66

Magic squares are a series of numerals placed in horizontal, diagonal, and vertical lines within a main square, that when added will give the same answer in all directions.

Magic squares have often been used to camouflage drill, and that is their purpose here. However, a new element has been added. "Do magic squares remain magic and workable if the original numbers are changed by addition, subtraction, multiplication or division? 13:24

Example A is magic--the sum of the number in all rows, columns andmain diagonals is 27. "Suppose we add 26 to the number written in each of the nine cells. What happens? Have the students add 26 to each of the numbers in Example A, forming a new square. The result is example B. The sum of the three numbers noted in each row column and main diagonal is 105. Of course this could have been anticipated since: 15 + 1 + 11 = 27 and (15 + 26) + (1 + 26) + 11 + 26) = 15 + 1 + 11 + 3 (26) = 27 + 78 = 105.

The numerals now contained in Example B are large enough now so that a subtraction exercise could executed. In Example C at left, the number 13 has been subtracted from the numbers previously found in B. Have students add the rows, columns and diagonals to test whether the magic square remains magic under the operation of subtraction. All sums should total 66.

D. A x 5

75	5	55	135
25	45	65	135
35	85	15	135
135	135	135	135

Example D is a study of the results of multiplying the number noted in each cell of A by 5. Now each row, column and diagonal will be (5 x 15) + (5 x 1) + (5 x 11) or 5 x (15 + 1 + 11) or 5 x 27 or 135 so in B, C and D the magic property is preserved.

E. A x 6

90	6	66	162
30	54	78	162
42	102	18	162
162	162	162	162

Carry the operations further. Example F is a study of what happens when the number noted in corresponding cells of B and D are added. Example G shows the results of subtracting the number noted in each cell of D from the corresponding cell of E and the results are again magic.

Addition and subtraction with one, two and three-place numerals

F. C + D

116	32	92	240
56	80	104	240
68	128	44	240
240	240	240	240

G. E - D

15	1	11	27
5	9	13	27
7	17	3	27
27	27	27	27

H. E + 18

108	24	84	216
48	72	96	216
60	120	36	216
216	216	216	216

Carry the operations further. Example F is a study of what happens when the number noted in corresponding cells of B and D are added. Example G shows the results of subtracting the number noted in each cell of D from the corresponding cell of E and the results are again magic.

In each of the magic squares, property is preserved.

"Can you think of any operations that would destroy the magic property?" (Multiply each number by itself--square it, and the magic property is destroyed.) This will lead to many considerations of what operations will and will not affect the magic property. Each assertion can be tested by considering several examples. Thus, while the focus is on studying an intriguing problem, an unusual amount of exercise in basic facts and algorithms is employed.

I. F ÷ 4

29	8	23	60
14	20	26	60
17	32	11	60
60	60	60	60

J. F - H

8	8	8	24
8	8	8	24
8	8	8	24
24	24	24	24

K. H ÷ 4

27	6	21	54
12	18	24	54
15	30	9	54
54	54	54	54

L. F + I

145	40	115	300
70	100	130	300
85	160	55	300
300	300	300	300

M. H + K

135	30	105	270
60	90	120	270
75	150	45	270
270	270	270	270

N. G x J

120	8	88	216
40	72	104	216
56	136	24	216
216	216	216	216

Addition and subtraction of whole numbers

21		8	18
10	16	15	13
14	12	11	17
9		20	6

Constructing magic squares

Draw blank magic squares on the chalk board before class. (This serves as additional motivation). Let the children discover that "Magic Squares" add up to the same sum in 8 different directions. They will think this very simple, so progress to squares with numbers left out as shown in the diagram. Some of the squares were not "Magic" because they would not have equal sums all 8 ways. Let them discover this themselves.

Have them make their own number square conbinations. They will find this much more difficult then they expect. Check them over and pick one or two who have made workable squares to put them on the chalk board, leaving 2 squares blank.

Choose one that isn't magic to put on the chalk board to see if the students will spot it.

Magic squares with an odd number of cells on a side can be made by:

1. Place a starting number in the middle cell of the top row. The number may be a whole number, fraction, or decimal.

2. Each number is obtained by adding an equal quantity to the last number entered. Each new number is placed one cell to the right and one cell above.

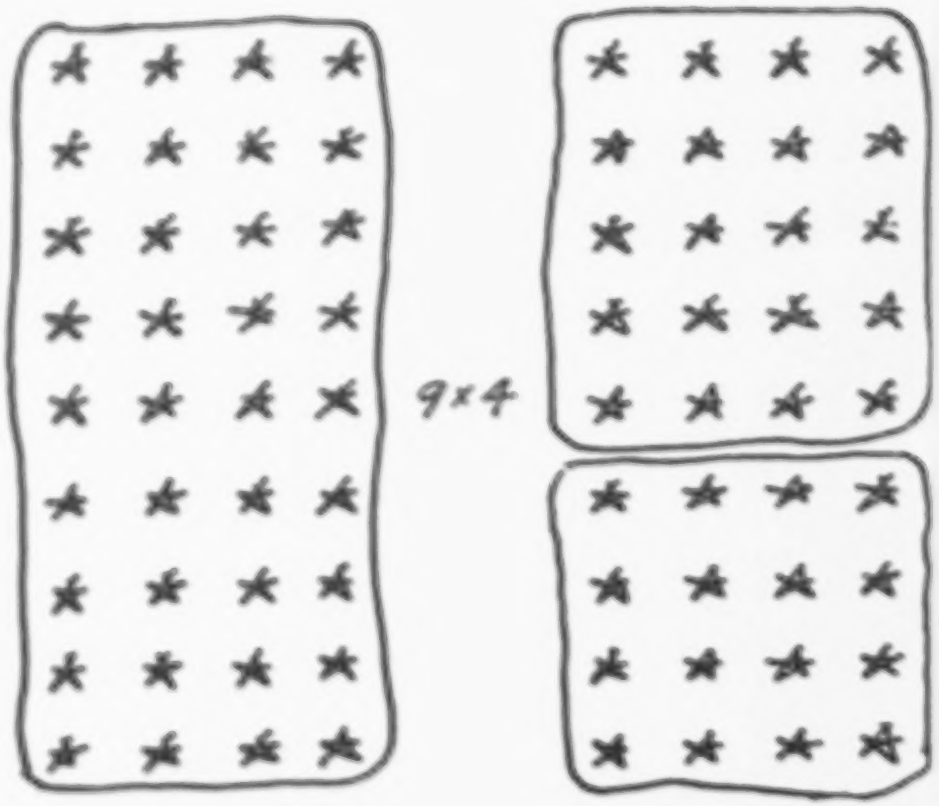
3. If a number falls outside the square, place that number at the opposite end of the row or column.

4. If a number falls in a cell already occupied, place the number in the cell below the last number entered.

5. If a number falls in a corner outside of the square, place the number in the cell below the last number entered.

Distributive property of multiplication over addition

We use devices to make up for our limited knowledge of multiplication facts. We generally use only facts in which neither factor is larger than nine, a multiple of ten that is not larger than 9 x 10, or a multiple of 100 that is not larger than 9 x 100. For the purpose of the lesson, we will "limit" our knowledge of combinations to the facts not larger than 5 x 5 and show how the distributive principle allows us to build the higher multiplication facts from those basic facts. 13:16-17



On flannel board or chalk board make a 4 x 9 array of squares or circular shapes. Ask students to find the total number of shapes using only multiplication facts "to the fives." Guide students to see how the set of shapes can be regrouped into two smaller sets, a 4 x 5 array and a 4 x 4 array. Since students know 4 x 5 = 20 and 4 x 4 = 16, they can quickly add the products 20 and 16 to find that 4 x 9 = 36.

Provide students with small objects to make their own 4 x 9 arrays. (bottlecaps, squares of graph paper, disks of construction paper etc.)

$$4 \times 9 = 4 \times (5+4)$$
$$= (4 \times 5) + (4 \times 4)$$
$$= 20 + 16$$
$$= 36$$

X	1	2	3	4	5	6
1	1	2	3	4	5	
2	2	4	6	8	10	
3	3	6	9	12	15	
4	4	8	12	16	20	
5	5	10	15	20	25	
6						

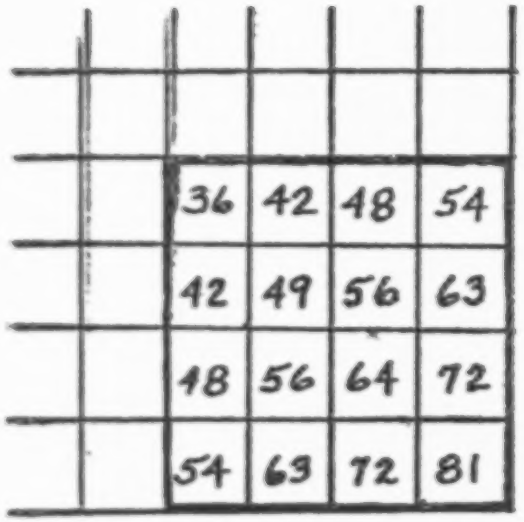
Guide students to see that 9 has been renamed as 5 + 4 and that the column of 9 markers has been regrouped into 5 + 4 markers. Place the number sentence 4 x 9 = (4 x 5) + (4 x 4) on the chalk board and ask students what it tells about their work. Encourage them to see that hard multiplication problems can be broken into two simpler multiplication problems whose products can then be added.

Provide students with graph paper to make multiplication tables. Have them fill in the products for the facts as large as 5 x 5 = 25. Ask students to make arrays on their desks that illustrate the "sixes" (1 x 6, 2 x 6, 3 x 6, 4 x 6, 5 x 6). Then ask students to regroup their arrays into two smaller arrays, thus renaming 6. Let students write number sentences on board to show the many ways a particular array may be regrouped. Thus a 2 x 6 array can be regrouped as:

a. $2 \times 6 = 2 \times (5 + 1)$
$= (2 \times 5) + (2 \times 1)$
$= 10 + 2$

b. $2 \times 6 = 2 \times (4 + 2)$
$= (2 \times 4) + (2 \times 2)$
$= 8 + 4$

c. $2 \times 6 = 2 \times (3 + 3)$
$= (2 \times 3) + (2 \times 3)$
$= 6 + 6$



		36	42	48	54
		42	49	56	63
		48	56	64	72
		54	63	72	81

d. $2 \times 6 = 2 \times (1 + 5)$
$= (2 \times 1) + (2 \times 5)$
$= 2 + 10$

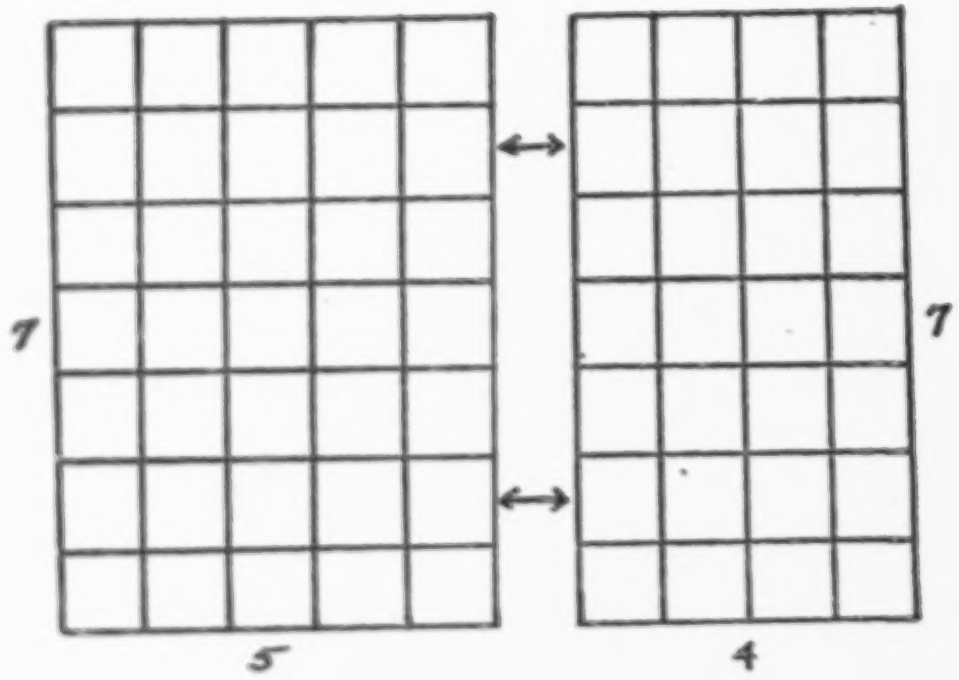
e. $2 \times 6 = 2 \times (2 + 4)$
$= (2 \times 2) + (2 \times 4)$
$= 4 + 8$

Continue with the "sixes," asking students to regroup arrays and rename 6 in various ways until $5 \times 6 = 30$ has been discussed. Have students fill in the "sixes" products thus found.

Use the distributive principle to build up the basic facts through $5 \times 9 = 45$. The students should have found all the products for their multiplication table except those to be placed in the 16 squares in the lower right-hand corner. To find these products both factors of the multiplication fact may be renamed using the distributive property and only the basic facts through $5 \times 5 = 25$. Or since the students should now know some of the "sixes," sevens," "eights," and "nines," they could continue renaming only one factor as before. Thus they now have two approaches to use in finding the product of 7×9:

1. Renaming only one factor.

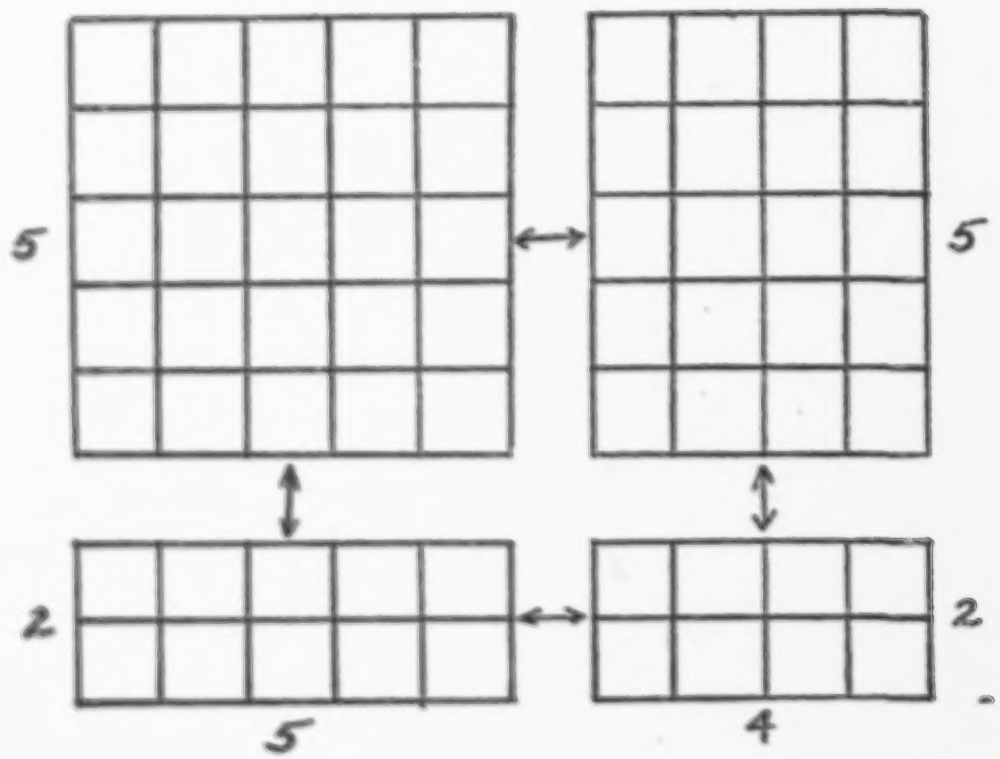
$7 \times 9 = 7 \times (5 + 4)$
$= (7 \times 5) + (7 \times 4)$
$= 35 + 28$
$= 63$



Note how the 7 x 9 array is regrouped into two smaller arrays whose products the students already know.

2. Renaming both factors.

$$\begin{aligned} 7 \times 9 &= (5 + 2) \times (5 + 4) \\ &= (5 \times 5) + (5 \times 4) + (2 \times 5) + (2 \times 4) \\ &= 25 + 20 + 10 + 8 \\ &= 63 \end{aligned}$$



This array is grouped into four smaller arrays whose products the students already know; 5 x 5, 5 x 4, 2 x 5, and 2 x 4.

When both factors are renamed let students cut up arrays made of 1 inch graph paper. Steps for the 7 x 9 fact would include:

1. Cut a 7 x 9 array.
2. Rename 7 as 5 + 2 by cutting the rectangle into two rectangles, 5 x 9 and 2 x 9.
3. Rename 9 as 5 +4 by cutting the 5 x 9 rectangle into two smaller rectangles, 5 x 5 and 5 x 4.
4. Rename 9 as 5 + 4 by cutting the 2 x 9 rectangle into two smaller rectangles, 2 x 5 and 2 x 4.

Let students cut arrays into smaller arrays whose products they know best. One student may choose to rename 6 x 7 as 6 x (5 +2) because he knows 6 x 5 and 6 x 2. Another child may rename 6 x 7 as 6 x (4 + 3) because he knows these basic facts best.

As the students explore these "harder" facts have them write the correct products in the lower right-hand corner of their multiplication tables.

Arrays may be used for drill on multiplication facts, flashing appropriate arrays on cards, flannel board, or overhead projector. When the illustrated fact is not quickly seen, divide the array into two smaller arrays that the student knows, using your hand, pointer, or some other device to divide the array. Eventually students should be able to give the multiplication facts quickly and easily in both written and oral form without the use of devices. The student stuck on 8 x 9 should be able to use the distributive principle mentally, reducing his problem to those facts he knows, such as

(8 x 5) + (8 x 4) or (4 x 9) + (4 x 9).

$$\begin{aligned} 2 \times 13 &= 2 \times (10 + 3) \\ &= (2 \times 10) + (2 \times 3) \\ &= \quad 20 \quad + \quad 6 \end{aligned}$$

$$\begin{array}{r} 13 = 10 + 3 \\ \times\ 2 \qquad\quad 2 \\ \hline 20 + 6 \end{array}$$

The distributive principle can be used again in multiplying two-place and three-place numbers. It is an invaluable aid to quick mental multiplication. Since the products of two-place numbers are large, it is difficult to give concrete experiences with set of objects at this level though some examples as 2 x 13 and 3 x 12 can be used. Let students regroup two-place numerals many ways at first. Have them write their renamings on the board for a given fact like this:

$$\begin{aligned} 3 \times 12 &= 3 \times (9 + 3) \\ &= (3 \times 9) + (3 \times 3) \end{aligned}$$

$$\begin{aligned} 3 \times 12 &= 3 \times (6 + 6) \\ &= (3 \times 6) + (3 \times 6) \end{aligned}$$

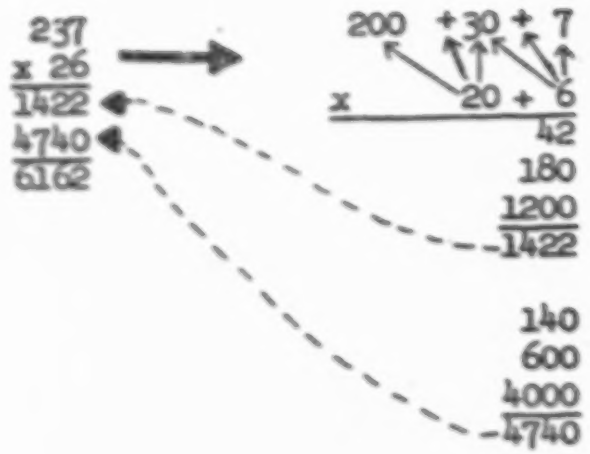
$$\begin{aligned} 3 \times 12 &= 3 \times (7 + 5) \\ &= (3 \times 7) + (3 \times 5) \end{aligned}$$

$$\begin{aligned} 3 \times 12 &= 3 \times (10 + 2) \\ &= (3 \times 10) + (3 \times 2) \end{aligned}$$

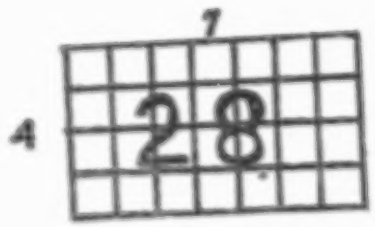
Guide students to see that computation is easier when 12 is renamed as 10 + 2. When using horizontal notation or asking students to multiply mentally, insist that they multiply left to right, multiplying tens first, then ones, then adding the products. Thus one would think in solving 4 x 23 = "Eighty (4 x 20) plus twelve (4 x 3) is ninety two." In conventional form or vertical notation we insist that students multiply right to left to simplify the "carrying" or regrouping process.

$$\begin{array}{r} 23 \\ \times\ 4 \\ \hline 12 \\ 80 \\ \hline 92 \end{array}$$

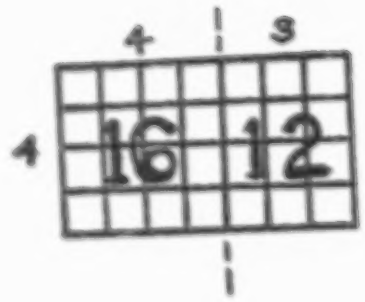
$$\begin{array}{r} 23 \\ \times\ 4 \\ \hline 92 \end{array}$$



A sense of the freedom which begins to emerge from the use of the distributive principle is found when the factors of a multiplication example can be regrouped into several addends. This regrouping is based on the convenience of the decimal system of numeration.



All the procedures using arrays and the distributive principle mentioned can also be used in teaching the division facts since division is the inverse of multiplication. The dividends are simply renamed so that the division problem is broken into two simpler division facts the student knows. Thus 28 ÷ 4 may be changed to 16 ÷ 4 and 12 ÷ 4 or 20 ÷ 4 and 8 ÷ 4.



$$28 \div 4 = (16 + 12) \div 4 = (16 \div 4) + (12 \div 4) = 4 + 3 = 7$$

The student may not remember that 5 x 7 = 35, so 35 ÷ 5 _____ is not simply a matter of memory. He knows that 25 = 5 x 5 and that 10 = 2 x 5 and that 35 = 25 + 10, so he regroups 35 as 25 + 10 and looks at the problem as:

$$(25 + 10) \div 5 = 25 \div 5 + 10 \div 5 = 5 + 2 = 7, \text{ or}$$

$$5\overline{)35} \quad 5\overline{)25 + 10} \quad \frac{5 + 2 = 7}{}, \text{ or } 5\overline{)35} \quad 5\overline{)20 + 15} \quad \frac{4 + 3 = 7}{}$$

PROBLEM SOLVING

Grade 4

Grade 4

PROBLEM SOLVING

Number Sentences with pictures and stories

Jim bought two balls Monday and found three balls today. How many balls does he have now?

Mathematics has many applications or analogies in the world of events. If the teacher goes to the trouble of teaching a child that 2 + 3 = 5, he hopes this fact will be useful. Each child should create a story problem of his own in the means of communication he prefers--telling a story, writing it, drawing it in pictures that illustrate it, manipulating objects that demonstrate it.

14: 43

The teacher guides the children through directed lesson from the board:

1) Introduce a real or contrived problem (attendance, game scores, etc.).

2) Guide students to set up an appropriate number sentence.

3) Write the story problem.

4) Pinpoint facts in the story problem with the class.

5) Evaluate workability.

6) Children make up their own story problems with their own number sentences.

The emphasis is turned from the answer to the number sentence for which the children create their own applications.

Pictures	Complete all sentences--Check the one suggested by
	5 + 2 = ◯ 3 + 2 = ◯ ✓ 2 + △ = 1 ◇ + 6 = 2

Could the picture at the left suggest another number sentence instead of 3 + 2 = 5? In what way does it suggest the other members of the basic fact team to which 3 + 2 = 5 belongs: 2 + 3 = 5; 5 - 3 = 2; 5 - 2 = 3?

As the story problems move up the scale of difficulty, the teacher stresses the central role of the number sentence. The teacher continues to call upon children to create stories that are consistent with a number sentence.

Phrasing more than one question about given data

For Playtime	
Baseball	$1.50
Bat	$2.25
Basketball	$2.00
Rocket Set	$2.50
Giant Hoop	$ .75

Some pupils may use the information at the left to make additional problems. For example:

1. If we have $10.00 to spend, could we buy one of each item?

2. If we are limited to two items of each, how many choices of purchases do we have?

1. Ted spent $3.50 for two things to play with. What did he buy and how much did he pay for each of them?

Does this sentence tell about the problem: $3.50 = □ + △ (Yes)

$1.50 Baseball $2.00 Basketball

2. Alice spent $3.25 for two things to play with. What did she buy? How much did the two things cost?

$3.25 = □ + △

$2.50 Rocket Set $.75 Giant Hoop

Problems may have more than one operation. To solve them one may need to add and to subtract. Which operation must one do first? Why?

Two-Step Problems

Solving a story problem often involves the use of more than one operation. In writing an equation for such a problem, we use parentheses to indicate which operation is to be performed first.

Place parentheses in such a way that each sentence makes a true statement.

42 - 21 + 17 = 28
87 - 43 + 19 = 25
63 - 27 + 31 = 5
59 - 34 + 17 = 42
94 - 59 + 23 = 12

On the board write these two equations: 9 - (3 + 2) = n and (9 - 3) + 2 = n. First, ask the pupils what the parentheses in each equation tell us. Then have two volunteers solve the equations, and have the pupils compare the answers.

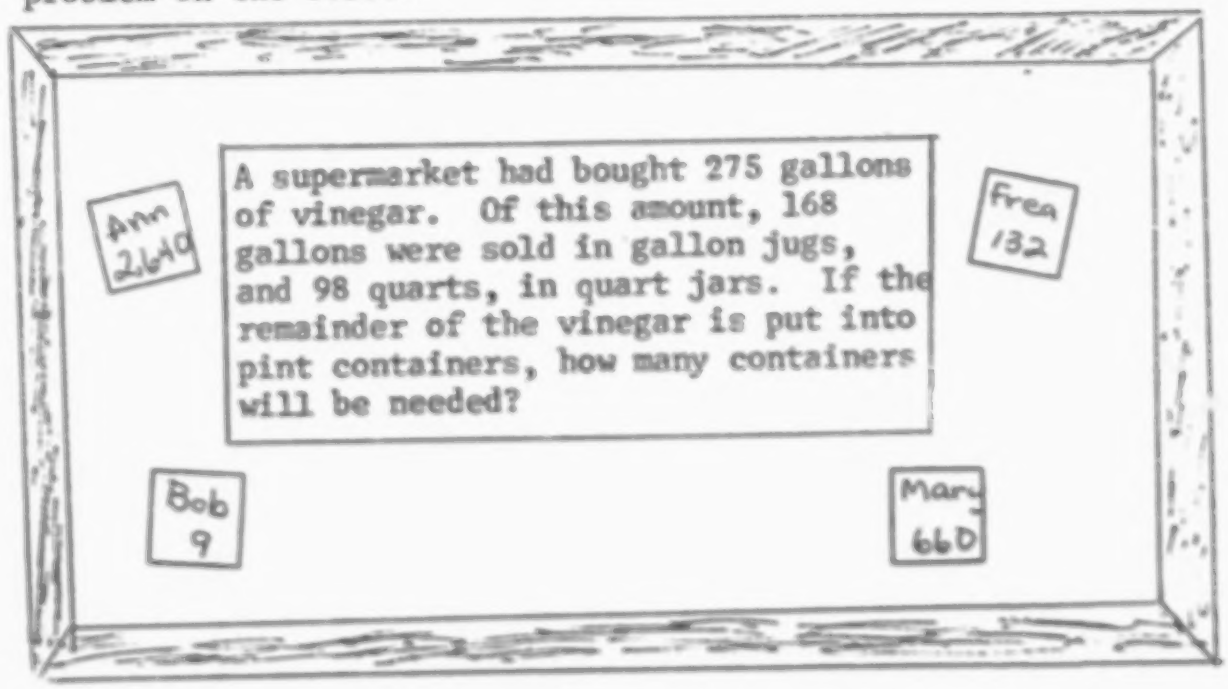
$165 - (24 + 35) = \square$

$(165 - 24) + 35 = 176$
$165 - (24 + 35) = 106$

The equation at the left came from the following problem:

Bob gave 24 of his stamps to Jim and 35 to Mike. He had 165 stamps before gave any away. How many did he have left?

The teacher may use some problems, one at a time, as problems of the day. The answer to each problem may be posted the next day, along with a new problem. As soon as the pupil gets what he thinks is the correct answer to the problem, he can write his name and the answer on a card and thumb tack the card beside the problem. The drawing below suggests some of the answers for the problem on the board. 12:29-31



(275 - 168 = 107; 98 ÷ 4 = $24\frac{1}{2}$ gal;
107 - $24\frac{1}{2}$ = $82\frac{1}{2}$; $82\frac{1}{2}$ x 8 = 660. or
275 - $192\frac{1}{2}$ = $82\frac{1}{2}$; $82\frac{1}{2}$ x 8 = 660 pts.

More problem examples for special challenge.

Read each problem carefully. Try to see the whole situation. Tell which equation or equations you would use in solving each problem. Reminder: The words dollars and cents are needed only in your final answer, not in the equation.

$75 - (16 + 8 + 25) = n$

1) Judy spent 16¢ for an ice cream cone, 8¢ for candy, and 25¢ for a notebook. She had 75¢ when she left home. How much money should she still have?

$120 - (53 + 17) = n$ (50 pgs.)

2) Mary read 53 pages of her 120-page book in the morning. In the afternoon she read another 17 pages. How many pages does she still have to read?

$144 - (61 + 65) = n$ (18 eggs)

3) Dick gathered 61 eggs on Monday and 65 eggs on Tuesday. He needs 144 eggs to fill a case. How many more eggs must he have before the case is filled?

$1 - \frac{1}{2} = \frac{1}{2}$
$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

4) Bonnie baked a pie. She took half of it to her grandmother. Then she saved half of the part that was left for her father. How much of the whole pie did he get?

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
or
$\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

5) Jean lives $\frac{1}{2}$ mile from Amy. One day when Amy had been at Jean's house, Jean walked half-way home with her. How far did each girl walk by herself?

One fourth mile. (Jean walked $\frac{1}{2} \times \frac{1}{2}$ or $\frac{1}{4}$ mile with Amy. Then Amy walked the remaining $\frac{1}{4}$ mile alone, and Jean walked the same $\frac{1}{4}$ mile back to her house.

(1 gal. = 4 qt. = 8 pt.
1 pt. = 2 cups;
2 x 8 = 16; 16 - 4 = 12)

6) We can make a gallon of ice in our ice cream freezer. There are four people in our family. If each of us eats the equivalent of one cup of ice cream, how many guests can we serve the same amount?

7) Three pies are cut as follows: one in fourths, one into fifths, and one into sixths. If all three pies are the same size, from which would you choose a piece? (The one in fourths if you want a large piece.)

(150 ÷ 5 = 30; 30 ÷ 3 = 10)

8) Figure the number of fives in 150. Divide this number by the number of bears in the Goldilocks story. Would you trade this number of pennies for a nickel?

(10 + 10=20; 20 - 5=15; 15 x 6 = 90)

9) To the number of dimes in a dollar add the number of your fingers or toes. From this subtract the number of pennies in a nickel and multiply by half a dozen. What is your answer?

(½ of 20=5; 5 = ½ of 10)

10) One half of what number is the same as one fourth of 20?

(1/7 of 210 = 30; 1/7 of 35 = 5;
1/7 of 49 = 7; 30 + 5 + 7 = 42)

11) One seventh of 210 added to the sum of one seventh of 35 plus one seventh of 49 gives what number?

30 - 15 = 15
15 ÷ 3 = 5

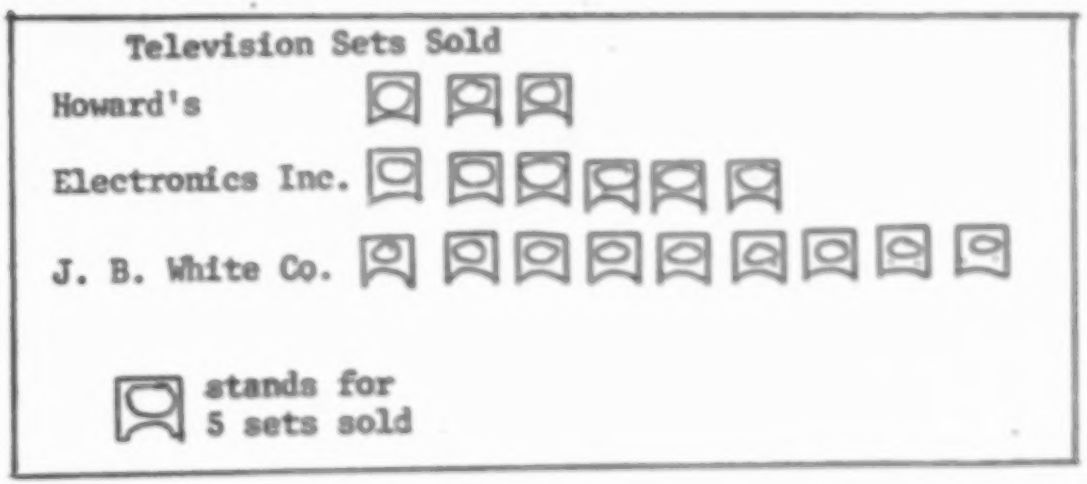
12) Mrs. McKee made thirty cookies and each of her children ate three. If there were fifteen cookies left, how many children had Mrs. McKee?

19 + 10 = 29
29 - 12 = 17

13) Bobby Smith had nineteen cents. He did an errand to earn ten more. How much was left for him to bank if he spent twelve cents at the store?

Problems using data from graphs

This picture graph shows the number of television sets sold in one week in three stores.



a. Which store sold the greatest number of sets? (Electronics Inc.)

b. Did any store sell as few as three sets? (No)

c. Compare the sales at Electronics, Inc. with the sales at Howard's. (45 to 15)

Stress the point that before one can read a picture graph, it is necessary to find the legend, or key.

Picture graphs are used to make quick comparisons and are not as accurate as a table of data.

Phrasing more than one question about given data.

Tables and graphs are excellent ways of showing relationships in mathematics. Problems should revolve around real life situations. These situations are often presented in tables or graphs which combine many details into manageable forms.

Here is a cook book recipe for cake icing sufficient for the tops and sides of two 9-inch layers.

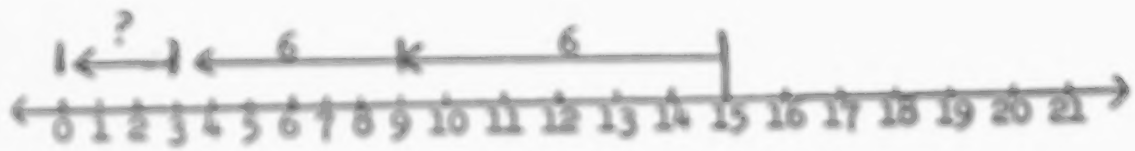
First, the recipe is rewritten in tabular form. Then, the recipe is doubled for four 9" layers, and then tripled. Why are the amounts in the bottom row the sum of the first two rows?

How much is a pinch? Suppose what was needed was enough for the top and sides of a three layer cake? How full should the cups and teaspoons be?

	Brown Sugar	Water	Egg Whites	Vanilla	Nuts	Salt
Tops and Sides of 2 - 9" layers	3 cups	1 cup	2	1 tps.	1 cup	1 pinch
Tops and Sides of 4 - 9" layers						
Tops and Sides of 6 - 9" layers						

Many factual situations can be more fully understood when they are presented in tabular form. Transportation schedules are published as time-tables. Interest tables are provided for bank clerks to eliminate difficult computations that might introduce error. Most books have an index. Height - weight relations are organized in tables. The grocery ad is a table. The shopping list, a recipe, some observations of scientists, and lists of batting averages are examples of tables or things that can be placed in tables.

Problems involving inequalities



$\square = (2 \times 6) \div \triangle$

The arrows show what happens when you divide 15 by 6. Can you show this division *any* other way on the number line?

7:130

This type of sentence helps pupils see the dividend in relation to the division, quotient, and remainder.

$\square \times 2 < 9$
$\square \times 5 < 17$
$\square \times 5 < 11$

In exercises such as the one at the left, pupils are asked only the <u>largest</u> number that will make the sentence true.

7:130-131

The concept of inequality is a very important one. For example, if we have <u>at least</u> enough money to pay for an article, we may have the exact amount but more likely, the cost of the article is less than the amount of money we have. Other examples would include: The football team has made <u>at least</u> enough yardage for a first down; we have <u>at least</u> enough seats on the ferryboat for each passenger.

What is the numeral for the largest number you can use to replace the frame in the following sentence and still have a true sentence?

$\square \times 30 < 278$

The pupils may suggest other situations in which the "is less than" or the "at least enough" concept is used.

"$5 \times 7 < 36$"

This sentence is read, "five times seven is less than thirty-six."

Packs of rubber bands cost 8 cents. Sue has 75¢. She wanted to buy as many packs of rubber bands as possible. How many could she buy? Would she have any money left?

a. Will the quotient mean rubber band packs or money?

b. Divide 75 by 8 just as you would divide by any pair of numbers.

c. Does the remainder mean rubber bands or cents?

Class discussion or demonstration may be necessary for pupils to develop correct interpretation of remainders.

A good illustration of the problem example is to have a pupil arrange 75 coins into groups of 8. Each group represents the amount of money needed to buy one pack of rubber bands. By counting the groups, the pupil will know how many packs of rubber bands he can purchase. He will see that the remainder represents the 3 coins he has left over.

Concrete examples can be an effective method of assisting pupils in understanding, but they should be encouraged, as soon as possible, to think through or mentally picture the situation to "see" what the remainder represents.

[redacted]

[redacted]

GRADE 4

Introduction to Geometry

Shapes to be found in classroom as, desk tops, windows, doors, chalk board.

Points Line Segment

Line

Ray

GEOMETRY

The approach to geometry in the elementary grades is based on an informal intuitive exploration. Children can develop geometric ideas through exploration of the world around them.

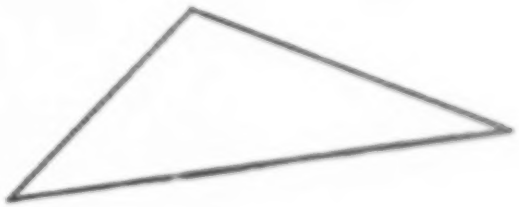
Many geometric figures can be found in the classroom, on the playground, and in nature. Always relate the study of geometric ideas to these physical models around us whenever you can. A list of some common geometric figures in our environment and a formal definition of geometry are given here for background material for the teacher.

Geometry is the mathematical study of space, shape and measurement. It tells us how to draw different shapes or figures and tells many facts about the relationships among these figures.

Geometric Figures

	Number of sides	Sum of measure of all angles	Number of (180°) straight angles in each sum
a. Triangle	3	180°	1
b. Rectangle	4	360°	2
c. Pentagon	5	540°	3
d. Hexagon	6	720°	4
e. Octagon	8	1080°	6

a.



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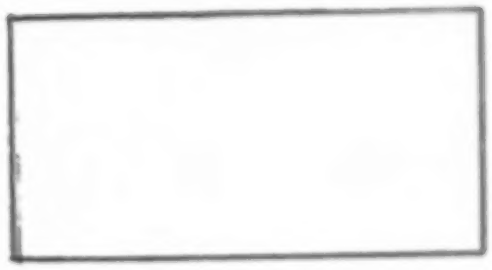

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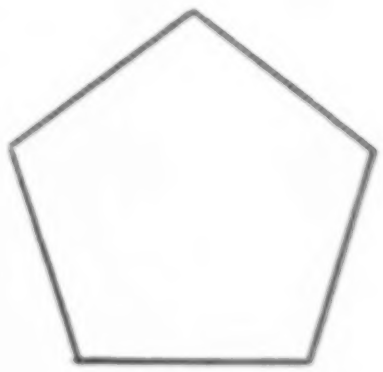

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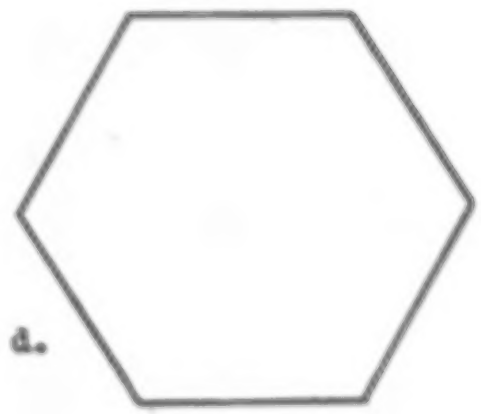
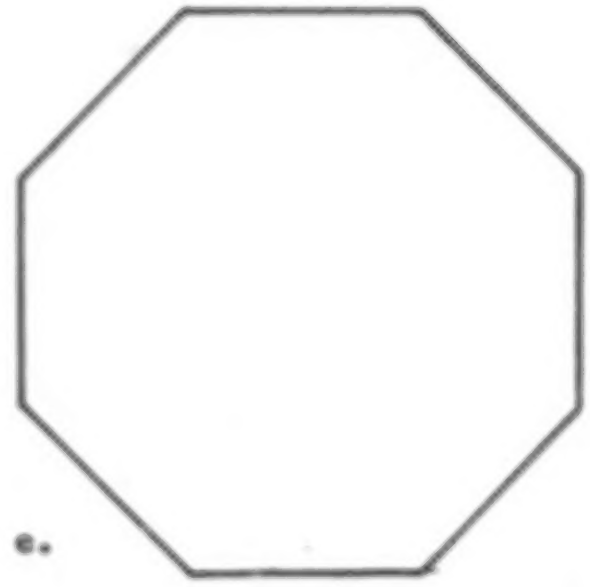
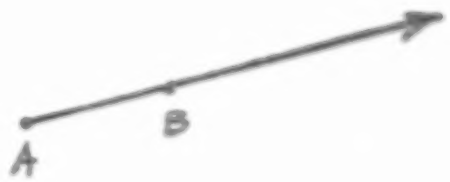
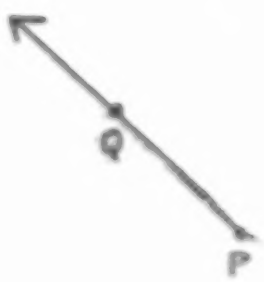


e.



Ray



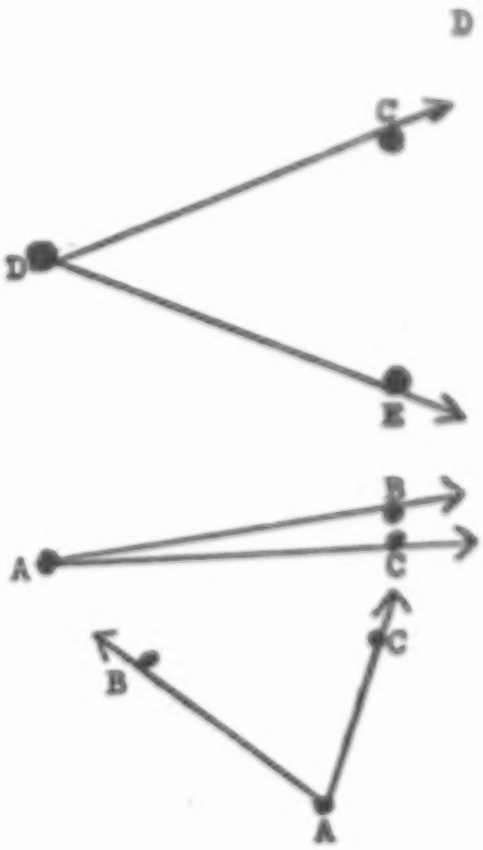


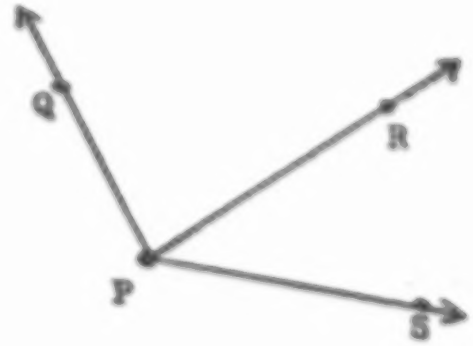
Ask students to draw a short line segment on their papers. "Label its endpoints A and B. Find a point C so that B is between A and C. (Or "extend line segment AB beyond point B") Find another point D so that C is between B and D. Continue extending line segment AB, keeping A as an endpoint. "What happens when your line segment reaches the edge of your paper? (It ends) Could we go on and on with our line segment? (Yes) Place an arrowhead on the line segment to show the end that goes on and on in space." Ask students to draw several line segments, label their endpoints, and then extend them beyond one endpoint. Show several positions, i.e., left to right, right to left, etc. Tell students simply "we call these extended line segments rays."

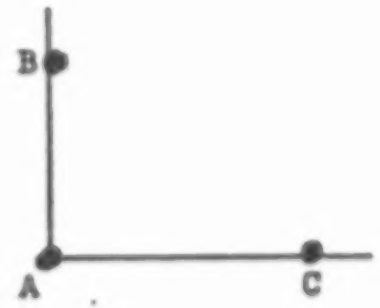
11:424-429
2:60
8:312-313
7:188-189

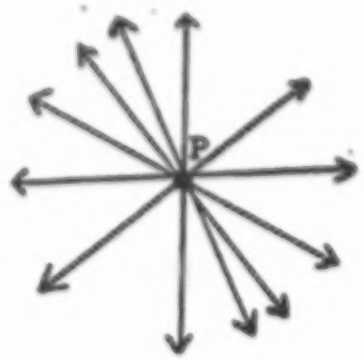
Hold up a ball of brightly colored yarn for the class to see. Ask -- "Who can use this ball of yarn to show a ray?" Encourage a variety of demonstrations, e.g., one student may hold the end while another, moving away, unwinds the ball. The end may be tied to some easily seen point and then unwound. If a small ball of yarn is used first, more balls can actually be tied on to show the extension of the line segment. Additional clarification of ray can be brought about through analogy. Ask how we might compare the rays of the sun with the rays we study in geometry. Also suitable for discussion would be the beam of a flashlight, the path of a bullet and the flight of a baseball hit for a home run.

Angle









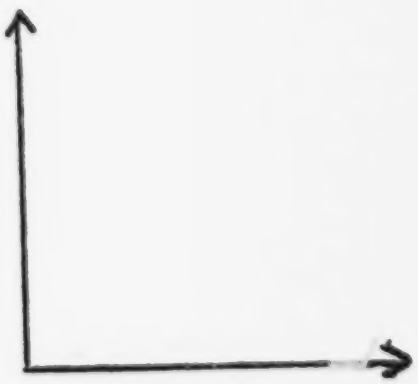
To student -- "Choose three points on your paper (not on a line) mark them and label them as shown on the chalkboard. Starting at D draw ray DC. Starting at D draw ray DE. What common point do ray DC and ray DE share? (D) Do they share more than one point? (No) Where does ray DE end? (It doesn't) This figure formed by two rays with the same endpoint is called an angle." 7:188-189 11:478-491

"Draw several angles of various sizes." Encourage students to draw angles between 0 and 180 and to draw them in various positions. Discuss possible ways of naming angles and rays. Explain that by agreement rays are named by saying their endpoints first. At left we find rays AB and AC. The angles at left would be named either angle BAC or angle CAB being sure to put in the middle the name of the endpoint shared by the two rays.

An activity reinforcement of these concepts concerning rays and angles might be developed in this way:

1. Label a point P on your paper.
2. Draw a ray PQ.
3. Draw a ray PR.
4. Draw a ray PS.
5. Can you name three angles in your figure? (Yes)
6. Give another name for angle QPS. (SPQ)
7. Name the endpoint of ray PQ. (P)
8. Name the endpoint of ray PR. (P)
9. Name a point on ray PS (P or S)
10. Draw 4 rays with P as an endpoint.
11. Draw 3 more rays with P as an endpoint.
12. How many rays are there with P as their common endpoint? (More than we can count)

Right angle



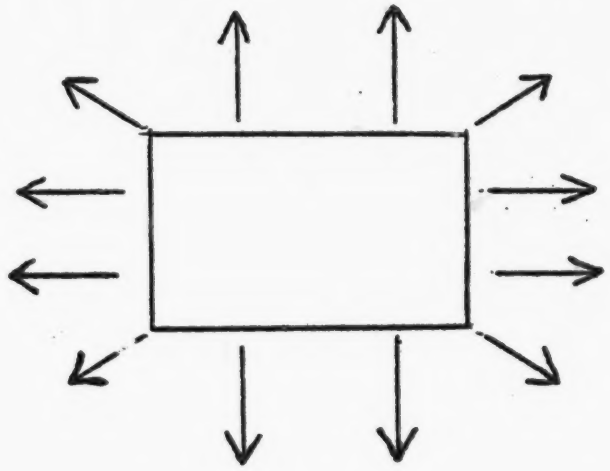
Review concept of "square"angle. Look for models of "square" angles in the classroom. Have students cut corners from sheets of construction paper or fold paper to construct models of right angles. (See construction in grade 3) Use the term right angle in referring to these models. Let students work in teams and use these models of right angles to measure certain angles in the classroom. Specific angles chosen could have their rays represented by masking tape. Ask student teams to decide whether the chosen angles are less than a right angle, equal to a right angle, or greater than a right angle. Teams then present their findings to the class for discussion.

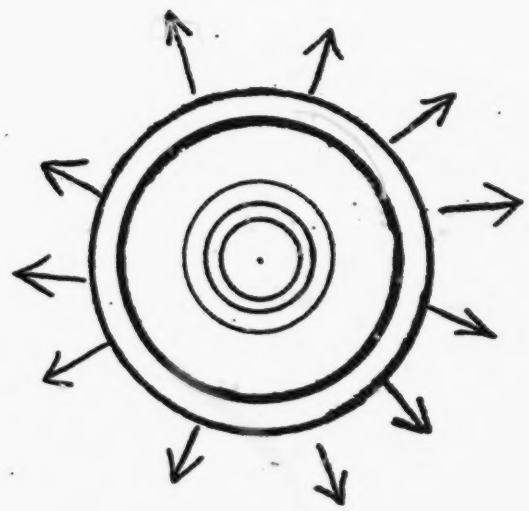
7:189

11:718-723

Provide students with sheets of paper containing various models of angles. Ask them to label each angle as being less than, greater than, or equal to a right angle. Insist that the measuring all be done by eye. When all angles are labeled, let students check their answers by measuring each angle with their right angle models.

Properties of a plane

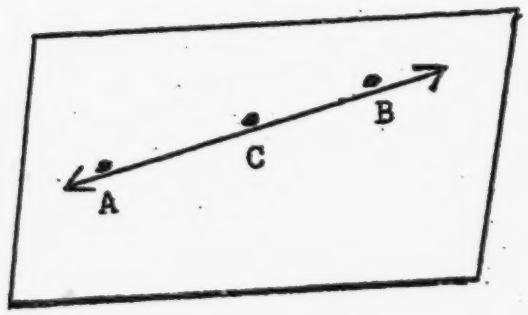


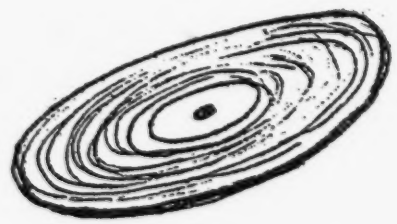


Discuss the flat surfaces we can see in the classroom. "Can we think of these surfaces as models of a set of points? (Yes) Place your finger on a point on the top of your desk. Touch ten points on the desk top. How many points are there on the desk top? (More than we can count) The set of points we think of when we look at a flat surface is called a plane. Your desk top represents only part of a plane." 7:193 11:430-436

Give students a large piece of white construction paper and crayons. "Draw a small circle in the middle of your paper. Color the interior of the circle green. Is this green region a picture of part of a plane? (Yes, discuss) Draw a larger circle around this region and color it green. Is this larger green region a picture of part of a plane?" (Yes) Ask students to continue drawing a larger and larger circle, each time coloring the interior. Conclude activity by asking if we could ever draw a picture of a whole plane. (No, a planes goes on without end) "Compare the piece of paper with the desk top. Which represents the larger part of a plane? (Desk top) Why?"

Have students turn paper over and mark two points in the plane of the paper. "Label the points A and B. Draw a line through A and B. Is A a point in the plane? (Yes) Is B a point in the plane? (Yes) Mark a third point C on line AB. Is C a point in the plane? (Yes) Is every point of line AB in the plane? (Yes) Think of a point not in

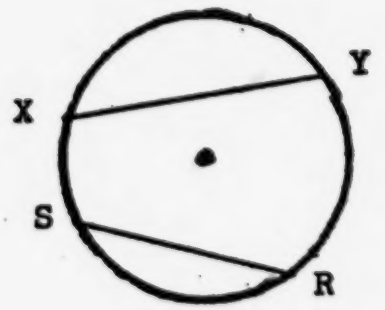




the plane of the paper. Name it. (A point of a pencil, toe of a shoe, the flag, etc.) Think of a line not in the plane. Name it." (Be careful here. Only a line parallel to the plane of the paper should be accepted, such as a line on the floor or the line represented by the intersection of the wall and ceiling.)

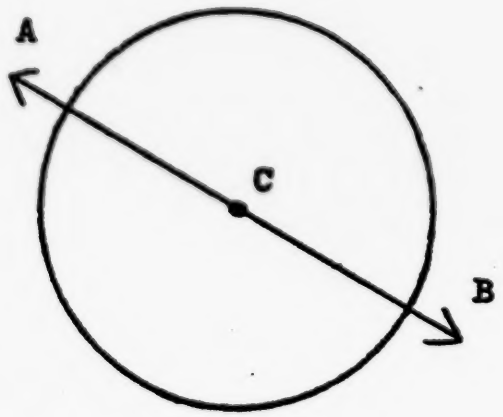
Place a large disk such as a phonograph record in the chalk tray. Ask the students to think of the whole plane that it represents. Ask-- "What objects or people in our room would be intersected (touched or met) by this plane? (Varies) What objects or people on our playground would be interesected by this plane? (Many answers) What objects or people in our community would be intersected by this plane?" Vary the position of the phonograph record so that the plane intersections will be different. If students have difficulty think of the plane ask them to imagine they are sitting in the middle of a giant pizza or pancake that goes on and on in all directions. It may help to set a small doll in the middle of some large flat surface and to ask the students to imagine being the doll and looking about them.

Chord



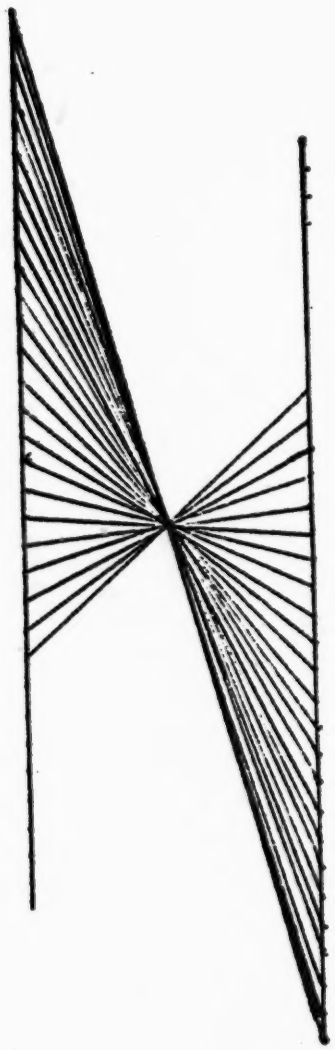
Have students construct a circle by tracing a model or using the compass. Ask them to choose any two points on the circle and connect them with a line segment. Tell them that the name for this line segment is chord. "Draw three more chords of the circle. Draw five more. How many pairs of points are on a circle? (More than we can count, remember?) How many chords can we draw in a given circle then? What is the shortest chord you can draw? (Varies with skill) What is the longest chord you can draw? (The chord that passes through the center of the circle.) 6:249

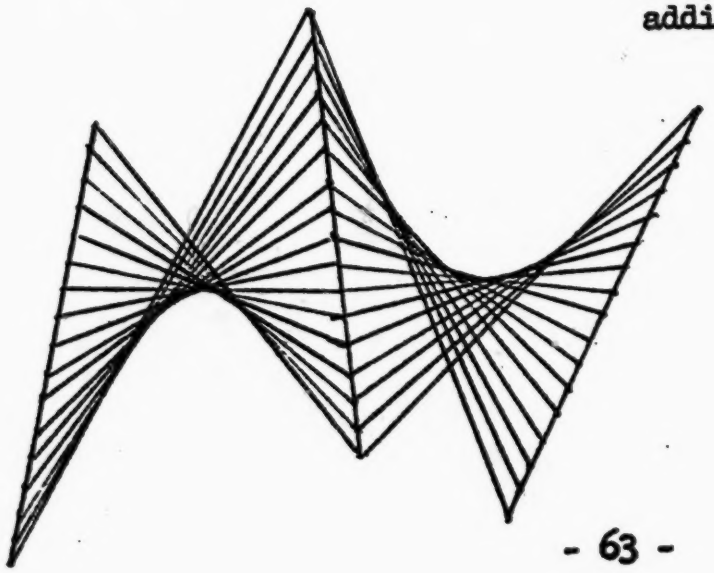
Diameter concept using term



Give students unlined paper and compasses. Have them construct circles of various sizes (after teaching the proper use of the compass). Instruct students to mark the center of a circle and draw a line which passes through the center and two points of the circle. Explain that line segment AB is called a diameter. "Use your pencil and ruler to draw another diameter in this circle. Can you draw two more diameters? Draw as many diameters as you can on the circle. How many diameters can you draw? (Varies) Are there more diameters on the circle than we can mark? (Yes) Measure several diameters. How do their lengths compare? (Equal in length) Is this true of all circles?" (Yes) 7:192 11:446-472

Designs using line segments





This design activity can be used either in geometry or measurement.

Materials needed:

1. unlined paper (any type)
2. colored pencils
3. ruler

For younger children graph paper may be used because they cannot measure eighths of an inch.

Procedure:

Draw two lines of equal length on the paper. They need not be parallel lines. Mark each line with dots that are an eighth of an inch apart. Using a ruler connect the top dot of one line with the bottom of the other. Connect the second dot down with the second dot up on the other line etc.

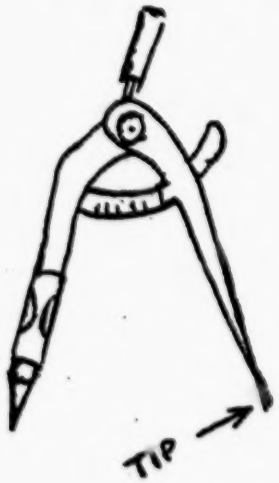
Straight lines give the illusion of being curved when the design is completed.

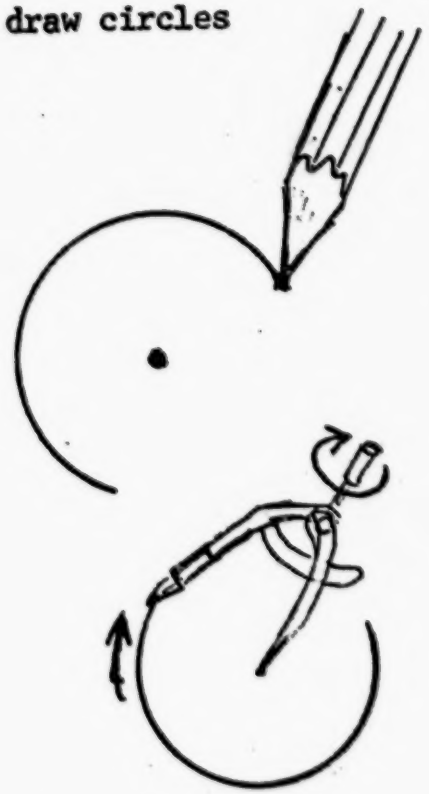
Many more complicated designs can be achieved by adding more lines to the original two.

MEASUREMENT

Grade 4

Using the compass to draw circles





A circle is the set of all points in a plane that are the same distance from a given point. The given point is called the center of the circle.

Draw a circle on the playground using a jumprope and chalk. One child holds the rope at a point on the ground while another walks the rope around dragging the chalk in its path. The rope is a model of a line segment from the center to the circle. This line segment is called a RADIUS.

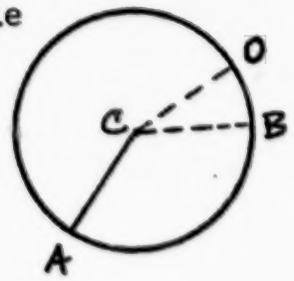
What is the model of a radius on a bicycle wheel?

Are there more than one?

The special tool for drawing pictures of circles is called a COMPASS.

Use a compass to draw a picture of a circle. Label The center of your circle C.

Measuring the radius of a circle



Draw a line from the center to a point on the circle. Call it $\overline{CA}$. Draw another radius on the same circle. Label it $\overline{CB}$. Measure $\overline{CA}$ and $\overline{CB}$ with a ruler. What is true of the lengths of these two line segments? Draw a third radius and call it $\overline{CO}$. Measure it. What can you say about the length of any radius of a circle?

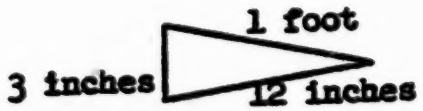
Perimeter of triangles

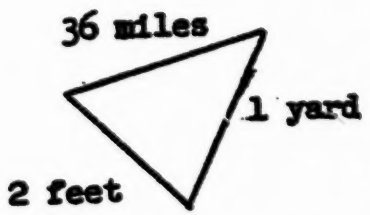
The length of measure which forms a simple closed curve is called a perimeter.

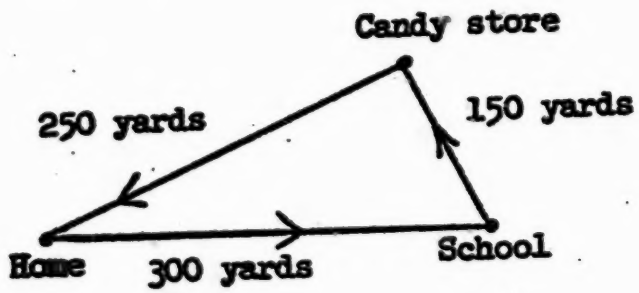
Alan went from home to school. Then he bought candy at a nearby store before he returned home. What was

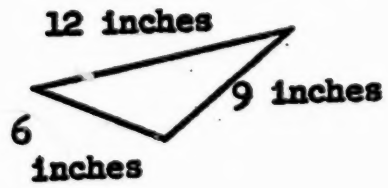
300 150 250 700

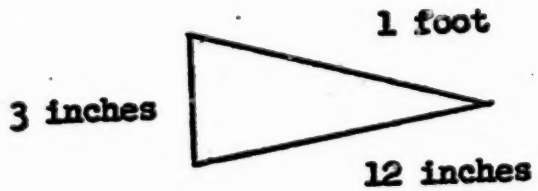
He walked 700 yards.





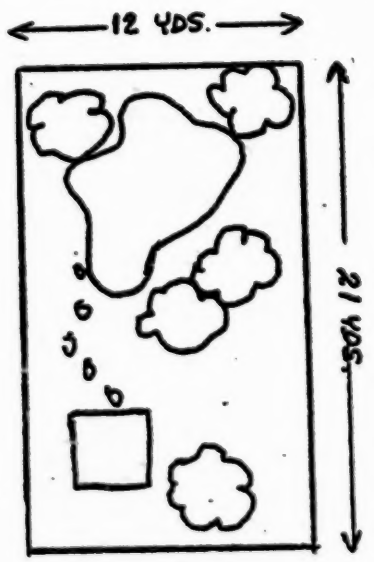






Provide the children with different triangles and ask them to find the perimeters.

Perimeter of rectangles and squares



12
12
21
21

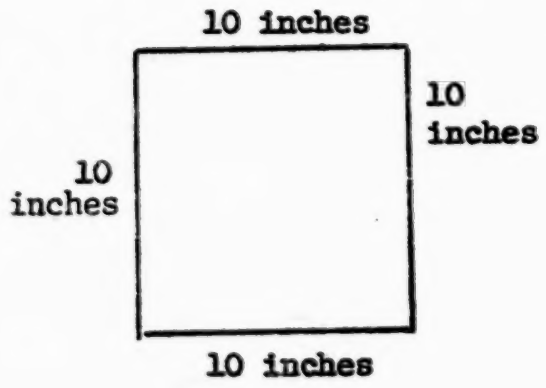
12 + 21 + 12 + 21 =

(2 x 21) + (2 x 12) =

2 x (21 + 12) =

If you walked around this park you would have walked on its perimeter. How far would you have walked?

Have the children show all the ways they could write this problem.



If you have a square, how would you find its perimeter? Let the children list ways they could use. Such as

☐ = 10" + 10" + 10" + 10"

☐ = 2 x (10" + 10")

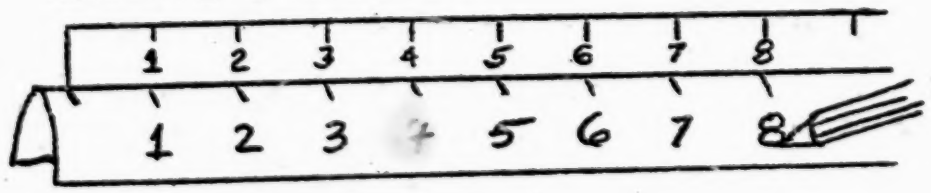
☐ = 2 x 20"

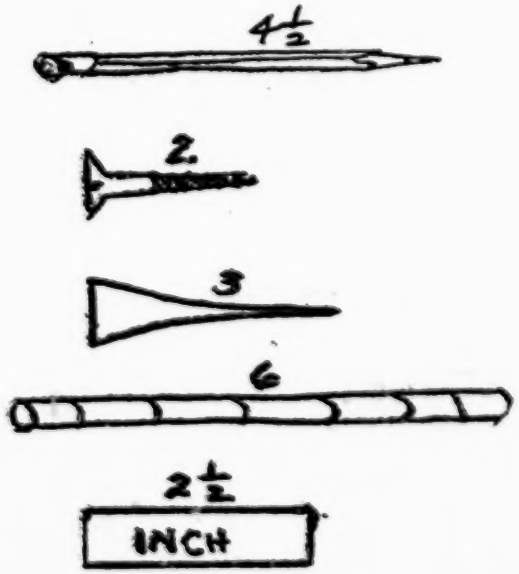
20"	20"	10"
+20	x 2	x 4

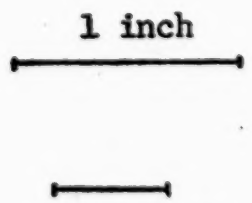
☐ = (2 x 10) + (2 x 10)

☐ = 4 x 10"

Measuring by centimeter







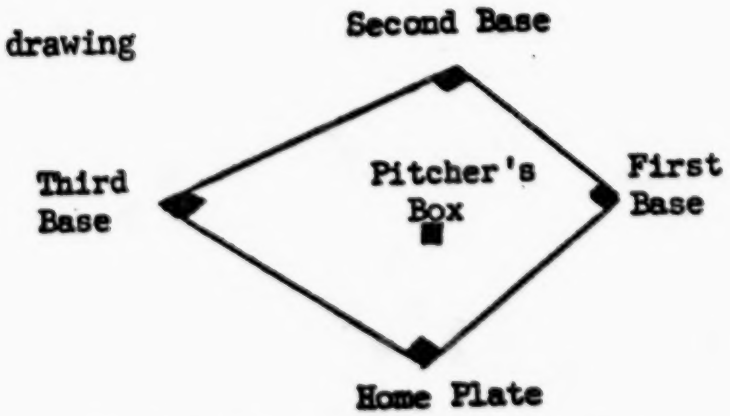
The metric system was developed in the 18th Century and is used all over the world. In the metric system the basic unit of length is the METER (approximately 39.3 inches or a little more than 1 yard) 11:774

1 centimeter is one hundredth of a meter. 1 centimeter is approximately 39.3 x .01 inches or .393 inches--this is nearly .4 inches or a little less.

The above information is for the teacher only and is not to be taught to the student. The fourth grade student should only need to measure by centimeters and compare them with inches in an approximate way.

Use the model at the left and have each child make his own centimeter ruler. Take a strip of stiff tagboard, hold it flush with the centimeter model, copy the marks on the tag. Then use the centimeter rulers to measure various objects and record in centimeters. 3:28-29

Scale drawing



Change so it will be suitable for blackboard work

Scale in Feet

Scale in Feet

Scale drawings by centimeter

one centimeter = ten miles on this map scale

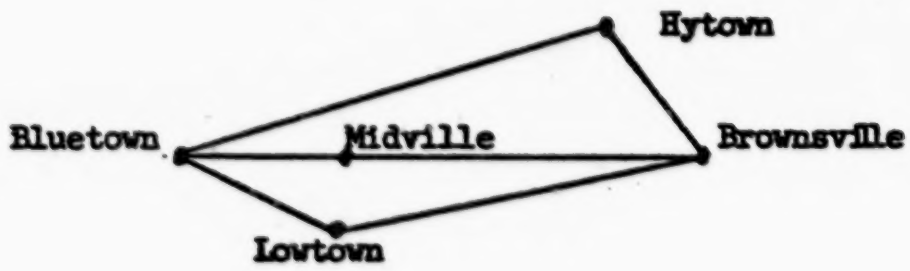
Some scale drawing and map reading was introduced in grade 3. It is further developed here.

Show the children a baseball diamond that has been drawn to scale on the blackboard. Then let them make up questions about the drawing and follow this by measuring and answering their questions. 10:354 7:252-253

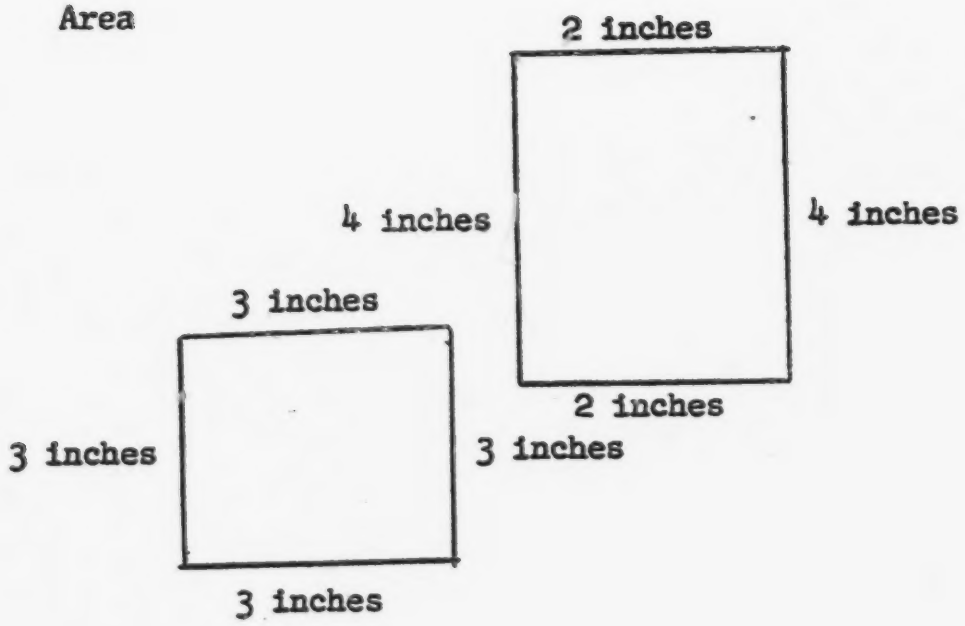
Give them another scale and let someone draw a diamond using it as a measure.

Ask the students to bring in as many local and state maps as possible. Discuss the scale and prepare projects for teams of children. Devote at least one math period to having these teams find distances between certain map points and make a chart of them.

Prepare a mimeographed map with the scale of 1 centimeter representing 10 miles. Have the children use the centimeter rulers they have made to answer questions concerning the map. 3:25



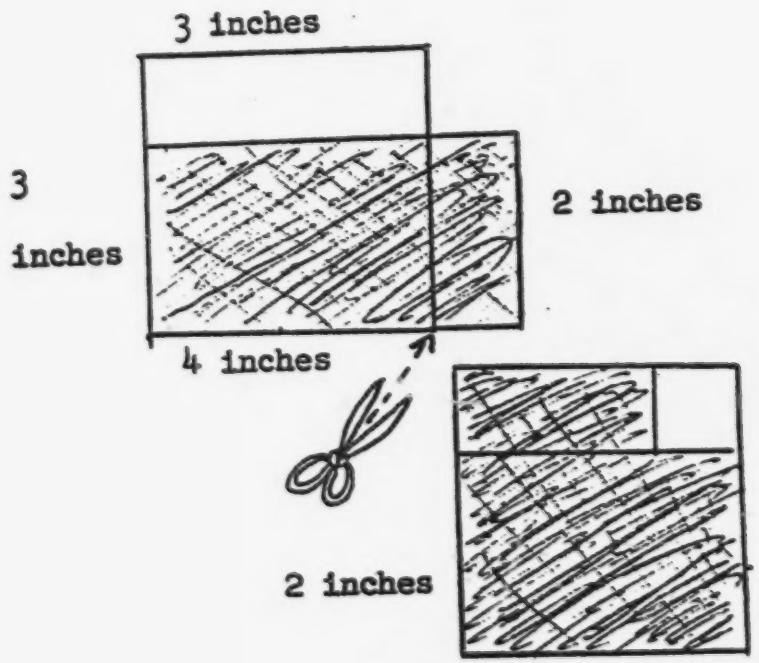
Area



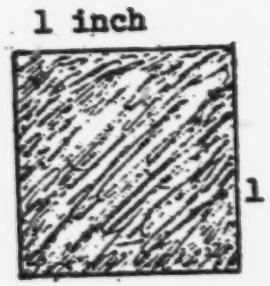
Draw two figures at the left on the board. Tell the class: Tom wanted to cover the fuselage on his model airplane. He had these two pieces of tissue paper. He wanted to use the largest piece. Which should he use? 9:340

Someone may say they are the same size because their perimeters are the same. Ask if there is another way to check which is larger. Lay a 2 x 4" sheet of paper on a 6 x 6" sheet and cut off in place.

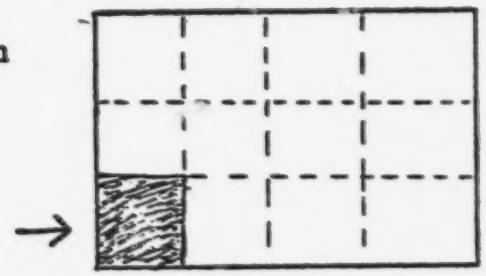
How could we solve our problem without cutting up paper? Some children may suggest putting a plastic grid over the figures as they did in third grade.

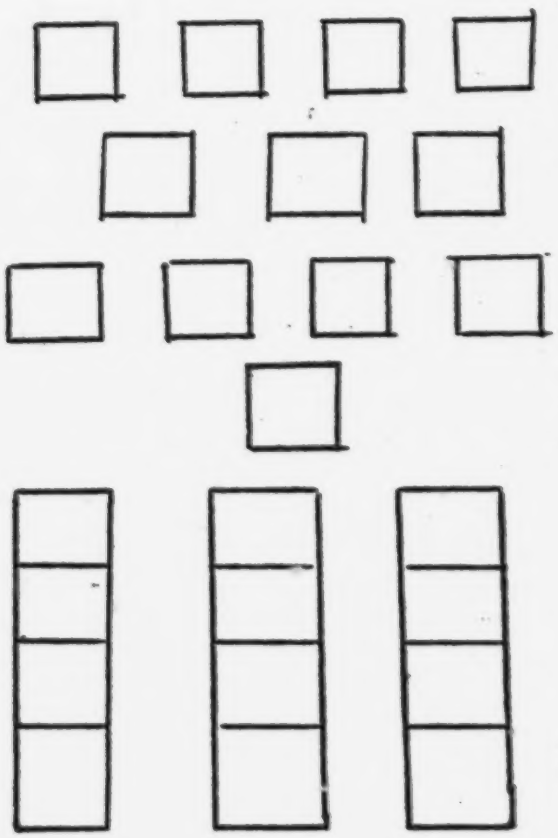


The square inch



1 square inch





Would it be better to have a good standard unit of area? We have already established standard units of length; standard units of area should be easy.

How many square inches would it take to cover the area at the left? Let's count them.

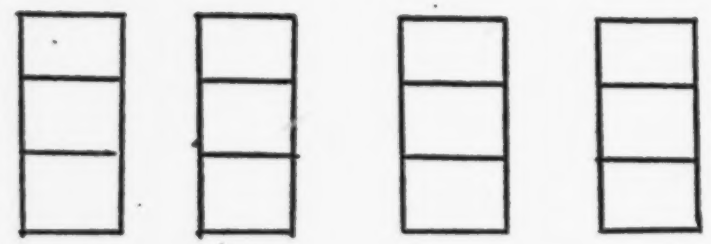
Provide each child with pencil, ruler, scissors and a 4 x 3" paper. Have them divide their paper into one inch squares then cut the 12 squares apart. Ask them to place the squares in three even rows. How many square inches in each row? (4) Then place them in 4 even rows of 3 each.

How can we find how many square inches are in a region without counting them one by one?

4 rows of three is 4 x 3

3 rows of four is 3 x 4

We can, therefore, multiply the number of units on one side of the figure (l) by the number of units on the other (w) side. Length x Width. If the children don't see this yet, give them other examples so they can discover it for themselves.



A = 8

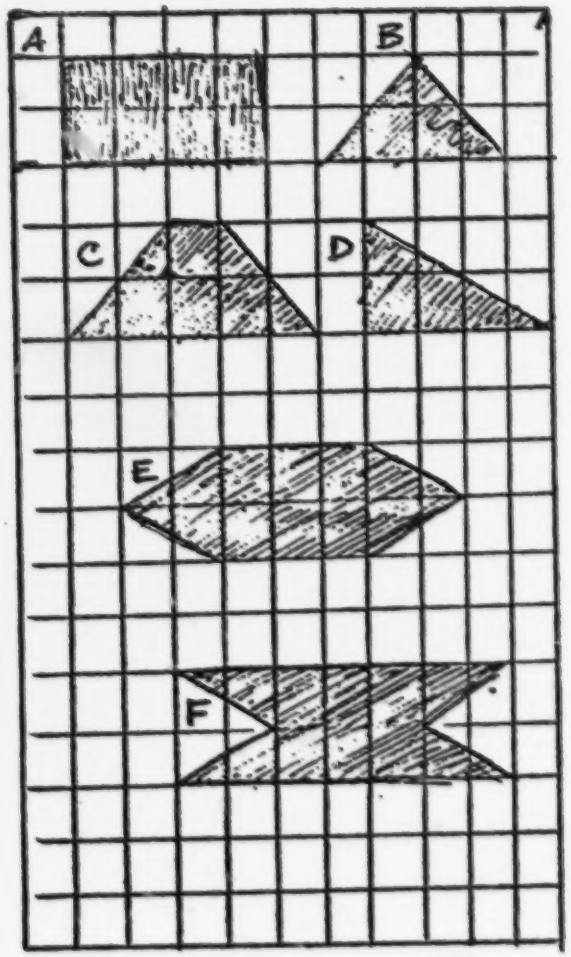
B = 4

C = 6

D = 4

E = 10

F = 10

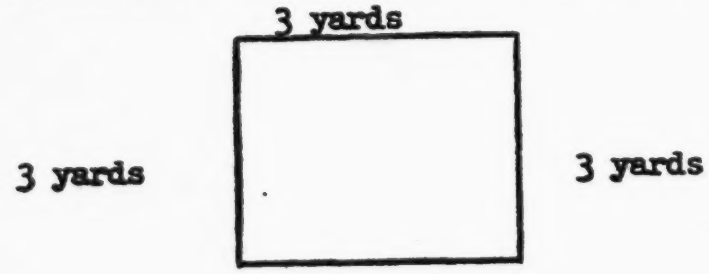


Find the number of square units (area) in each shaded region

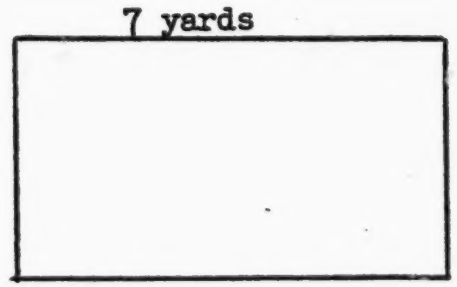
Have on hand floor tiles and use them to find various area measurements. Have the math teams estimate the number of tiles it would take to cover the classroom floor. Verify by measuring.

Provide the children with work papers and ask them to find the areas involved. Laying sheets of plastic graph paper over the worksheet is one method; mimeographing the shapes on a grid is another.

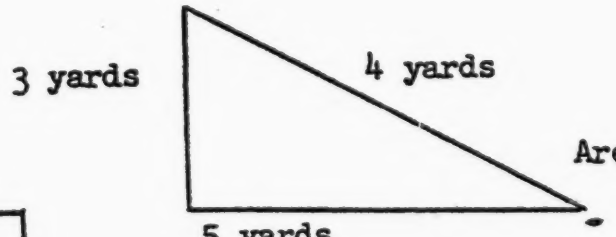
Square yard Area = 9 sq. yards



Line off a square, a rectangle, and a right triangle on the playground with chalk. Have the children, working in teams, estimate the areas and then verify by using the formula. Take along a model of a square yard made from chipboard to double check their answers.

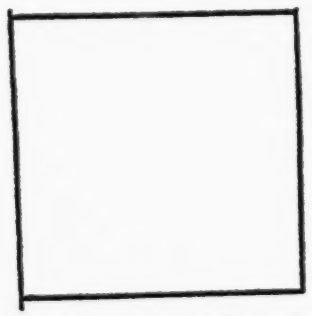


Area = $7\frac{1}{2}$ sq. yds. Area = 21 sq. yds.



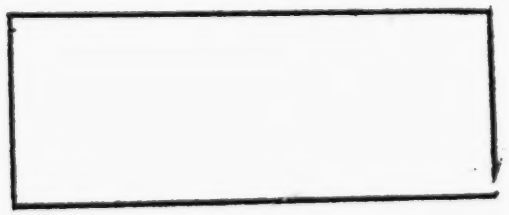
Square Centimeters

(A)



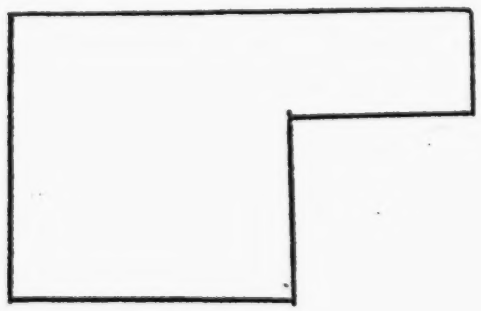
Area = 9 sq. cm.

(B)

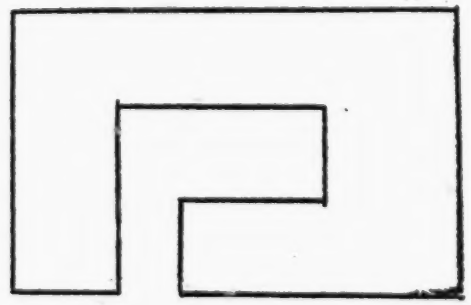


Area = 10 cm.

Have the children use their centimeter ruler to find the area of each region. Make sure for this exercise that there is no grid available. You can easily see from this less-structured problem those children who are still having difficulty grasping the concept of square measure.



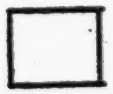
Area = 11 cm.

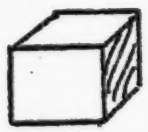


Area = 11 cm.

Volume

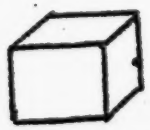
(1)

(2) 

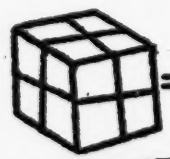
(3) 

Cubic inch and cubic centimeter

Each side is one square inch



Each side is one square centimeter



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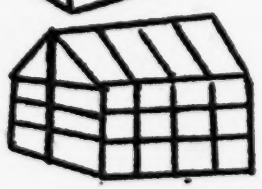


= 28

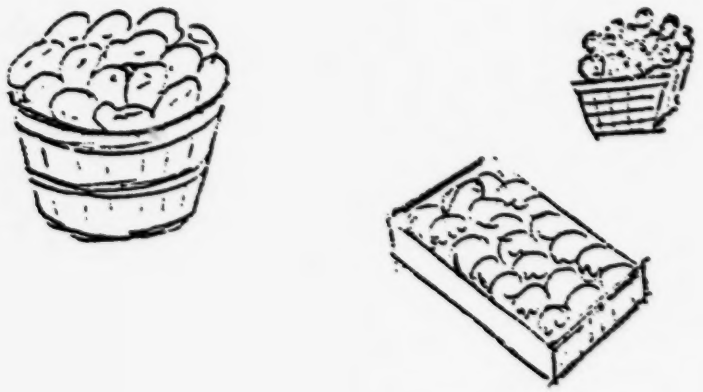
Find the number of cubic units (volume) in each figure

A unit for finding length is a segment, a unit for finding area is a square. What would we use to find volume? What is volume? How much space can you fill in these two boxes? Provide two boxes, one of which is larger than the other. Provide wooden or plastic play blocks (cubes) and have the children fill the boxes with these cubes. (You can make your own boxes out of cardboard and masking tape if none the right size are available.) 3:38-39

Prepare worksheets which will graphically present problems using cubic inch and cubic centimeter measures. Have the children build the forms with cubes if they cannot figure it out on their own.

We are not attempting to teach the formula for volume, but are only trying to lead toward an intuitive feeling for the formula for the volume of a box.

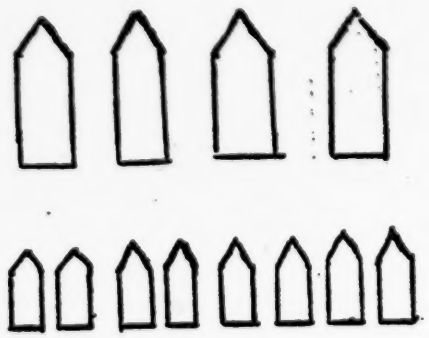
The study of dry measure need not be emphasized due to its dwindling use. Dry measures are inaccurate and their use is not practical since Orange County is no longer a rural area.



2 pints (pt.)	1 quart (qt.)
8 quarts	1 peck (pk.)
4 pecks	1 bushel (bu.)

Changing from one unit of measure to another

When we change to a smaller unit of measurement, more units than before will occur so we usually <u>multiply</u>



Units of dry measurement are the pint, the quart, the peck, and the bushel. The <u>pint</u> and the <u>quart</u> have the same names as in liquid measurement. If you have the students compare these dry measure containers with liquid measure containers, they will discover that they don't hold the same amount. They will find the dry-quart container slightly larger than the liquid-quart container, to allow for space required in packing. The same is true for the dry-pint container.

There is nothing new of mathematical importance 5:126-127 to learn when studying dry measures. We hope, however, to get over the idea that:

1. Dry measures, more than any other measures, are approximate.

2. When the equal sign is used with dry measures, it means "are equal in value to" or "are equivalent to."

Provide the children with both liquid and dry containers. Have teams of students do pouring and measuring experiments and record their results into tables.

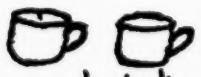
Ask leading questions of the teams after they have experimented on their own. Ask, "If we empty liquid from 4 quart containers into <u>pint</u> containers, how many pint containers will we use?" In this case the children will multiply 2 by 4 to get the total number of pint containers.

$4 \times 2 = 8$

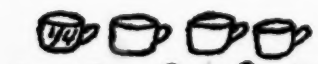
When we change to a larger unit of measurement, fewer units will be needed, so we usually <u>divide</u>

Liquid ounce

= 8 oz.



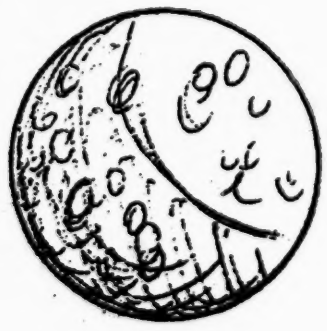
4 + 4 = 8 oz.



2 + 2 + 2 + 2 = 8 oz.

cup = 1 oz.

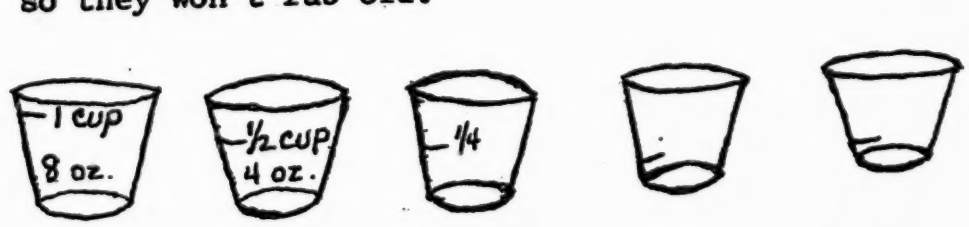
Weight



Ask the teams to convert from 6 pint containers to quart containers. They will divide to find how many 2's in 6.

$$2\overline{)6}\quad 3$$

Bring a set of measuring cups into class for each team to use as a model for their experiment. Tell them they are to create their own set of measuring cups, using ounces as the unit, from a set of jelly (or other) glasses. They can mark and label their glasses with felt tip pen. Then cover the marks with scotch tape so they won't rub off.



Bring in a set of nurse's scales and weigh the children. Have the children record their own weight on a card as well as on a large class chart.

Tell the children they are going to take a trip to 3:144 the moon. Since the earth is six times as large as the moon, and gravity which controls weight is determined by the size of the planet, they will weigh six times as much on the earth as they do on the moon. Have the children record their weight rounded off in pounds.

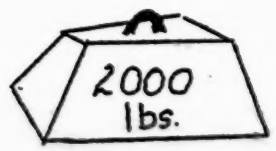
WEIGHT		
Name	Earth	Moon
Tom	48 lbs.	8 lbs.
Art	60 lbs.	10 lbs.
Sally	34 lbs.	Between 5-6 lbs.
Jim	54 lbs.	9 lbs.
Betty	42 lbs.	7 lbs.
Sue	43 lbs.	

Have teams of children devise some interesting questions to ask other teams concerning comparisons of weights of people and objects on earth and on the moon. The teacher should council with each team to see that their questions are reasonable before having a quiz among teams.

Ton

16 ounces = 1 lb.

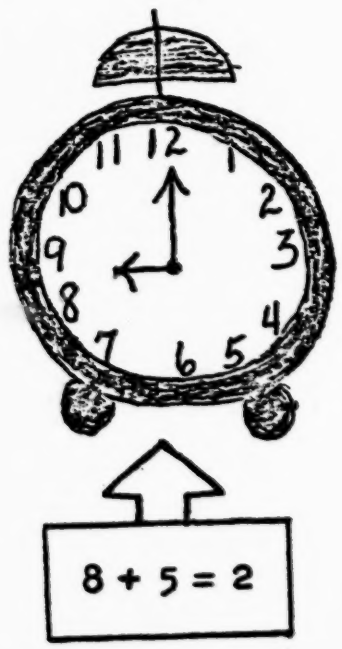
2000 pounds = 1 ton (T.)



We have worked with ounces and pounds. How would we weigh gold for jewelry making? (Ounces) How would we weigh potatoes? (Pounds)

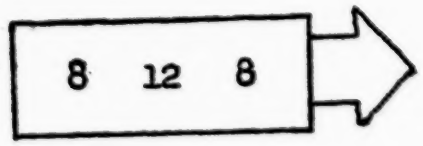
To weigh rocks brought into the classroom, use both a regular kitchen scale and a simple balance scale. Have several one-pound weights and one-ounce weights available. How would we weigh an elephant? A truck? We would need a larger scale. We would weigh them in pounds or tons. If there are 2000 pounds in a ton, how many pounds in two tons, three tons, etc.

Time



Using a demonstration clock and the clock on the classroom wall, review facts concerning the clock.

Clock addition is different, since numbers go only to 12; there are 24 hours in a day, 60 minutes in an hour, etc. "If it's 8 o'clock now, what time will it be in 5 hours?" "In 5 hours? Let's count--9, 10, 11, 12, 1. It will be 1 o'clock."



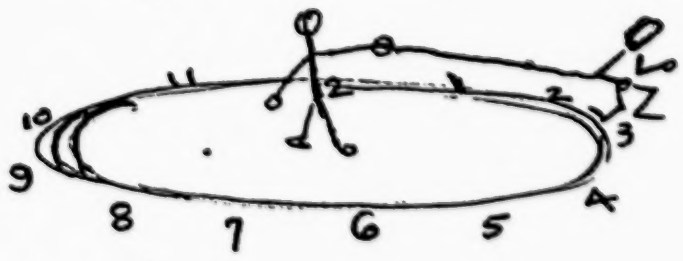
If you write an equation to go with this conversation it would read 8 + 5 = 2. This equation is correct when it concerns clocks. For example:

8 + 12 = 8

Discuss the second hand in the classroom, and then suggest they act out the movements of a clock.

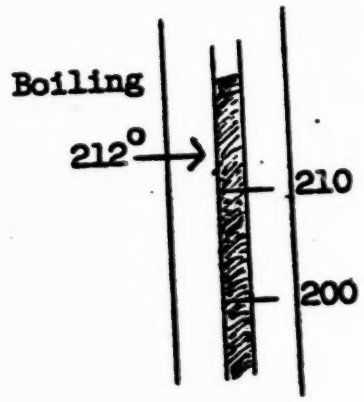
The second hand

Take a watch and a stopwatch onto the playground. Mark with chalk a large clock and put in marks for minutes and write the numerals. Have one child stand in the exact center of the "clock" holding a jump rope (which stands for the second hand) and pivot on his foot as the second hand turns moved by a second child who runs from minute mark to minute mark as the class chants the seconds. One member keeps tract on the stopwatch.



They play this game a second time, adding 2 students who move appropriately as they play act the hour hand and the minute hand.

Temperature

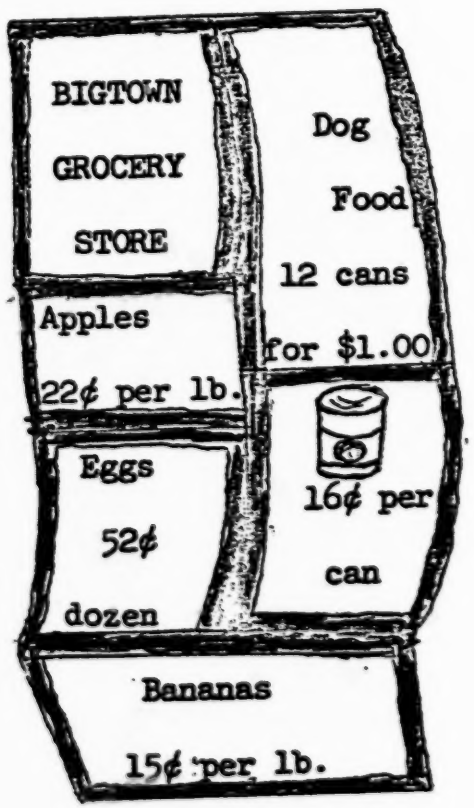


Remember the temperature at which water freezes is 32°. How warm must the water get to boil? (At sea level) Provide a hot plate and a candy thermometer. Heat a pan of water with the candy thermometer inserted and check to see the boiling point (212°).
5:124-5

Meat thermometer
Outdoor thermometer
Indoor thermometer
Candy thermometer
Clinical thermometer
Etc.

Bring in as many kinds of thermometers as possible. Compare and discuss their scales. Why are they different?

Money



The teacher cuts grocery store advertisements out of the newspaper--using the ads from several different stores. She then makes out a different shopping list for each team of children. Included on each list is the amount of money the team has to spend.

The teacher posts the various grocery ads in different parts of the room, and the teams go "shopping." The team members discuss the purchases with one another, but keep a record separately so they can check together later to avoid errors.

The division operation

SHOPPING

LIST

2 dozen eggs
4 cans dog food
5 lbs. sugar
2 lbs. hamburger (ground round)
1 head lettuce
2 lbs. tomatoes
1 loaf bread
2 cans cleanser
1 giant size laundry soap
3 lbs. bananas

A typical shopping list will include the use of addition, subtraction, multiplication, and division operations. If the ad reads two cans of tomatoes for 26¢ and only one can is on the list children would divide by 2. If a box of cereal costs 32¢ and the shopping list reads 4 boxes the children would multiply, etc. Interesting discussion should arise from a 3 for $1.00 or 12 for $1.00 ad.

Money

When a team completes its shopping list, each child totals his own list, figures the amount of change he has coming, breaks down the change into numbers of pennies, nickels, dimes, etc., he will receive, and compares the results with other team members. The teacher will check for accuracy each "team" report handed in to her. She can then report to each team as a whole their progress in relation to the last "shopping trip."

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O.C.S.E.I.P. SYLLABUS

Grade 5

SE 006 069

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PREFACE

The Orange County Science Education Improvement Program (O.C.S.E.I.P.) is sponsored by the National Science Foundation and hosted by U.C. Irvine. It is a cooperative venture undertaken by the University of California, Irvine, California State College at Fullerton, the Orange County Schools Office and local school districts throughout Orange County. This syllabus was written by O.C.S.E.I.P. to help teachers teach the best aspects of recent mathematics programs. It is not meant to be another textbook for a new program. Instead, it is meant to be a sharing and synthesis of effective teaching methods. The outline of topics is a minimum coverage which is common to all schools in Orange County. Topics adequately covered in the majority of texts in use are given a minimum treatment in the syllabus.

The first draft of this syllabus was written during an 8 week session at University of California, Irvine during the summer of 1966 by:

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We wish to thank all the participants in this program for their hard work and fine cooperation.

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NUMBERS AND NUMERATION

Grade 5

NUMBERS AND NUMERATION

Grade 5

Understanding sets of numbers through hundred millions.

Purpose: To reinforce the places in the decimal system.

When first presenting this lesson, put the work on the board and do the work with the students.

Say, "Here is a set of whole numbers."

$$S = \{26,\ 4{,}321,\ 3001,\ 14,\ 139,\ 3,\ 709,\ 111,\ 17,\ 89,\ 9,\ 4,\ 173,\ 83\}$$

"Construct subsets of S."

Set A: All members of S that have an even number in the ones place. $\{26, 14, 4\}$

Set B: All members of S that have an odd number in the hundred's place. $\{4{,}321, 139, 709, 111, 173\}$

To reinforce place value, write numerals through 999,999,999 in the original set and have the students construct subsets from it. The follow-up work could be on a ditto.

(a) 6789243

(b) 1083543

(c) 567 thousand, 234

(d) 124,267

(a) List number and have pupils place commas where they are needed.

(b) List numbers and have pupils rewrite arranging the digits to make largest or smallest possible number. (Explain that each 3-number group can be considered a "family." Each comma designates a new family)

(c) Rewrite, using all numbers.

(d) Write, using sentences.
(i.e. One hundred twenty-four thousand, two hundred sixty-seven)

(e) seven hundred ninety-six million five hundred thirty-four thousand, one hundred twenty-six.

(e) Write, using numerals.

(List numbers on board and have pupils practice reading.)

Reading and writing through 999,999,999.

Materials: Place value chart, 3"x 2" slips of paper numbered 0 through 9.

(a)

100's	10's	1's
2	4	5

Place 5 in the 1's place. Have class identify it ("five ones").
Place 4 in the ten's place and identify it ("four tens").
Place 2 in the hundred's place. Identify it ("two hundreds").
Read the 3 digits aloud: "Two hundred forty-five".

(b)

100,000's	10,000's	1,000's	100's	10's	1's
3	7	6,	2	4	5

Repeat 235 as in Figure A and extend the places to 1,000, 10,000, and 100,000. Place 6 in the thousand's place. Identify it as "Six thousands." Place 7 in the ten thousand's place. Identify it as "Seven ten-thousands." Place 3 in the 100,000's place and identify it as "Three hundred-thousands." Read the entire number by family groups as 376 thousand 245. Identify the families (thousands and ones or units).

(c)

100,000,000's	10,000,000's	1,000,000's	100,000's	10,000's	1,000's	100's	10's	1's
1	2	3,	3	7	6,	2	4	5

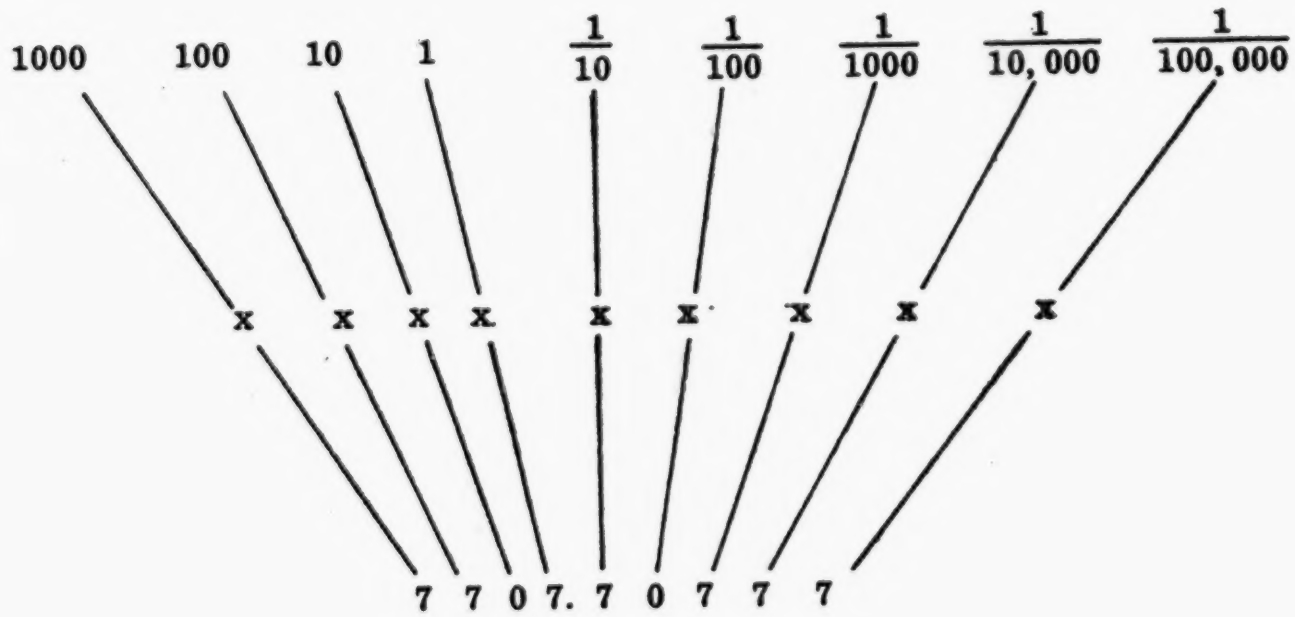
Repeat the introduction of numerals 1, 2, and 3 in the millions family. Finally read the numbers by families, 123 million 376 thousand 245.

Understanding fractional numbers through thousandths.

Decimal fractions are extremely convenient and an efficient way to represent fractional parts. For this reason, decimals are widely used in science and industry, and form the numerical basis of our monetary system.

The concept of place value is fundamental to an understanding of decimals.

In using these two examples to teach the concept of place value, emphasize the fact that the decimal point is the pivot point, not the one's place.



7000.
700.
7.
.7
.007
.0007
+ .00007
7707.70777

Does each 7 in this numeral have a different value? The last 7 is in hundred-thousandth's place. . Its value is $7 \times \frac{1}{100,000}$ or $\frac{7}{100,000}$.

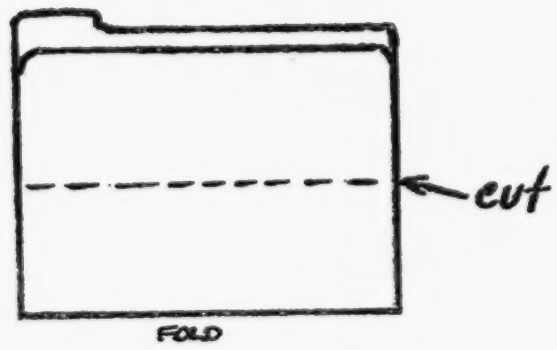
Which 7 has the value of 700? Which 7 has the value of $\frac{7}{1000}$?

Place value charts are effective aids in teaching decimals.

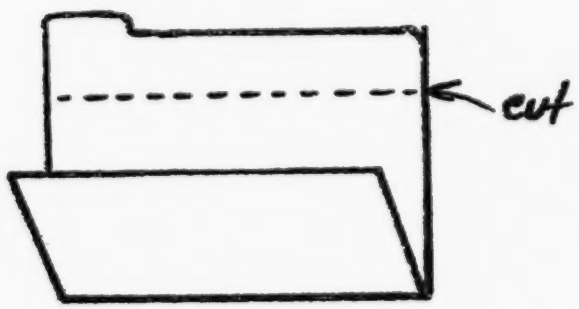
A pocket chart can easily be made from a manila folder, the kind used for filing. Other materials needed: Stapler, scissors, and marking pens. Proceed as follows:

(a) Illustrated at left
(b) Illustrated at left
(c) Illustrated at left
(d) Cut the extra pieces of the folder into strips for tallies.
(e) If you wish to work with other bases, cut a strip of the same width as the part of your chart where the place-value's names are written. Attach this with paper clips. These charts can be made appealing by using coat fabric to look like kangeroo pockets, or piggy bank with appropriate name (place-value) slots.

(a)



(b)



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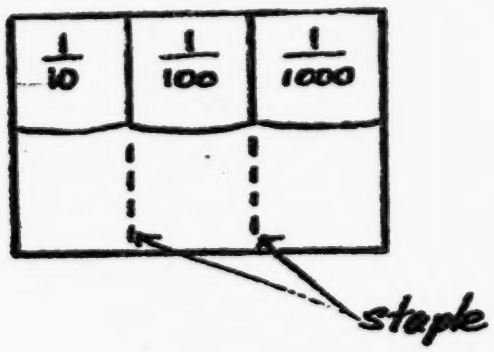
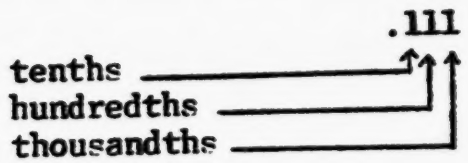


Place value of decimal fractions.



As you move to the right of the decimal point, each place has 1/10 the value of the place to its left. The first place to the right of the decimal point has 1/10 the value of the ones place, 1/10 x 1 - 1/10 (tenths): the second place has 1/10 the value of the tenths place, 1/10 x 1/10 = 1/100 (hundredths); the third place has 1/10 the value of the hundredth's place, 1/10 x 1/100 = 1/1000 (thousandths).

Write the correct numeral:

.58 means __?__ tenths and __?__ hundredths.

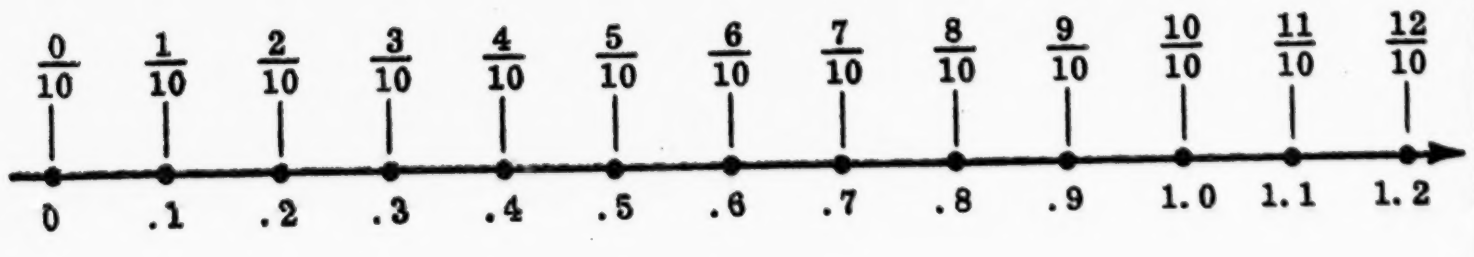
.309 means __?__ tenths and __?__ hundredths and __?__ thousandths.

Relationship of decimal fractions with common fractions on the number line.

Sentences with a variable give children experience in noting greater-than or less-than relationships between common fractions, common and decimal fractions, or decimal fractions. Ask students to place $<$, $>$, or $=$ in each $\square$ to make each sentence true. Use the number line for help.

$.9 \square \frac{3}{10}$

$1.1 \square 1\frac{4}{10}$



Decimal fraction sequence.

.15, .17, .19, ___, ___

.43, .48, .53, ___, ___

Many children who seem to have a good understanding of whole numbers often have difficulty in learning about decimal fractions. They fail to see the relationship between the set of whole numbers, their properties and operations and the set of decimal fractions. Every opportunity should be given to draw analogies between the two so that the pupils can transfer the learning from whole numbers to decimal fractions.

Renaming common fractions as decimal fractions.

$\frac{2}{10}$ = ◯

$\frac{75}{100}$ = ◯

$\frac{10}{100}$ = ◯

Expanded Notation through seven-place numerals.

10,000's	1,000's	100's	10's	1's
2	1	2	3	7

Show place values in 785 on the chalkboard.

$$785 = (7 \times 100) + (8 \times 10) + (5 \times 1)$$

(Present each student with ditto sheet - Place place value grid on board for presentation).

$21,237 = (2 \times 10,000) + (1 \times 1,000) + (2 \times 100) + (3 \times 10) + (7 \times 1)$.

Here we show we have: two 10,000's added to one 1,000, added to two 100's added to three 10's added to seven 1's.

When combined we read it 21,237.

one millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
9,	3	6	2,	7	4	8

Give students expanded numbers like this one to read aloud and then write in grid or chalkboard.

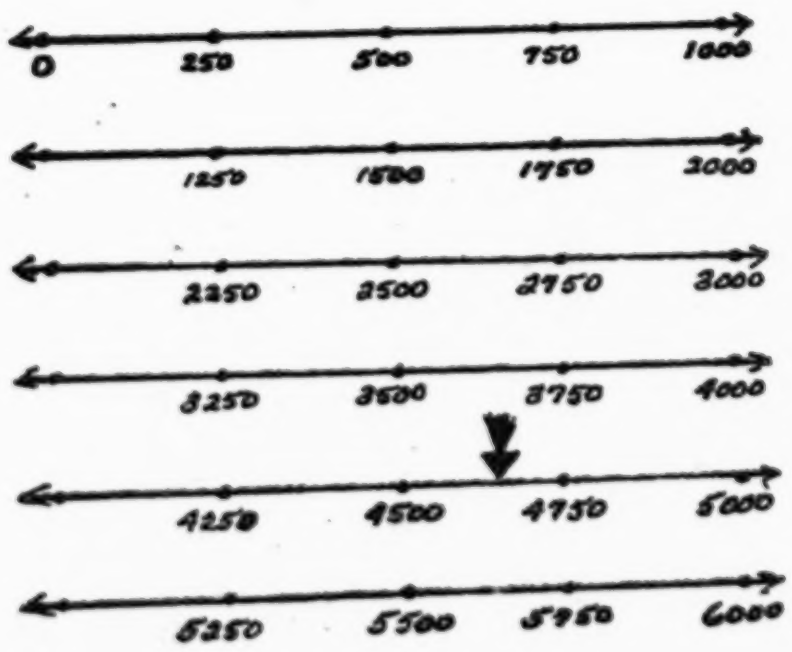
(9 x 1,000,000) + (3 x 100,000) +(6 x 10,000) + (2 x 1,000) + (7 x 100) + (4 x 10) + (8 x 1).

Follow-up: Have ditto sheets of similar problems to give each child.

Rounding to nearer thousand.

It is recommended that rounding to the nearer 10 and the nearer 100 be reviewed before presenting rounding to the nearer 1,000.

Folded Number Line



Make a large folded number line as shown in the example, but label more of the points than is shown.

Locate 4,674 on the folded number line. By counting determine whether this number is closer to 4,000 or 5,000.

Repeat the above procedure with other numbers until the students become comfortable with the concept. Dittoed sheets of number lines without numerals can be given students so they may number their own and work directly on the lines while rounding.

When students can readily handle rounding three-place digits to thousands, extend presentation to include rounding five, six, and seven-place numbers to thousands.

Round to the nearer thousand:

1. 43,261 (43,000)
2. 543,261 (543,000)
3. 9,543,261 (9,543,000)

To make the point that it is necessary only to examine the numerals in thousand's place and the numerals to the right, cover the numerals to the left of thousand's place with hand or paper. Ask students to prove answers on their number lines. Example 1 above might be proven thus:

Prime numbers and prime factors through

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

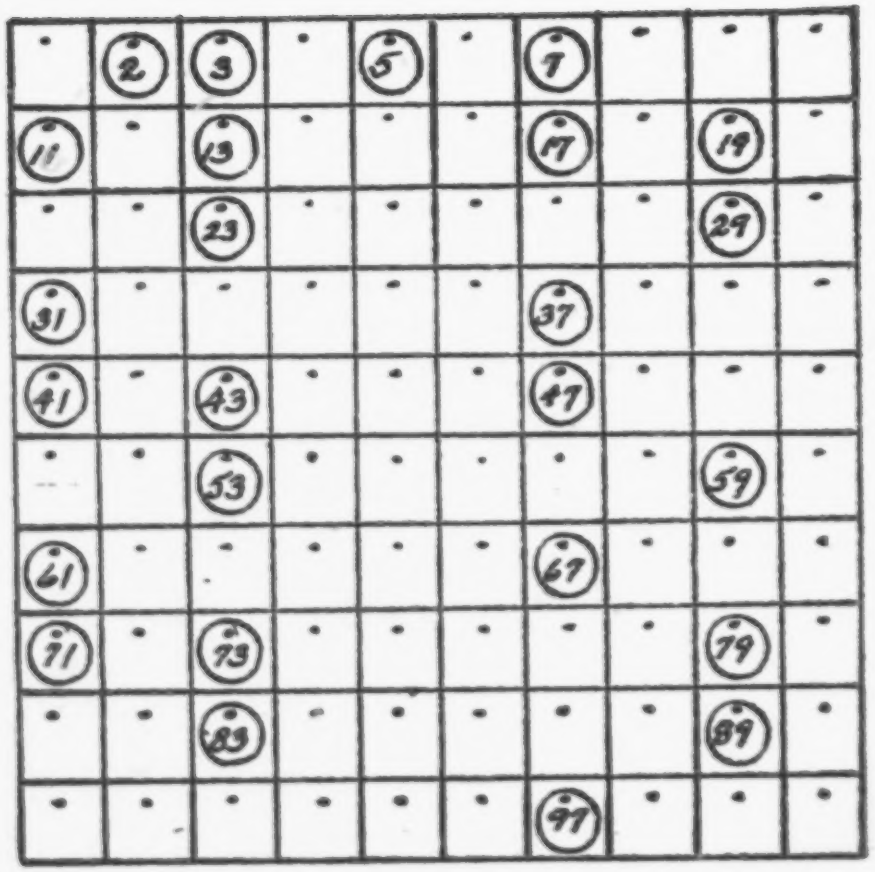
Students who know the prime numbers will be more easily able to find the greatest common factor and the least common multiple when working with fractions. Encourage students to memorize the primes through 100 after this lesson.

A prime number has only one and itself as factors. All numbers not prime are called composite. Mathematicians agree that one, by definition, is neither prime nor composite.

Let students "discover" the primes by making a device called the Sieve of Eratosthenes out of graph paper. Ask them to insert numerals 1 to 100 as shown and give the following directions:

> "Use this Sieve of Eratosthenes to find the prime numbers less than 100. Two has no other factor but one and itself. Therefore, it is prime. Now cross out all multiplies of two. The next numeral is three. It is prime because it has no other factors than itself and one. Cross out all multiples until only those remain which have only themselves and one as factors."

These are the prime numbers less than 100:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37,
41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83,
89, and 97.



Another visual aid for discovering primes is the hundred's chart.

A hundred's chart, 10" x 10", may be made on poster paper, but a more durable and more versatile variety may be made from wood with a cup hook, or a screw which looks like an L, place in each square. Number key tags may be hung on the hooks in any way the occasion may warrant. Signs for operations and relations may also be printed on some tags and the chart used in working with open number sentences. Rubber bands stretched around appropriate hooks may demonstrate triangles and other polygons.

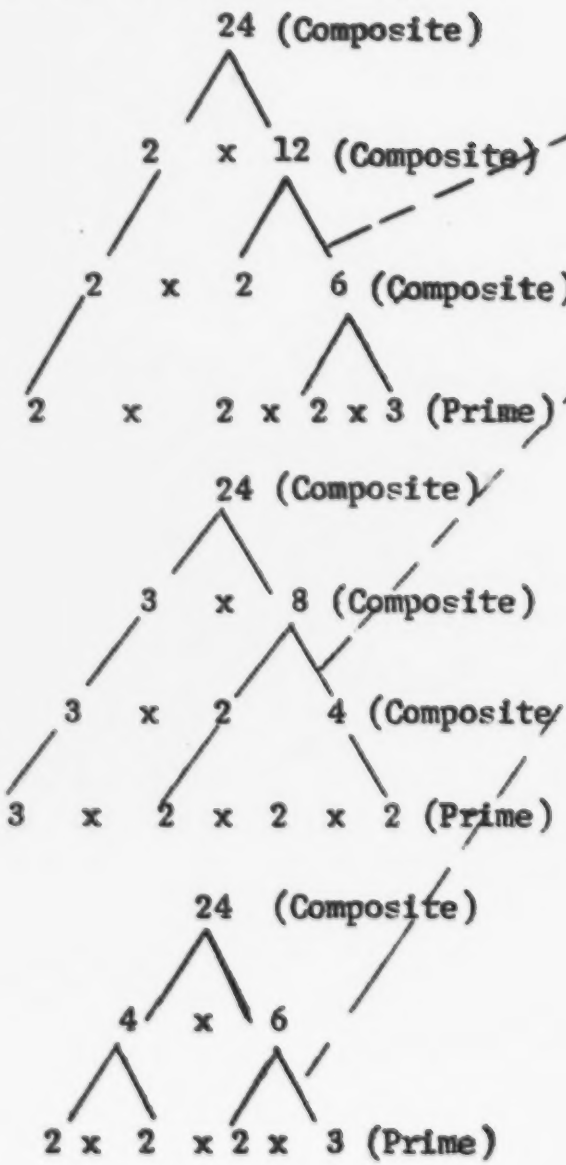
Once 2 is identified as a prime, the teacher could ask students to remove the tags containing multiples of two, and so on with multiples of 5, 7, 11, etc. until only primes are remaining. While this demonstration is presented before the class, the students could be crossing out their own multiples on the charts made from graph paper.

Another way to approach primes is to have students name all possible factors of each number from 1 to 100. When finished, have them circle those numerals that have only themselves and 1 as factors.

Write the set of all factors of each number.

(2)	{1, 2}	12		22	
(3)	{1, 3}	13		23	
4	{1, 2, 4}	14		24	
(5)	{1, ____}	15		25	
6	{1, 2, ____, 6}	16		26	
(7)	{____, ____}	17		27	
8		18		28	
9		19		29	
10		20		30	
11		21		31	

Prime factors of composite numbers.



A composite number is one which has other factors than itself and one. For instance, 24 can be renamed as 2 x 12, 3 x 8, 4 x 6. The factor tree illustrates that each of the composite numbers can be shown by the product expressions 2 x 2 x 2 x 3. Because of the commutative property of multiplication, the fact that they are arranged in a different order is not important.

Children will see that factor trees look different from each other. Nevertheless, the same set of factors is to be found in the last product expression.

Prime factorization.

When a composite number is expressed as a product of prime numbers, it is often referred to as prime factorization.

By using prime factorizations students can arrive at the greatest common factor and the least common multiple of two numbers.

(1) $42 = 2 \times 21$
$= \boxed{2 \times 3 \times 7}$

(2) $45 = 3 \times 15$
$= \boxed{3 \times 3 \times 5}$

(3) $50 = 2 \times 25$
$= \boxed{2 \times 5 \times 5}$

(4) $56 = 2 \times 28$
$= 2 \times 2 \times 14$
$= \boxed{2 \times 2 \times 2 \times 7}$

Roman numberals D and M.

The Roman system of numeration is probably the most familiar to us of all the older systems. We still see Roman numerals used for book systems, clock numerals, and formal instructions. The Roman system was not a place-value system and did not have a symbol for zero. The most common Roman symbols are listed in Example A.

A.

Roman Numeral	Value of Numeral
I	1
V	5
X	10
L	50
C	100
D	500
M	1,000

B. XVI = 10 + 5 + 1 = 16
LXVIII = 50 + 10 + 5 + 1 + 1 + 1 = 68

To find the value of a number, the Romans used an additive principle. See B. Have students write some numbers using M and D, making sure that the students only add the values.

C. MDCCCCLXVII

However, the Roman numeration system was not based entirely on simple addition of number values represented by numerals in any order. Although the Roman system did not have place value, numerals of greater value were usually placed to the left of numerals of lesser value. See C.

A numeral of lesser value placed to the left of (or preceding) numerals of greater value, indicated that the numeral of lesser value was to be subtracted from the numeral of greater value. See D.

D. IX = 10 - 1 = 9

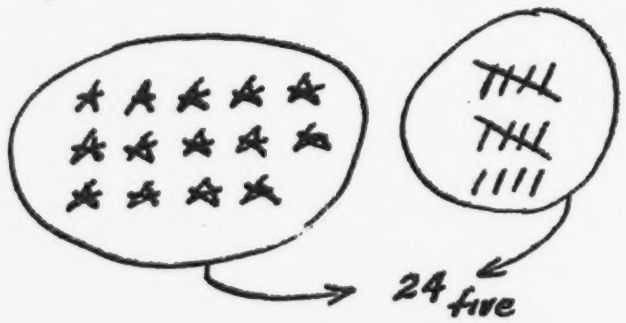
IV = 5 - 1 = 4

XCIV = (100-10)+(5-1) = 94

Have students write numbers using M, D, and this subtraction concept.

Suggested Activity: Divide the class in half and play baseball at the seats with Roman Numeral-writing the "pitched-ball". The youngsters at "bat" write the Roman numerals for: 300, 92, 505, 123, 1001, 1111, 223, 1214, etc. or, have the youngsters write the Hindu-Arabic numerals for: MCX, MDCIV, MIII, etc.

Grouping sets of objects, counting, and recording in base five numeration system through 100 five.



The operations that students perform with numerals are much better understood through the use of expanded notation utilizing place value. Working with place value in base five helps the student "see" the process of the operation, the importance of place value, and the beauty of our base ten system.

Have the students count sets of objects easily seen by the class such as books, erasers, rulers, etc. As objects are counted, have two students record the count: one using

base x base x base	base x base	base	ones
			1
			2
			3
			4
		1	0
		1	1
		1	2
		1	3
		1	4
		2	0
		2	1

tally marks grouped in fives and one using base five numerals. 12_{five} is read "one,two, base five." 314_{five} is read "three, one, four, base five." Continue counting practice until students can easily read and write up to 100_{five} (twenty-five in base ten). When students can count in base five easily, have four students count a set of twenty-five objects: one using tally marks grouped in fives, one using base five numerals, one using tally marks grouped in tens, and the last using base ten numerals. Let students compare the two different bases. How are they alike?

Ask students to make both addition and subtraction tables for base 5. Do not ask them to memorize facts but let them use tables to solve simple problems involving some "carrying" and "borrowing."

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	10
2	2	3	4	10	11
3	3	4	10	11	12
4	4	10	11	12	13

Addition in base five.

One group of 5 x 5, no groups of 5 and 3 units.

$$\begin{array}{r} 32_{five} \\ +\ 21_{five} \\ \hline 103_{five} \end{array}$$

Subtraction in base five.

One group of 5 and 1 unit.

$$\begin{array}{r} 32_{five} \\ -\ 21_{five} \\ \hline 11_{five} \end{array}$$

A great deal of time should not be spent on base five. Fast learners will enjoy the experience and will no doubt like to explore other bases. The average performers will benefit by the experience and apply more fervor into understanding the decimal system. Few students performing below fifth grade will make an analogy between the base five and base ten but will think of base five as a frustrating assignment. If this seems to be the case, the base five experience will be of no value to the students and should be discontinued for that group.

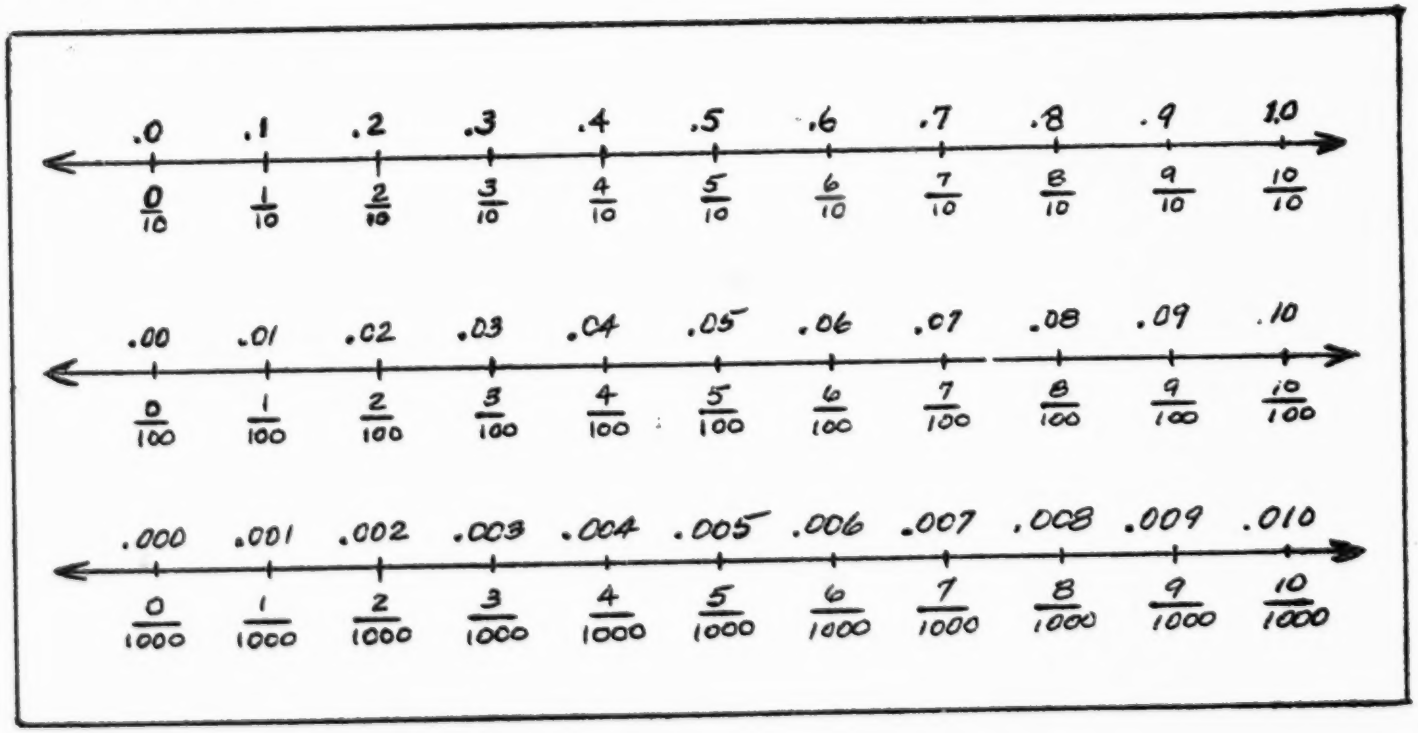
ADDITION AND SUBTRACTION

Grade 5

Grade 5

Addition and subtraction of decimal fractions through thousandths.

A chart such as this one will be helpful in leading students to see relationships between decimal and common fractions.

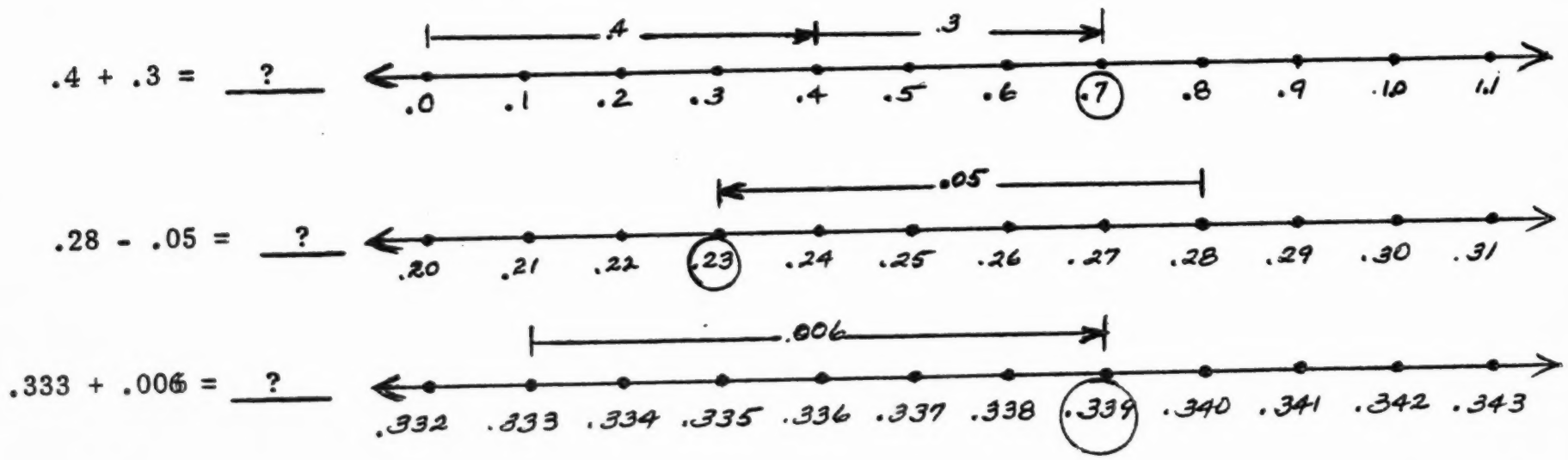


Since the students know how to add and subtract "like" common fractions, the transition to decimals can be bridged by comparing decimals and fractions in examples like these:

$$\begin{array}{r} .1 \\ +\ .3 \\ \hline .4 \end{array} \qquad \begin{array}{r} \frac{1}{10} \\ +\ \frac{3}{10} \\ \hline \frac{4}{10} \end{array} \qquad \begin{array}{r} .57 \\ +\ .22 \\ \hline .79 \end{array} \qquad \begin{array}{r} \frac{57}{100} \\ +\ \frac{22}{100} \\ \hline \frac{79}{100} \end{array} \qquad \begin{array}{r} .327 \\ +\ .041 \\ \hline .368 \end{array} \qquad \begin{array}{r} \frac{327}{1000} \\ +\ \frac{41}{1000} \\ \hline \frac{368}{1000} \end{array}$$

Give students dittoed number lines without numbers and have them show simple addition and subtraction problems. The problems can be given in horizontal form since these problems would have equal denominators (like fractions). Have students show their solutions on chalkboard for class discussion.

12:57
6:240-258



Addition and subtraction of decimal fractions having different denominators (unlike fractions) can best be presented by building on student knowledge of common fractions. Show how a common denominator is found for 10 and 100, 10 and 1,000, and 100 and 1,000.

a. $\frac{3}{10} = \frac{30}{100}$ $\quad .3 = .30$ annex 1 zero to change tenths to hundredths.

$\underline{+\ \frac{2}{100}} \quad \underline{= \frac{2}{100}} \qquad \underline{+\ .02} \quad \underline{= .02}$

$\frac{32}{100} \qquad .32$

b. $\frac{7}{10} = \frac{700}{1000}$

$+ \frac{14}{1000} = \frac{14}{1000}$

$.7 = .700$

$+ .014 \quad .014$

$.714$

Annex 2 zeros to change tenths to thousandths.

c. $\frac{55}{100} = \frac{550}{1000}$

$+ \frac{222}{1000} = \frac{222}{1000}$

$\frac{772}{1000}$

$.55 = .550$

$+.222 \quad .222$

$.772$

Annex 1 zero to change hundredths to thousandths.

d. $\frac{3}{10} = \frac{30}{100}$

$- \frac{19}{100} \quad \frac{19}{100}$

$\frac{11}{100}$

$.3 = .30$

$.19 = .19$

$.11$

Annex 2 zeros to change tenths to thousandths.

e. $\frac{9}{10} = \frac{900}{1000}$

$- \frac{325}{1000} = \frac{325}{1000}$

$\frac{575}{1000}$

$.9 = .900$

$- .325 \quad .325$

$.575$

Annex 2 zeros to change tenths to thousandths.

f. $\frac{72}{100} = \frac{720}{1000}$

$- \frac{511}{1000} \quad \frac{511}{1000}$

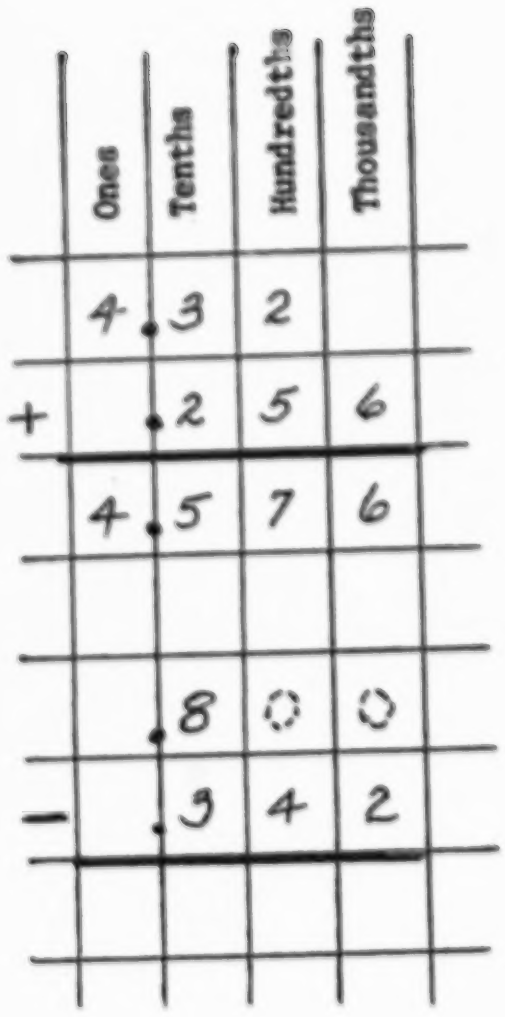
$\frac{209}{1000}$

$.72 = .720$

$- .511 \quad .511$

$.209$

Annex 1 zero to change hundredths to thousandths.



Lead students to see that zeros are added to the numerators of the decimal fractions so that the denominators will become equal. Number lines may be used to show that in example a., .30 is another name for .3 and thus they are equivalent fractions.

When place value is reviewed, it can be easily shown that they need not annex the zeros when adding (since they are simply placeholders). Remind students that thousandths, hundredths to hundredths, and tenths to tenths, thus we usually align the decimal points. Dittoed place value grids or graph paper can be used to aid students in aligning place values of unlike decimals as shown at left. Remind them that zeros may have to be annexed when the minuend has four places than the subtrahend as .8 - .342 = .458. Make sure that the class "sees" that annexing zeros to a decimal fraction simply renames it and does not change its value. A number line can be used to show that .8 and .800 name the same point on a number line as does $\frac{8}{10}$ and $\frac{800}{1000}$; thus, they are equivalent fractions or different names for the same number. Another way to show this is:

$\frac{8}{10} \times 1 = \frac{8}{10}$ .8 x 1 = .8

$\frac{8}{10} \times \frac{10}{10} = \frac{80}{100}$ .8 x 1.0 = .80

$\frac{8}{10} \times \frac{100}{100} = \frac{800}{1000}$ .8 x 1.00 = 800

Addition and subtraction of fractional numbers with sums and minuends greater than 2.

a. $15\frac{1}{4} + 27\frac{7}{10} =$

$15\frac{5}{20} + 27\frac{14}{20} = 42\frac{19}{20}$

b. $11\frac{7}{12} = 11\frac{35}{60}$

$3\frac{2}{5} = 3\frac{24}{60}$

$8\frac{11}{60}$

c. $5\frac{11}{12}$

$17\frac{9}{12}$

$+ 14\frac{7}{12}$

$36\frac{27}{12} + 2\frac{3}{12} = 38\frac{1}{4}$

Previous work with fractions whose sums were less than 2 and subtraction where the minuend was less than 2. These skills are extended at this grade level to include working with larger mixed fractions such as those in the examples at left.

New concepts to be learned include:

1. Addition - horizontal form (example a.). Discuss a problem in both vertical and horizontal forms.

$15\frac{1}{4} + 27\frac{7}{10} = 15\frac{5}{20} + 27\frac{14}{20} = 22\frac{19}{20}$

$15\frac{1}{4} + 27\frac{7}{10} = (15 + 27) + (\frac{1}{4} + \frac{7}{10})$

$= (15 + 27) + (\frac{5}{20} + \frac{14}{20})$

$= 42 + \frac{19}{20}$

$= 42\frac{19}{20}$

2. Subtraction - minuend greater than 2 (example b.). No special difficulties here except when "borrowing" as below.

$$\begin{array}{r} 27\frac{1}{8} \\ -16\frac{2}{3} \\ \hline \end{array} = \begin{array}{r} 27\frac{3}{24} \\ 16\frac{2}{3} \\ \hline \end{array} = \begin{array}{r} 26\frac{27}{24} \\ 16\frac{16}{24} \\ \hline 10\frac{11}{24} \end{array}$$

$$\begin{array}{r} 34 \\ -27\frac{3}{4} \\ \hline \end{array} = \begin{array}{r} 33\frac{4}{4} \\ 27\frac{3}{4} \\ \hline 6\frac{1}{4} \end{array}$$

Place some problems in horizontal form on the chalkboard for class discussion.

$$\begin{aligned} 27\tfrac{1}{8} - 16\tfrac{2}{3} &= (27 - 16) + (\tfrac{1}{8} - \tfrac{2}{3}) \\ &= (11) + (\tfrac{3}{24} - \tfrac{16}{24}) \\ &= (10 + \tfrac{24}{24}) + (\tfrac{3}{24} - \tfrac{16}{24}) \\ &= 10 + (\tfrac{24}{24} + \tfrac{3}{24}) - \tfrac{16}{24} \\ &= 10 + (\tfrac{27}{24} - \tfrac{16}{24}) \\ &= 10 + \tfrac{11}{24} \\ &= 10\tfrac{11}{24} \end{aligned}$$

3. Addition - sum of fractions may be greater than 2 when adding three or more fractions (example c.). This type of problem might be preceded by work with improper fractions as:

$1 = \frac{\square}{4}$ $2 = \frac{\square}{4}$ $3 = \frac{\square}{4}$ $4 = \frac{\square}{4}$

$1 = \frac{\square}{6}$ $2 = \frac{\square}{6}$ $3 = \frac{\square}{6}$ $4 = \frac{\square}{6}$

$1 = \frac{\square}{7}$ $2 = \frac{\square}{7}$ $3 = \frac{\square}{7}$ $4 = \frac{\square}{7}$

$\frac{5}{4} = \frac{4}{4} + \frac{\square}{4}$ $\frac{10}{4} = \frac{4}{4} + \frac{4}{4} + \frac{\square}{4}$ $\frac{17}{4} =$ __+__+__+__+__

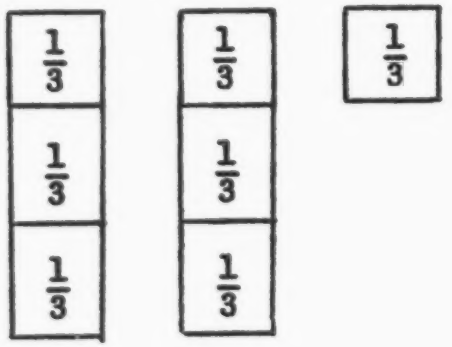
$\frac{9}{6} = \frac{6}{6} + \frac{\square}{6}$ $\frac{15}{6} = \frac{6}{6} + \frac{6}{6} + \frac{\square}{6}$ $\frac{21}{6} =$ __+__+__+__+__

$\frac{13}{7} = \frac{7}{7} + \frac{\square}{6}$ $\frac{14}{7} = \frac{7}{7} + \frac{7}{7}$ $\frac{24}{7}$ __+__+__+__+__

$\frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{1}{3} =$ ____ $\frac{8}{8} + \frac{8}{8} + \frac{8}{8} + \frac{8}{8} + \frac{7}{8} =$ ____

$\frac{5}{5} + \frac{5}{5} + \frac{4}{5} =$ ____ $\frac{12}{12} + \frac{12}{12} + \frac{12}{12} + \frac{6}{12} =$ ____

Students having trouble with renaming improper fractions as mixed fractions may need to work with paper fraction-kits and show regroupings as they work. Thus $\frac{7}{3}$ can be shown at student desk as:



$$\frac{3}{3} + \frac{3}{3} + \frac{1}{3} = 2\frac{1}{3}$$

Readiness for addition and subtraction of negative integers.

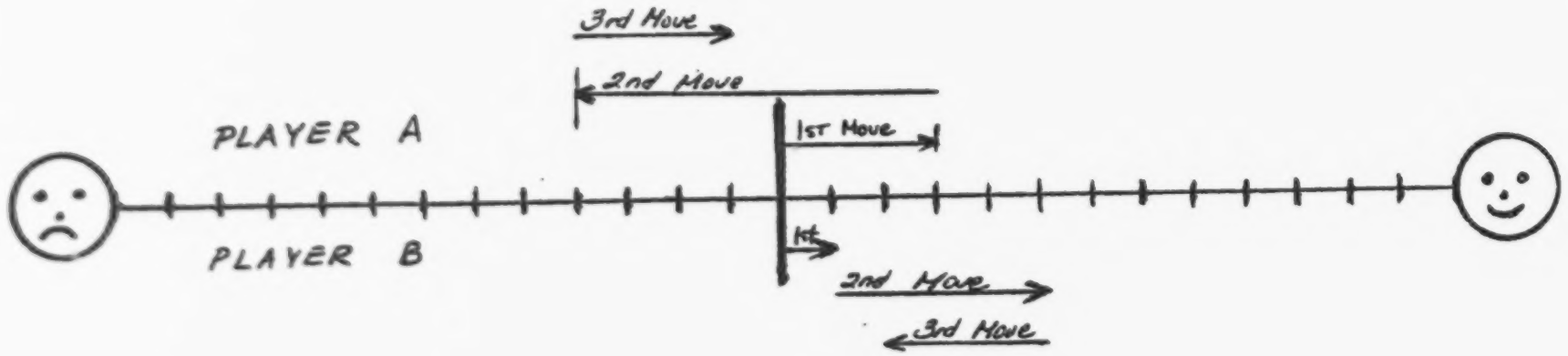
Put a number line on the chalkboard as shown in the following example.

On the board write numerals 3, 1, 7, 4, 2, 5 in this order. The direction of moves is determined by flipping a coin. Heads moves toward the winner's and tails moves toward the loser's end.

Two students go to the board. One student makes his moves on the top of the number line and the other student marks his moves on the bottom of the number line.

For example:

Player A gets heads. (He moves 3 spaces to the right.)
Player B gets heads. (Moves 1 left)
Player A gets tails. (Moves 1 left)
Player B gets heads. (Moves 4 right)
Player A gets heads. (Moves 2 right)
Player B gets tails. (Moves 5 left)



In selecting the numerals to use for determining the moves, use any number of numerals. The selection of these numerals is arbitrary.

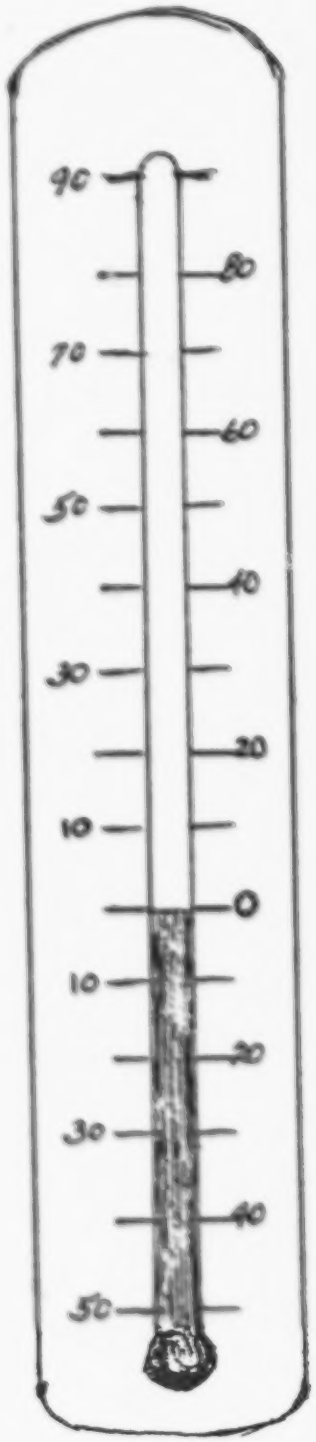
Yards Gained	Yards Lost	Position

Boys (and perhaps girls) will enjoy these football games in subtraction for negative numbers.

a) In a football game Team A was on their 40 yard line when the ball was snapped. The player was tackled and brought down at Team B's 35 yard line. How many yards were made in the play?

b) At the next play, the ball was stopped on the 24 yard line. However, Team A was penalized 15 yards. Where was the ball placed?

Chart these scores on the score card or a chalkboard number line.

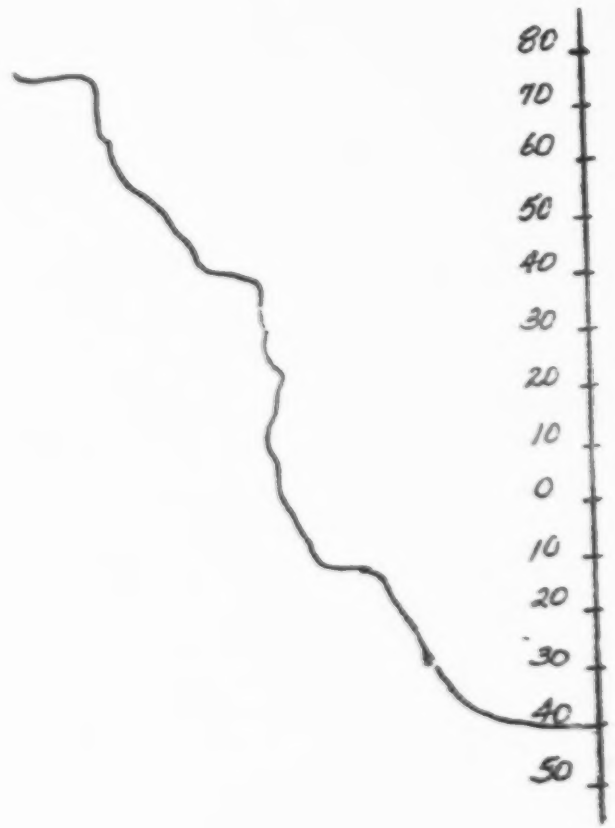


Use a thermometer to point out the negative integers below zero. Make a picture of a thermometer of tagboard. For the mercury indicator, use a strip of red and white elastic sewn together at the ends. Insert this at top and bottom so it can be moved up and down to indicate various temperature readings.

Have the students count the degrees as the indicator is moved from one temperature reading to another.

a) If the temperature registers 62° and drops to 12° below 0° how many degrees did the temperature drop? (74 degrees)

b) The temperature registered 28° at two o'clock. By eight o'clock it had dropped 36°. What was the new temperature reading? (-8 degrees).



Problems dealing with elevation provide experiences with negative integers. On a trip one summer Jim noticed an elevation sign reading 48' below sea level. Later he saw another sign which read, 10' above sea level. How many feet had they climbed? (58 feet)

After reaching 80' above sea level, Jim noticed that they were descending again. He kept a record of the number of feet of descent. On reaching the bottom of the pass, he found they had descended 215'. What was the elevation at the bottom of the pass?

Have students make up story problems dealing with elevations and exchange their stories with other class members. Encourage them to construct vertical number lines to aid them in solving these problems.

On the chalkboard or on a large piece of tagboard draw a football field. Mark off the yardlines by 2.

Divide the group into two teams. On the board write five even numbers.

Start the game on the 20 yard line. The team in possession of the ball gets 4 downs to make 10 yards.

The amount of yardage the ball moves on each down is determined by a toss of a coin. If the coin comes up heads, the ball moves forward. If the coin is tails, the ball moves back. The amount of yardage on each toss of the coin in determined by the 5 numerals which are listed on the board. Move through the five numerals consecutively. Variations may be made to suit the participating group.

Softball "500"

Fly ball = 100
One bounce = 75
Two bounce = 50
Grounder = 25

Pupils add for balls they catch, and subtract for errors (only if they actually touch the ball, but don't hang on to it.)

SAMPLE SCORING

1. Fly ball caught; worth +100 pts: Total = 100

2. Ground ball dropped; worth -25 pts: Total = 75

3. Fly ball dropped; worth -100 pts: Total = -25

4. One-bounce caught; worth +75 pts: Total = 50

5. Fly ball caught; worth +100 pts: Total = 150

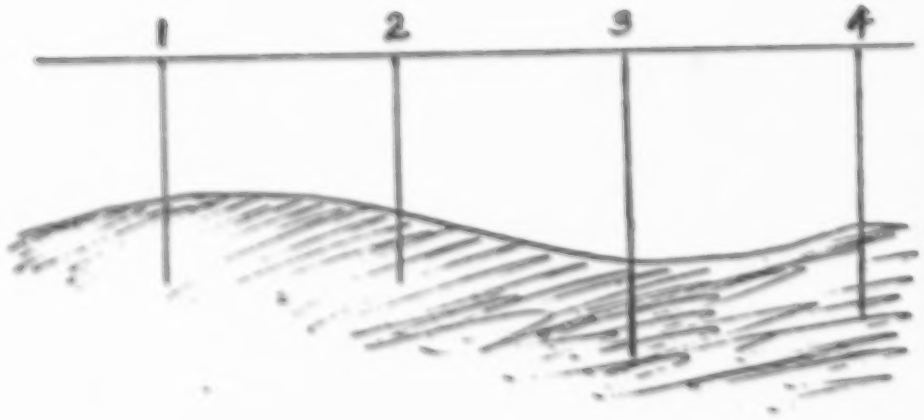
6. Fly ball caught; worth +100 pts: Total = 250

7. One-bounce caught; worth +75 pts: Total : 325

To build a foundation for work with negative numbers, you might try "500," a Physical Education game. Many pupils will already know this game, and will play it at recess. Others will need to have you explain the rules. "500" is a softball lead-up game, with a batter and several who try to work up to a score of 500 so they can become the next batter. There are two basic rules. A ball that is caught - even if it has hit the ground first - adds to a child's score. A ball that is dropped takes away from that score. (Only one person may score on each hit. A dropped ball is "dead"). Pupils keep a running total. (They'll check up on each other, and help to figure one anothers' scores.) A kickball may be substituted for ball and bat.

Other illustrations of negative numbers include distances above and below sea level; height and depth of a pole whose lower end is sunk in the ground; distances on either side of a point; count-down before rocket launching; and various number line activities.

Examples of some of these approaches are given on the pages following.



"A farmer was building a high fence around his pasture. He wanted the poles to be level with each other, even though the ground was not level. His first pole (#1) was 10' long, and 6' showed above ground. How much was below ground? (4') The next pole he drove was at the corner of the pasture (#4). He sighted across a level to get it the right height, and strung a cord between the two end posts to help his keep the others straight. On this chart is a record of some of his work:

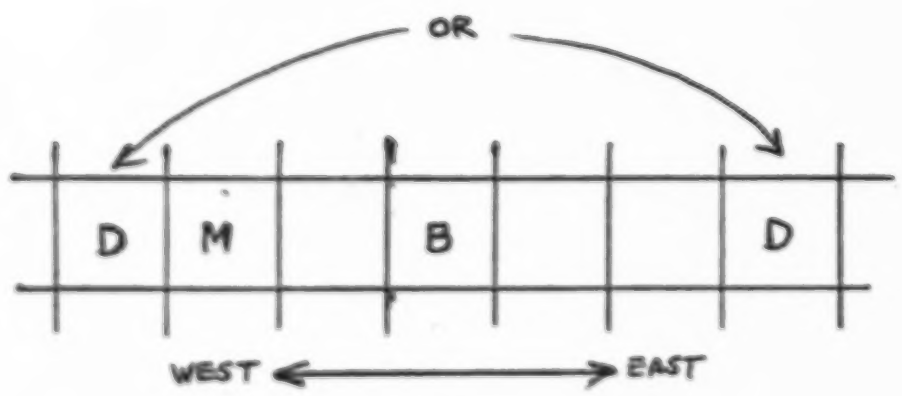
	Length of pole	Depth below ground	Height above ground
1.	10'	(4')	6'
2.	10'	3'	(7')
3.	(13')	4'	9'
4.	12'	(4')	8'

(Answers are shown in parentheses)

"Using ground level as "zero," how far above and below ground is each pole? Fill in the blank spaces on the chart."

"In one of California's valleys, the level of the ground is about 100 feet below sea level. Imagine you are there, and want to know your elevation. There are signs that tell when the level is at certain elevations. You are 25' higher than the 100' below sea level sign. What is your true elevation? (75' below sea level.) If you go 20 feet higher, are you getting closer to "0," or farther from it?"

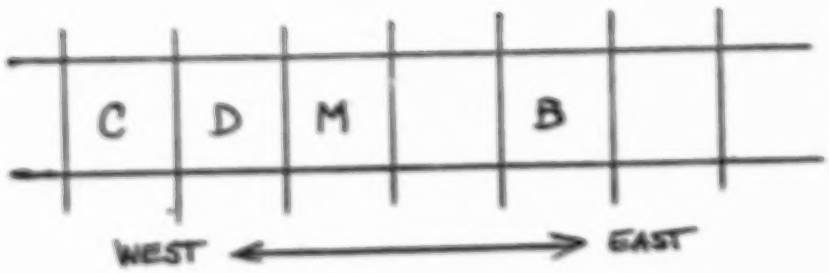
Give pupils some more examples, and let them make up some of their own. You might have them look up the height of Mt. Whitney, Bad Water, Death Valley, and find what the total difference in elevation is. (Have you tried giving "Extra point" problems for only the willing ones?)



To further the meaning of "negative numbers," the idea of distances in two directions can be shown.

"Bob, Carl, and Don live on the same straight street that the market is on. Bob lives two blocks east of the market, and Don lives three blocks away from Bob. How far away from the market is Don's house? (Try for two different answers: 5 blocks and one block)

Lead pupils to find some sign to show direction from a "zero point." Arrows above the numeral may be more meaningful than + or - signs, which will be used in 6th grade. Have a pupil draw a diagram to show his solution to Don and Bob's housing map. If another student has a different solution, let his show that.



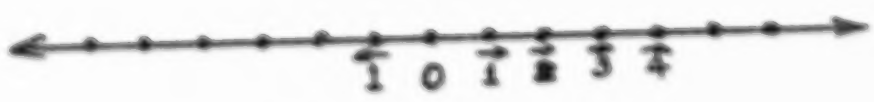
"Carl's house is 4 blocks west of Bob's. How far is it from Carl's house to the market? (2 blocks) Which direction must he walk to get to the market (the shortest way)? (East)"

"Let's agree that Don's house is between Bob's and Carl's. Can someone map the facts we now have? Use letters for the Market, Bob's, Carl's, and Don's homes."

"Tell where Bob would be if he made the following moves: (#1) He started at the market and went 2 blocks west. Where would he be? (Carl's house). (#2) From there, he walked 4 blocks east to (Bob's) (His own house)".

Try "shorthand" notation: The starting point will be given first, followed by the number of moves in the direction shown by the arrow. Pupils may find the diagram helpful, and perhaps now is the time to show negative values on a numberline. 12:46

M $\overleftarrow{2}$, $\overrightarrow{4}$ ⇝B
(Read "Starting at the market, he walked 2 blocks west and 4 blocks east to Bob's House.)
If Shorthand is confusing, go back to words:
"From Market, West 2 blocks, then East 4 blocks, to Bob's."



Draw a numberline, with spacing dots but no numerals. Fill in only zero, and $\overleftarrow{1}$ and $\overrightarrow{1}$. Ask pupils to label any point on the numberline with the correct numeral. "No one can put up more than one numeral. You may change someone's numeral only if you can tell why you think it is in the correct place."

For now, use raised ($\overrightarrow{3}$) arrows to show the direction "away from zero." Give problems of this sort: "Start at zero. Go two moves to the right, and four to the left. What is the name of the point where you finished?" ("left 3," or $\overleftarrow{3}$). Shorthand for that would be: "Start at zero. $\overleftarrow{2}$, $\overrightarrow{4} \rightsquigarrow 3$."

The students will be familiar with the "countdown" before satellite launching. Work up some problems based on this idea. For example, lead-in questions may be used; 12:46

1. "What is the count like <u>before</u> blast-off?" (numbers get smaller).

2. "What is it like <u>after</u> blast-off?" (numbers get larger)

3. "What is the 'zero point'?" (blast-off, ignition, "fire," etc.)

4. "How long is it between calls?"

First Call	2nd Call	Seconds between
minus 5	minus 3	(2)
minus 7	Blast-off	(7)
minus 6	minus 1	(5)
Blast-off	plus 4	(4)
minus 5	plus 5	(10)

START	MOVE	MOVE	END
0	$\overleftarrow{3}$	$\overrightarrow{3}$	($\overrightarrow{2}$)
0	$\overleftarrow{5}$	$\overleftarrow{6}$	($\overleftarrow{11}$)
$\overrightarrow{5}$	$\overleftarrow{2}$	$\overleftarrow{3}$	(0)
$\overrightarrow{2}$	$\overleftarrow{1}$	$\overleftarrow{2}$	($\overleftarrow{1}$)
0	$\overrightarrow{4}$	$\overrightarrow{3}$	($\overrightarrow{7}$)
($\overrightarrow{4}$)	$\overrightarrow{4}$	$\overrightarrow{4}$	$\overrightarrow{12}$
$\overleftarrow{3}$	($\overrightarrow{1}$)	$\overleftarrow{3}$	$\overleftarrow{2}$

(Answers in parentheses would be blank on worksheet)

A chart may be used to provide "drill" on the work with positive and negative values. From any starting point, "jumps" may be made in either direction. Pupils fill in the "directed numerals" to record the results of moves, or the missing moves. Pupils will need to make a suitable numberline.

ADDITION AND SUBTRACTION DRILLS AND GAMES

Grade 5

Casting our nines in addition

Purpose: To give the pupil practice in "casting our nines."

Procedure: Add the column of numerals, then add each row of numerals and keep adding until there is only one digit left.

Now, add the column down until only one digit is left. Add the sum row until only one digit remains. The two digits should be the same if the problem is correct.

Example:

```
 96742 = 28 = 10 = 1
 74932 = 25 =      7
 52957 = 28 = 10 = 1
 32704 = 16 =      7
257,335 = 25 = (7)  16 = (7)
```

Problems:

a) 276495 + 765298 + 736923

b) 438695 + 243769 + 329478

c) 36542 + 50489 + 34297

d) 764398 + 874209 + 283984

e) 438752 + 432094 + 296023

f) 298503 + 432981 + 239482

Patterns in addition

Complete the addition tables below.

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

+	10	20	30	40	50	60	70	80
2								
3								
4		24	34	44	54			
5								
6								

+	8	18	28	38	48
2					
3		21	31	41	51
4					
5					
6					

Story problems of addition

Purpose: to furnish drill for addition facts.

Work Space

Father gave Jane and Jerry cyclometers for their bicycles.

1. During the first month the children had their cyclometers, Jerry rode 134 miles, and Jane rode 129 miles. How many more miles did Jerry ride than Jane?

2. When Jerry rode to Paradise Lake, his cyclometer read 199 miles. When he returned, it read 239 miles. How many miles did he ride on the trip?

3. Carl told Jerry that at the beginning of the previous summer his cyclometer had read 168 miles. At the end of the summer it read 544 miles. How many miles had he ridden during the summer?

4. When Jerry's cyclometer read 239 miles, Jane's read 178 miles. How many more miles had Jerry ridden than Jane?

5. The Martin family made several short drives in their new car during the weekend. The speedometer read 753 miles at the beginning of the weekend and 833 miles at the end. How many miles did the family drive during the weekend?

6. Jerry and Jane rode 78 miles in the car while they were visiting in the country and 8 miles more when they returned home. How many miles did they ride during the weekend?

Applying addition and subtraction skills

Prices may be found in advertisements, catalogues, newspapers, or by visiting a store.

Fill in the blanks and add each list.

Clothing for a Girl

1 Coat	$_____
1 Dress	_____
1 Sweater	_____
2 pr. Stockings	_____
1 Hat	_____
1 pr. Shoes	_____
Total	$

Clothing for a Boy

1 Suit	$_____
2 Shirts	_____
1 Sweater	_____
3 pr. Stockings	_____
1 Tie	_____
1 pr. Shoes	_____
Total	$

Living Room Furniture

1 Rug	$_____
1 Living Room Suite	_____
1 Television Set	_____
2 Lamps	_____
1 Coffee Table	_____
2 Chairs	_____
Total	$

Brainteasers

Think twice before you answer:

1. Mike and Max were on their bicycles 30 miles apart. Mike started riding toward Max at 10 miles an hour. Max started riding toward Mike at 5 miles per hour. A fly starting on Mike's bicycle flew back and forth between the bicycles until they met. The fly's

rate was 30 miles per hour. How far did the fly travel? (60 miles) (It took the boys 2 hours to meet, so the fly flew 2 hours)

2. The number of eggs in a basket doubles every minute. The basket is full in half an hour. When was it half full? (it was half full in 29 minutes)

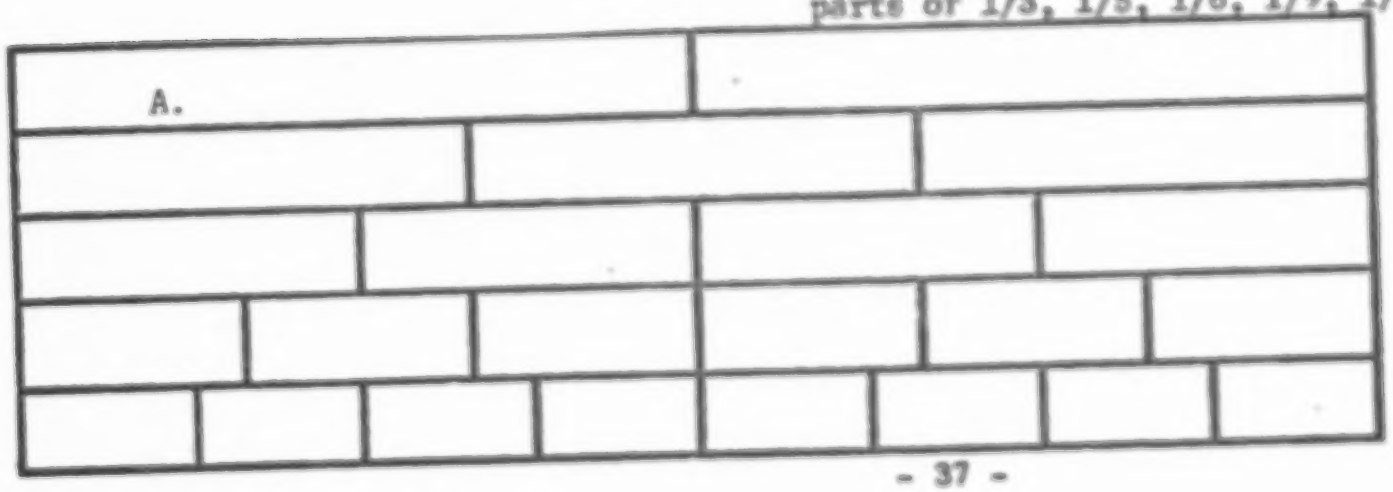
3. A paper bag contains 10 white socks. While blindfolded, what is the least number of socks you can remove from the bag before you are sure you have either a black pair or a white pair? (3 socks)

4. An electric train is going 100 miles per hour against a 100-mile-an-hour head wind. Which way will the smoke from the engine blow? (There will be no smoke from an <u>electric</u> train)

Comparing fractions

Write the correct fraction in each rectangle to the left.

Pupils should study these diagrams before attempting to fill in the fractions. Then have them prepare diagrams depicting the whole divided into equal parts of 1/3, 1/5, 1/6, 1/9, 1/10, 1/15.



(1) $\frac{1}{2} > \frac{1}{3}$ ____

(2) $\frac{1}{6} < \frac{1}{4}$ ____

(3) $\frac{1}{8} < \frac{2}{3}$ ____

(4) $\frac{3}{4} > \frac{5}{6}$ ____

(5) $\frac{1}{2} < \frac{3}{6}$ ____

(6) $\frac{3}{6} = \frac{4}{8}$ ____

(7) $\frac{7}{8} < \frac{3}{4}$ ____

(8) $\frac{2}{4} \neq \frac{3}{8}$ ____

(9) $\frac{8}{8} = \frac{6}{6}$ ____

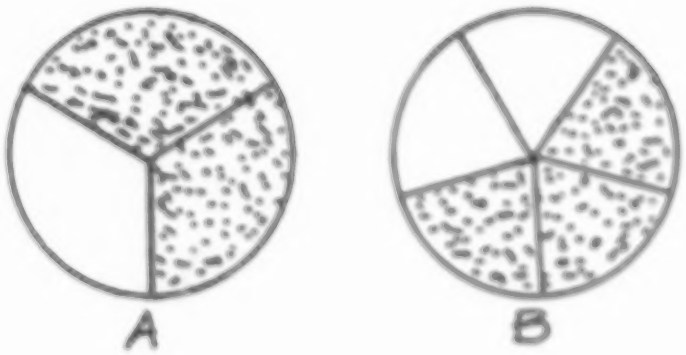
(10) $\frac{5}{6} < \frac{3}{4}$ ____

(11) $\frac{3}{4} \neq \frac{6}{8}$ ____

(12) $\frac{4}{6} > \frac{5}{8}$ ____

Write a T after each true sentence and a F after each sentence that is false. Some pupils may be able to complete most of these exercises without the aid of the diagram. Others will find it necessary to use the diagram model to answer each question.

Parts of a whole

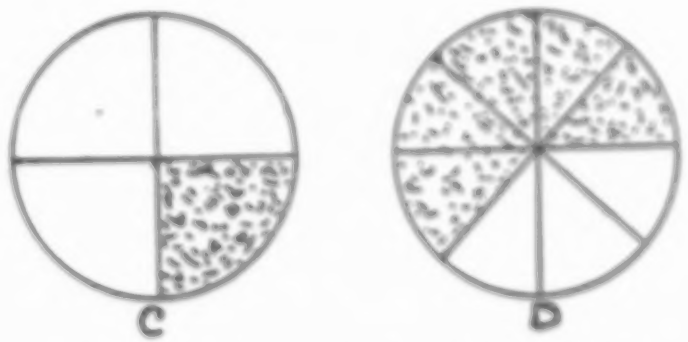


Use this device to maintain understanding of fractions as: 1) a part of a whole, and 2) part of a set or collection of things.

A. 3 parts in all; 2 parts are shaded.
B. 5 parts in all; ____ parts are shaded.
C. ____ parts in all; ____ parts are shaded.
D. ____ parts in all; ____ parts are shaded.

A. 2/3 of A is shaded; 1 parts are white.
B. ____ of B is shaded; ____ parts are white.
C. ____ of C is shaded; ____ parts are white.
D. ____ of D is shaded; ____ parts are white.

A. 1/3 of A is white.
B. ____ of B is white.
C. ____ of C is white.
D. ____ of D is white.



Each whole region above has been divided into equal parts. Look at the regions and complete the table.

At this point relate the concept of Parts of a Set to Parts of a Whole by preparing pictures of sets on chalkboard or on paper. Have them identify number of members in set, number of members shaded, fractional part of set shaded.

Addition of like fractions

1 penny is 1/100 of a dollar

1 nickel is 1/20 of a dollar

1 dime is 1/10 of a dollar

1 quarter is 1/4 of a dollar

1 half is 1/2 of a dollar

This game is based on a version of "fewest coins." A fractional amount of a dollar is given. This fraction is to be written as the sum of fractions, each fraction used indicating the coins required to make up the indicated amount.

Amount as fraction of a dollar	Fewest coins written as fractions of a dollar
7/100	= 1/20 + 1/100 + 1/100
35/100	= 1/4 + 1/10
3/4	= 1/2 + 1/4
81/100	= 1/4 + 1/20 + 1/100

The first example refers to 7/100 of a dollar or 7¢. This can be made with the fewest coins by one nickel and two pennies. 7/100 = 1/20 + 1/100 + 1/100 or if some say 7/100 = 1/20 + 2/100, they should not be told they are wrong. If the first form is preferred, then a rule has to be made for this game that each fraction used must suggest a single coin. Thus, the number of addends would indicate the "number of coins' -- the fact in which we are interested.

3/4 + 4/20 = 1/2 + 1/4 + 1/10 + 1/10
3/8 + 1/8 = 1/2
2/3 + 1/12 = 1/2 + 1/4
1/2 - 1/20 = 1/4 + 1/10 + 1/10

Another version of this game can be played with much the same rules. The initial amount may be written as a sum of fractions - to be rewritten to indicate the fewest coins.

A.

1/4	1/4	1/2
1/4	1/4	1/2
1/2	1/2	1

B.

1/8	1/8	1/4
3/8	3/8	3/4
1/2	1/2	1

The rule for this page is "Addition" (and subtraction) of common fractions. It may come as a surprise to some that all examples lead to one in the check box. It might be argued that insufficient information is provided for E. However, the fractions given in the basic four cells are 5/8 and 3/8. Their sum is 1 and the check box indicates that the sum of numbers written in all four cells is one. Therefore, the empty cells can contain 0. 12:60

C.

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$
$\frac{1}{4}$	0	$\frac{1}{4}$
$\frac{3}{4}$	$\frac{1}{4}$	1

D.

$\frac{7}{8}$	$\frac{1}{8}$	1
0	0	0
$\frac{7}{8}$	$\frac{1}{8}$	1

E.

$\frac{5}{8}$	0	$\frac{5}{8}$
0	$\frac{3}{8}$	$\frac{3}{8}$
$\frac{5}{8}$	$\frac{3}{8}$	1

F.

$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{8}$
$\frac{1}{2}$	$\frac{1}{8}$	$\frac{5}{8}$
$\frac{3}{4}$	$\frac{1}{4}$	1

G.

$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$
$\frac{3}{8}$	$\frac{1}{4}$	$\frac{5}{8}$
$\frac{1}{2}$	$\frac{1}{2}$	1

Addition-Subtraction Tables

Complete the addition table of fractions. Students may have to review the use of the addition table of whole numbers (or the multiplication table may help their understanding).

In the table, the top row and the first column

+	$\frac{1}{3}$	$\frac{3}{4}$		$\frac{2}{3}$
$\frac{5}{6}$		$\frac{19}{12}$	$\frac{9}{6}$	
	$\frac{5}{3}$			

+	$\frac{1}{3}$	$\frac{5}{6}$	$7\frac{1}{8}$	$\frac{1}{4}$
$\frac{1}{2}$				
$2\frac{1}{3}$				
$2\frac{1}{4}$				

+	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{7}{12}$	$\frac{1}{4}$
$\frac{5}{6}$				
$\frac{25}{12}$				
$7\frac{2}{3}$				

contain addends, the other squares contain sums. Seeking a sum reinforces addition of fractions. Seeking a missing addend reinforces subtraction of fractions.

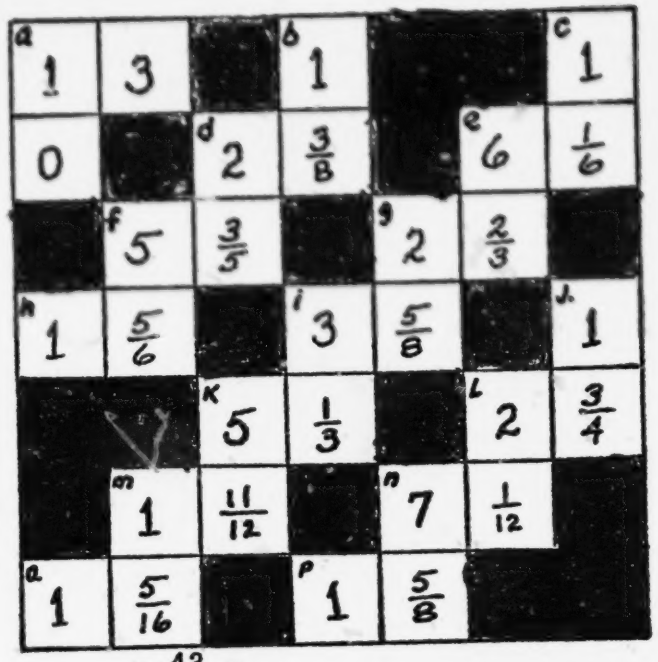
Subtraction - the missing addend

Complete the table.

Follow the example given in the first row.

Addends	Sum	Addition Equation	Subtraction Equation	n
$\frac{7}{8}$, n	$\frac{9}{10}$	$\frac{7}{8} + n = \frac{9}{10}$	$\frac{9}{10} - \frac{7}{8} = n$	$\frac{1}{40}$
$\frac{5}{6}$, n	$\frac{17}{12}$			
n, $\frac{6}{3}$	$\frac{24}{12}$			
$\frac{2}{3}$, n	$3\frac{2}{3}$			
n, $\frac{2}{9}$	$1\frac{1}{6}$			
$\frac{2}{6}$, n	$2\frac{1}{3}$			

Addition and subtraction of of fractions without limiting sums and minuends - horizontal form



DOWN

a.) $4\frac{1}{2} + 3\frac{1}{5} + 2\frac{3}{10}$

b.) $\frac{1}{2} + \frac{1}{4} + \frac{5}{8}$

c.) $2\frac{1}{2} - 1\frac{1}{3}$

d.) $1\frac{1}{10} + 1\frac{1}{2}$

e.) $2\frac{2}{9} + 3\frac{1}{3} + 1\frac{1}{9}$

f.) $3\frac{2}{3} + 2\frac{1}{6}$

g.) $3\frac{3}{8} - \frac{3}{4}$

i.) $5 - 1\frac{2}{3}$

j.) $\frac{11}{12} + \frac{5}{6}$

k.) $2\frac{1}{6} + 3\frac{3}{4}$

l.) $7\frac{1}{3} - 5\frac{1}{4}$

m.) $2 - \frac{11}{16}$

n.) $2\frac{1}{6} + 1\frac{1}{8} + 4\frac{1}{3}$

ACROSS

a.) $5\frac{1}{2} + 3\frac{1}{3} + 4\frac{1}{6}$

d.) $4\frac{5}{8} - 2\frac{1}{4}$

e.) $2\frac{7}{12} + \frac{1}{3} + 3\frac{1}{4}$

f.) $9 - 3\frac{2}{5}$

g.) $8\frac{5}{12} - 5\frac{3}{4}$

h.) $\frac{1}{3} + \frac{1}{2} + \frac{1}{6} + \frac{5}{6}$

i.) $9\frac{3}{8} - 5\frac{3}{4}$

k.) $7\frac{5}{6} - 2\frac{1}{2}$

l.) $5 - 2\frac{1}{4}$

m.) $4\frac{1}{3} - 2\frac{5}{12}$

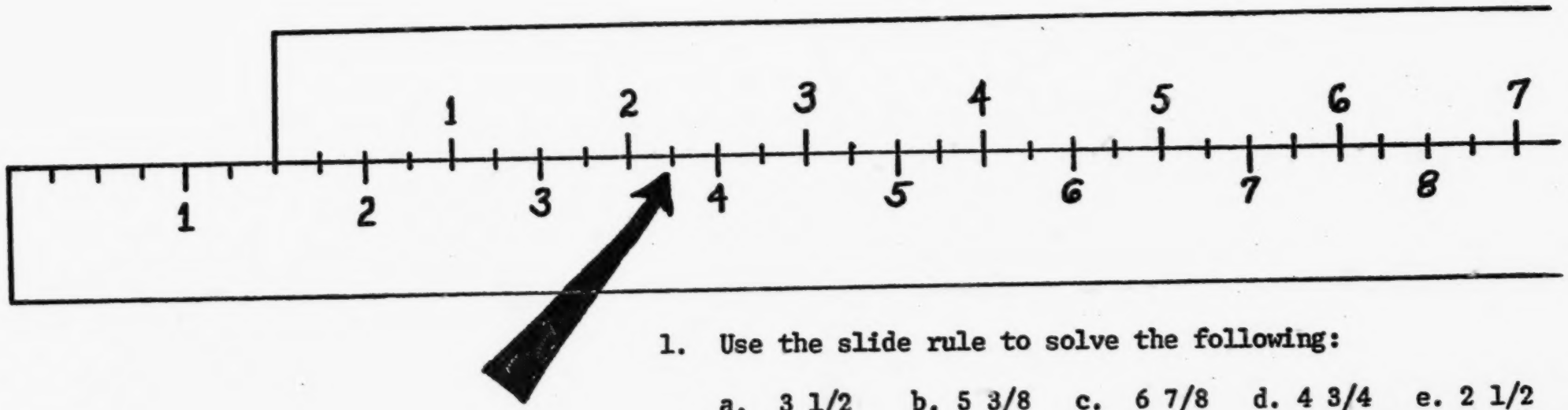
n.) $2\frac{1}{3} + 3\frac{1}{4} + 1\frac{1}{2}$

o.) $\frac{7}{16} + \frac{5}{8} + \frac{1}{4}$

p.) $18\frac{7}{8} - 17\frac{1}{4}$

Slide rule number lines

Have students make a cardboard slide rule 10 inches long for use with fractions. Make it with divisions between the whole numbers to show halves, fourths, and eighths. The picture below shows the slide rule set for $1\frac{1}{2} + 2\frac{1}{4}$. What is the sum? (3 3/4)



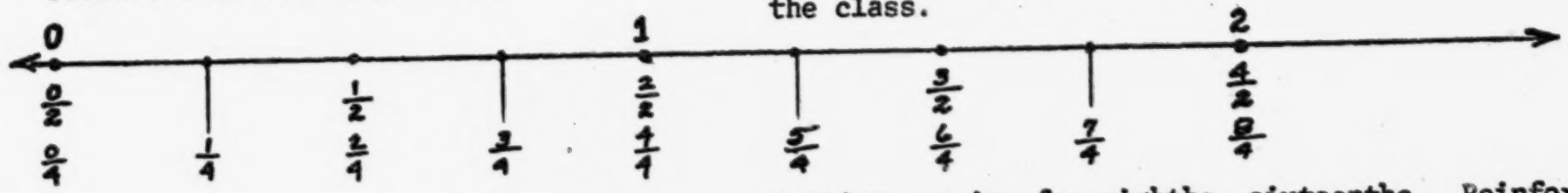
1. Use the slide rule to solve the following:

a.	b.	c.	d.	e.
3 1/2	5 3/8	6 7/8	4 3/4	2 1/2
2 1/4	1 1/4	2 1/2	3 5/8	6 3/4

2. On your slide rule, find remainders for these examples:

a.	b.	c.	d.	e.
7 3/4	8 5/8	6 1/4	5 1/8	9 1/2
-2 1/2	-3 1/2	-3 1/2	-2 3/8	-3 7/8

Fractions on the number line



Have pupils study this number line. Discuss with the class.

Continue naming for eighths, sixteenths. Reinforce here that points on the number line can be named by several fractions. Thus, 1 is also called 2/2, 4/4, 8/8, and 16/16. We say that 2/2, 4/4, 8/8, and 16/16 are equivalent fractions. They are equal in value because they name the same numbers. 3/4, 6/8, and 12/16 are equivalent fractions. Can you find other sets of equivalent fractions?

There are several such as:

$\left\{\frac{1}{4}, \frac{2}{8}, \frac{4}{16}\right\}, \left\{\frac{3}{2}, \frac{6}{4}, \frac{12}{8}, \frac{24}{16}\right\}$

Have pupils identify several sets of equivalent fractions.

Equivalent fractions.

Complete the sentences below:

$1 = \frac{}{2}$	$1 = \frac{}{4}$	$1 = \frac{}{8}$	$1 = \frac{}{16}$
$1 = \frac{}{50}$	$1 = \frac{}{77}$	$1 = \frac{}{123}$	$1 = \frac{}{1000}$
$\frac{3}{8} \times 1 = \_\_$	$23 \times 1 = \_\_$	$279 \times 1 = \_\_$	$\frac{1}{4} \times 1 = \_\_$
	$\frac{5}{6} \times 1 = \_\_$	$\frac{2}{3} \times 1 = \_\_$	$\frac{1}{5} \times 1 = \_\_$
$\frac{1 \times 1}{4 \times 1} = \_\_$	$\frac{1 \times 2}{4 \times 2} = \_\_$	$\frac{1 \times 3}{4 \times 3} = \_\_$	$\frac{1 \times 4}{4 \times 4} = \_\_$
$\frac{1 \times 5}{4 \times 5} = \_\_$	$\frac{1 \times 6}{4 \times 6} = \_\_$	$\frac{1 \times 7}{4 \times 7} = \_\_$	$\frac{1 \times 8}{4 \times 8} = \_\_$

$\frac{1}{3} \times \left(\frac{1}{1}\right) = \frac{1}{3}$

$\frac{1}{3} \times \left(\frac{2}{2}\right) = \frac{2}{6}$

$\frac{1}{3} \times \left(\frac{3}{3}\right) = \frac{3}{9}$

$\frac{1}{3} \times \left(\frac{4}{4}\right) = \frac{4}{12}$

Stress that equivalent fractions name the same point on the number line. Since 2/2 and 3/3 are equivalent to 1, the products 1/3, 2/6, and 3/9 name the same point on the number line. They are different names for a number. Have pupils make a point on the number line. Discuss this idea in detail.

Find the missing equivalent fractions in each set.

$B = \left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \_\_, \_\_, \_\_\right\}$

$W = \left\{\frac{1}{2}, \frac{2}{4}, \_\_, \_\_, \frac{5}{10}, \_\_\right\}$

$D = \left\{\frac{1}{6}, \_\_, \_\_, \frac{4}{24}, \_\_, \frac{6}{36}\right\}$

$M = \left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \_\_, \_\_, \_\_\right\}$

$R = \left\{\frac{2}{5}, \_\_, \_\_, \frac{8}{20}, \frac{10}{25}, \_\_\right\}$

$G = \left\{\frac{5}{6}, \_\_, \_\_, \frac{20}{24}, \_\_, \frac{30}{36}\right\}$

Have pupils look for patterns to help complete sets.

$R = \left\{\frac{1}{4}, \frac{2}{8}, \_\_, \frac{4}{16}, \_\_\_, \_\_\_\right\}$

$E = \left\{\frac{1}{5}, \_\_\_, \frac{3}{15}, \_\_\_, \frac{5}{25}, \_\_\_\right\}$

$K = \left\{\frac{1}{8}, \_\_\_, \frac{3}{24}, \_\_\_, \_\_\_, \frac{6}{48}\right\}$

$P = \left\{\frac{3}{4}, \frac{6}{8}, \_\_\_, \frac{12}{16}, \_\_\_, \_\_\_\right\}$

$O = \left\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \_\_\_, \_\_\_, \_\_\_\right\}$

$N = \left\{\frac{7}{8}, \frac{14}{16}, \_\_\_, \_\_\_, \_\_\_, \frac{42}{48}\right\}$

Write the set of all the fractions equivalent to $\frac{1}{2}$:

$Z = \left\{\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{7}{14}, \frac{8}{16} \ldots\right\}$

Ask the pupils if this set has a last member. This is an infinite set.

Lower and Higher Terms

$G = \left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \frac{6}{18}\right\}$

1 and 3 are the terms of the fraction 1/3. 2 and 6 are terms of the fraction 2/6. Each fraction in Set G has higher terms than the one before it. 4/12 has higher terms than 3/9. 2/6 has higher terms than 1/3. Pupils should realize that 4/12, 3/9, 2/3 all name the same number.

$\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$ $\frac{1}{3} \times \frac{3}{3} = \frac{3}{9}$

$\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$ $\frac{1}{3} \times \frac{5}{5} = \frac{5}{15}$

$\frac{1}{3} \times \frac{6}{6} = \frac{6}{18}$

In each case we have multiplied 1/3 x 1. But 1 has been renamed as 2/2, 3/3, 4/4 etc.

$B = \left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \ldots\right\}$

Is Set B an infinite set? (Yes) Write the set of all fractions that have lower terms than 18/30. $\left\{\frac{3}{5}, \frac{6}{10}, \ldots\right\}$ Write four fractions that have higher terms than 9/15. What fraction in Set H has the lowest terms of all? (3/5)

Lowest Terms

$\frac{12}{16} \div \frac{2}{2} = \frac{6}{8}$

$\frac{12}{16} \div \frac{4}{4} = \frac{3}{4}$

12/16 has been changed to the equivalent fraction 6/8 by dividing the terms by 1 or 2/2. Is there another fraction equivalent to 12/16 that has lower terms than 6/8? (Yes, 3/4)

For each fraction write an equivalent fraction in lowest terms.

$\frac{4}{6} =$ ____ $\frac{3}{9} =$ ____ $\frac{6}{30} =$ ____ $\frac{21}{24} =$ ____ $\frac{50}{60} =$ ____

$$\frac{5}{10} = \frac{1 \times 5}{2 \times 5}$$

$$= \frac{1}{2} \times \frac{5}{5}$$

$$= \frac{1}{2} \times 1$$

$$= \frac{1}{2}$$

Notice that 5 is a factor of the numerator and denominator. 5 is a common factor of the terms of 5/10.

$$\frac{50}{80} = \frac{5 \times 10}{8 \times 10}$$

$$= \frac{5}{8} \times \frac{10}{10}$$

$$= \frac{5}{8} \times 1$$

$$= \frac{5}{8}$$

What number is a common factor of both 50 and 80? (10)

Do 5 and 8 have a common factor other than one? (No)

Complete each equation. (Pupils should understand that 2/2, 9/9, 4/4, etc. are names for 1. Multiplying or dividing a number by 1 does not change it.

$\frac{6}{8} = \frac{3}{4} \times \frac{2}{2}$ $\quad \frac{6}{6} = \frac{1}{1} \times \_\_\_$ $\quad \frac{21}{30} = \_\_\_ \times \frac{3}{3}$ $\quad \frac{10}{12} = \frac{5}{6} \times \_\_\_$ $\quad \frac{28}{44} = \_\_\_ \times \frac{4}{4}$

$\frac{26}{39} = \_\_\_ \times \frac{13}{13}$ $\quad \frac{9}{36} = \frac{1}{4} \times \frac{9}{9}$ $\quad \frac{8}{24} = \frac{1}{3} \times \_\_\_$ $\quad \frac{16}{20} = \_\_\_ \times \frac{4}{4}$ $\quad \frac{10}{15} = \_\_\_ \times \frac{5}{5}$

$\frac{28}{40} = \_\_\_ \times \_\_\_$ $\quad \frac{200}{300} = \_\_\_ \times \_\_\_$

$$\frac{24}{36} = \frac{8}{12} \times \frac{3}{3}$$

$$= \frac{2}{3} \times \frac{4}{4} \times \frac{3}{3}$$

$$= \frac{2}{3}$$

Sometimes you may not be able to find the largest common factor in one step. Study the problem at the left. In this problem two common factors of 24 and 36 were found. What were they? (3 and 4)

$$\frac{24}{36} = \frac{2 \times 4 \times 3}{3 \times 4 \times 3} \quad \text{or} \quad \frac{24}{36} = \frac{2 \times 12}{3 \times 12}$$

Write the lowest-terms fraction for each fraction below. Use the factoring approach used above.

$\frac{7}{21} =$ ____ $\frac{42}{60} =$ ____ $\frac{14}{84} =$ ____ $\frac{36}{54} =$ ____ $\frac{18}{36} =$ ____ $\frac{12}{32} =$ ____

$\frac{16}{28} =$ ____ $\frac{42}{72} =$ ____ $\frac{75}{105} =$ ____ $\frac{126}{162} =$ ____ $\frac{165}{183} =$ ____

MULTIPLICATION AND DIVISION

GRADE 5

Division with two-place divisors through five-place dividends, with or without remainders

Division with two-place divisors should be carefully preceded with a review of the multiples of ten and some instruction in mental multiplication.

Multiples (and all multiplication facts) can be easily reviewed by asking students to find the missing numerals in sets such as these:

A = {2, 4, 6, 8, __, __, __, __, __, __}

B = {5, 10, 15, __, __, __, __, __, __}

C = {__, __, __, 28, 35, 42, __, __, __, __}

D = {__, 18, __, 36, __, 54, __, __, __, __}

E = {__, __, __, __, 50, 60, __, __, __, __}

Next lead students to multiply quickly using some multiple of ten as one of the factors. Build on the basic facts using pattern approaches:

2 x 3 = __	3 x 2 = __
2 x 30 = __	30 x 2 = __
20 x 3 = __	3 x 20 = __
20 x 30 = __	30 x 20 = __
4 x 7 = __	7 x 4 = __
4 x 70 = __	70 x 4 = __
40 x 7 = __	7 x 40 = __
40 x 70 = __	70 x 40 = __

When students can work such patterns quickly and efficiently. (Be sure to cover all basic facts through 9 x 9 or 10 x 10) have them rework similar sets of equations where only one factor and its product is given and the missing factor is a multiple of ten. This underscores the meaning of division and its relationship to multiplication. In multiplication we multiply two factors to find their product; in division we divide the product of two numbers by one factor to find the missing factor.

Write the multiple of ten that makes each sentence true.

2 x ___ = 60	___ x 2 = 60
___ x 3 = 60	3 x ___ = 60
___ x 20 = 600	20 x ___ = 600
4 x ___ = 280	___ x 4 = 280
___ x 7 = 280	7 x ___ = 280
___ x 70 = 2800	70 x ___ = 2800

Then progress to inequalities:

A. Write the <u>largest</u> numeral that makes each sentence true.

a. 40 x ___ < 86	c. 70 x ___ < 439
b. 20 x ___ < 191	d. 90 x ___ < 685

B. Write the largest multiple of ten that makes each sentence true.

a. $30 \times \_\_ < 785$

b. $50 \times \_\_ < 2679$

c. $80 \times \_\_ < 5177$

d. $60 \times \_\_ < 1930$

Students can now begin simple division problems where the two-place divisor is some multiple of ten. Encourage students to use inequalities to estimate quotients.

```
   5  R 23
30 173
  -150   30 x 5
    23
```

$30 \times \underline{?} < 173$

```
   57  R  28
30 1738
  -1500   30 x 50
    238
   -210   30 x 7
     28
```

$30 \times \underline{?} < 1738$

$30 \times \_\_\_ < 238$

To extend division to a five place dividend the students must stop here for a review of multiples of 100 and pattern work involving multiples of 100 similar to those used for tens.

```
   579  R  16
30 17386
  -15000   30 x 500
    2386
   -2100   30 x 70
     286
    -270   30 x 9
      16
```

$30 \times \_\_\_\_\_ < 17{,}386$

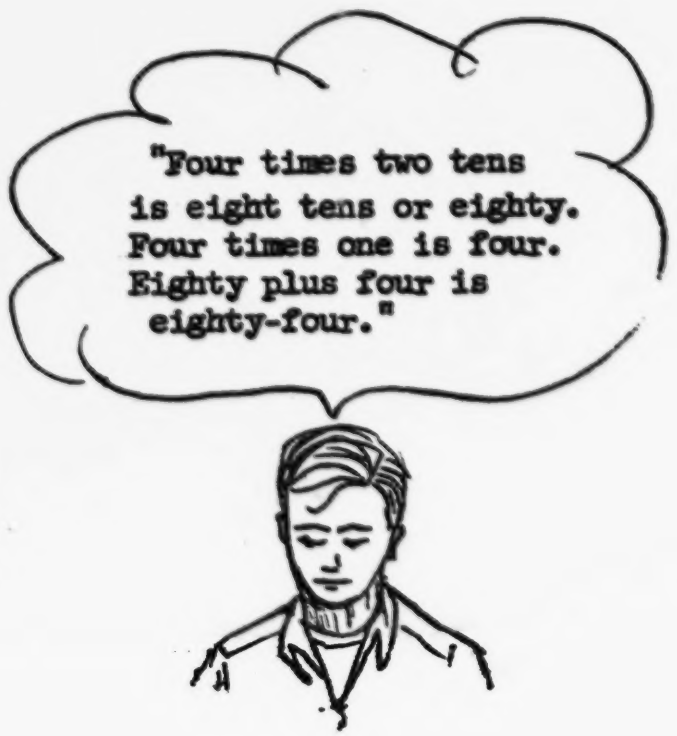
$30 \times \_\_\_\_\_ < 2386$

$30 \times \_\_\_\_\_ < 286$

4 x 21 =

(4 x 20) + (4 x 1) =

80 + 4 = 84



It may be necessary to review and extend the rounding of numbers at this point to aid the students to solve the inequalities more efficiently. Thus, in the example above the student could be led to think of the inequalities as:

30 x ___ < 17,386 30 x ___ < 2386

30 x ___ < 17,000 30 x ___ < 2400

When students can divide five place dividends by multiples of ten they are ready for the more difficult task, dividing by any two-place divisor such as 21, 57 or 98. Students can save much time and many erasures if this task is preceded with instruction in mental multiplication so that a trial divisor can be tested without writing any numerals.

In mental multiplication we multiply left-to-right, finding the tens' product first, the ones' product second, and then the sum of the tens and ones products: (Writing the problems in horizontal notation helps to break the habit of multiplying ones first.)

Use mental multiplication to find these products. Remember to multiply tens first!

a. 3 x 16 = ___ c. 24 x 5 = ___ e. 43 x 6 = ___

b. 7 x 34 = ___ d. 4 x 84 = ___ f. 9 x 91 = ___

With mental multiplication skills these inequalities can be more easily solved.

Write the largest numeral that will make each sentence true.

a. $17 \times \_\_\_ < 59$

b. $34 \times \_\_\_ < 256$

c. $24 \times \_\_\_ < 134$

d. $84 \times \_\_\_ < 349$

Then move to simple problems involving three-place dividends:

$$\begin{array}{r} 3\ R\ 8 \\ 17\overline{)59} \\ -51 \\ \hline 8 \end{array} \qquad 17 \times 3$$

$17 \times \_\_\_ < 59$

$$\begin{array}{r} 7\ R\ 18 \\ 34\overline{)256} \\ -238 \\ \hline 18 \end{array} \qquad 34 \times 7$$

$34 \times \_\_\_ < 256$

Then to four-place dividends (Review multiples of ten)

$$\begin{array}{r} 52\ R\ 30 \\ 39\overline{)2058} \\ -1950 \\ \hline 108 \\ 78 \\ \hline 30 \end{array} \qquad \begin{array}{l} 39 \times 50 \\ \\ 39 \times 2 \end{array}$$

$39 \times \_\_\_ < 2058$

$39 \times \_\_\_ < 108$

And five-place dividends (Know multiples of hundred plus rounding skills to solve inequalities.)

The conventional algorithm in division

```
        647   R 12
32 ) 20,716 |
     19,200 |  32 x 600
      1,516 |
     -1,280 |  32 x 40
        236 |
       -224 |  32 x 7
         12 |
```

32 x ___ < 20,716
30 x ___ < 20,000
32 x ___ < 1516
30 x ___ < 1500
32 x ___ < 236
30 x ___ < 240

The "scaffolding" or "subtraction" forms for division are used to underscore insight and understanding in the division operation. When students can perform the computation with understanding they should be encouraged to throw away elaborate notations and develop streamlined approaches to division that are efficient and time saving. Thus, when dividing by a one-digit number we would hope that the "short division" algorithm would be developed.

A.

```
      2806   R 3
7 ) 19645 |
   -14000 |  7 x 2000
      5645 |
     -5600 |  7 x 800
        45 |
       -42 |  7 x 6
         3 |
```

In example A the complete notation is shown for the scaffold form of division. It is worth mentioning that the writing of the quotient over the dividend is usually the last step in the computation, performed after collecting the partial products at the right side of the scaffold (2000 + 800 + 6) and the remainder. (3)

B.

```
    2806 R 3
7 |19645|
  -14000|
    5645|
   -5600|
      45|
     -42|
       3|
```

C.

```
    2806 R 3
7 |19645|
  -14   |
    56  |
   -56  |
      4 |
     -0 |
     45 |
    -42 |
      3 |
```

Example B shows these partial products missing, indicating that the student performed these multiplications mentally and wrote the products only under the dividend or partial dividend from which he was subtracting.

The conventional "long division" algorithm shown in example C streamlines the computation process by omitting the zeros in the partial products to be subtracted. It is at this point that a great deal of understanding might be lost if students are not occasionally reminded that the unwritten zeros are understood.

The "14" just below the dividend is, in fact, 14 thousand; the "56" is 56 hundreds or 5 thousand 6 hundred.

The student who really understands the division operation can give the place values of these numerals readily. To prevent loss of understanding it might be well to have students work out problems on graph paper with columns marked with place value names. (See example D)

D.

	ten-thousand	thousands	hundreds	tens	ones	
		2	8	0	6	R 3
7	1	9,	6	4	5	
-	1	4				
		5	6			
		-5	6			
				4		
				-0		
				4	5	
				-4	2	
					3	

E.

$$7\overline{)19{,}645} \quad \begin{matrix} 2806 \\ \text{R } 3 \end{matrix}$$

Example E illustrates "short division" where all multiplication and subtraction operations are carried out mentally and only the successive digits of the answer are written from left to right above the dividend. The short division form is generally used only when dividing by a one-place divisor or a two-place divisor ending in zero.

Some students, of course, will not be able to streamline their division algorithms at the fifth grade level. Some will be better able to attempt it in sixth grade. Some never. Students generally know when they are able to move to short cut methods and will be eager to try such streamlining methods if they are introduced. Teachers should check students periodically to see that shortcuts are performed with understanding.

Students having difficulty may profit from this activity. Follow the division process and fill in the blanks:

A.

34 | 54321			
-	20321	1000	1000 x 34 = _____
-	3321	500	500 x 34 = _____
-	1621	50	50 x 34 = _____
-	261	40	40 x 34 = _____
-	57	6	6 x 34 = _____
-	23	1	1 x 34 = _____
		1597	

$54,321 \div 34 = (34 \times 1597) + 23$

The youngsters are now ready to try the above method on problems such as B.

B.

1. $24\overline{)78965}$

2. $31\overline{)21552}$

3. $79\overline{)84093}$

4. $92\overline{)10534}$

Expressing remainders in fractional forms

Ex. 1

$$4\overline{)274}\quad 68\ 2/4 \text{ or } 68\ \tfrac{1}{2}$$

24

34

32

2

Ex. 2

$$4\overline{)274}\quad 68 \text{ R } 2$$

24

34

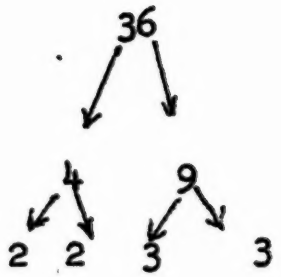
32

2

1. Example 1 is appropriate when students are dividing something that is divisible into fractional parts, for example, apples, pies, time, money, measurements, etc.

2. Example 2 is appropriate when students are working with something that is not divisible into fractional parts, eg. people, automobiles, furniture, etc.

Prime factorization



Factor Tree

The initial steps of prime factorization of numbers are introduced at this grade level. Essentially, the development is of a "readiness" nature, the identification of prime and composite numbers and exploration of simple factoring of a number through the use of "factor trees." This material will be found in Chapter I, Numbers and Numeration.

MULTIPLICATION AND DIVISION

GAMES AND DRILL

GRADE 5

GRADE 5

MULTIPLICATION AND DIVISION

GAMES AND DRILL

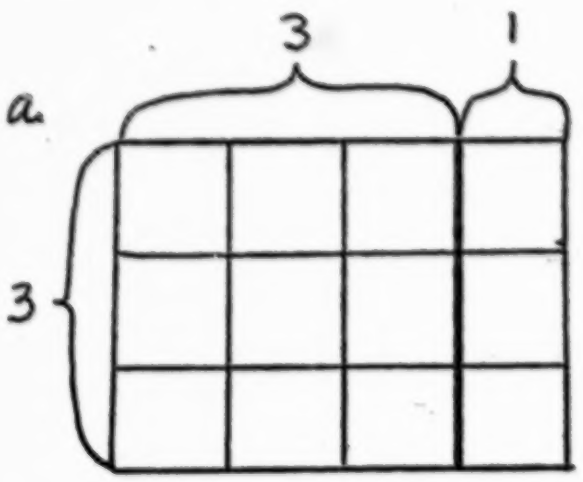
Examples with one and two place divisors

$3\overline{)12} = 3\overline{)9+3}^{3+1} = 4$

$\frac{12}{3} = \frac{9+3}{3} = \frac{9}{3} + \frac{3}{3} = 3 + 1 = 4$

$12 \div 3 = (9 \div 3) + (3 \div 3) = 3 + 1 = 4$

Examples with one and two place divisors



It took the finest mathematicians centuries to devise a simple shortcut to performing such divisions as 37250 ÷ 215 = _____. Slowly a means was evolved that, today, is part of the mathematical heritage we pass on to children in the elementary schools. 13:27

Teachers are sometimes so anxious to reveal this beautiful technique that they hurry to present the division algorism as a series of steps to be memorized. The price is a lack of understanding for some that may never be overcome.

Take time to play some division games which focus the spotlight on meaning.

The following games provide useful drill and gradually lead toward both "subtractive" and "traditional" division algorisms. These games assume that all children would understand that the statements to the left are true.

Discuss problems like the following:

"I have to divide 12 pieces of fruit among 3 children. If I have 9 apples and 3 pears, what would be a good way to do it?" 13:27

Each child of course gets 3 apples and 1 pear; 4 pieces in all. Now the principle can be brought out. How is it possible to break a group of 12 pieces of fruit into two groups? How does the picture to the left suggest what has been done?

If there is any uneasiness about the example a., don't hurry: Try dozens like it. Make up stories. Draw pictures. Go back to manipulative material!

Children must have a deep understanding of the fundamental relation between addition and multiplication along with their inverses, subtraction and division, because the algorism of division uses all four operations.

A game is invented as soon as there is agreement on a rule or set of rules. In the games to follow, the special rule for each game is stated.

Game I: The rule is that the player may use no term in the dividend that has more than a single digit. An example which violates the rule is $2\overline{)12} = 2\overline{)10+2}$ because "10" is a term in the dividend and has 2 digits.

$$2\overline{)12} = 2\overline{)\overset{4+2}{8+4}} = 2\overline{)\overset{2+1+3}{4+2+6}} = 2\overline{)\overset{3+3+3-6}{6+6+6-6}}$$

$$3\overline{)12} = 3\overline{)\overset{2+2}{6+6}} = 3\overline{)\overset{2+3-1}{6+9-3}} = 3\overline{)\overset{3+3-1-1}{9+9-3-3}}$$

$$4\overline{)12} = 4\overline{)\overset{3+1}{8+4}} = 4\overline{)\overset{3-1+3}{8-4+8}} = 4\overline{)\overset{1+1+3-1}{4+4+8-4}}$$

Game II: The rule is that no term in the quotient may have more than a single digit:

$$2\overline{)24} = 2\overline{)\overset{6+6}{12+12}} = 2\overline{)\overset{4+4+4}{8+8+8}} = 2\overline{)\overset{3+3+3+3}{6+6+6+6}}$$

$$3\overline{)24} = 3\overline{)\overset{7+1}{21+3}} = 3\overline{)\overset{3+3+2}{9+9+6}} = 3\overline{)\overset{4+3+3-1}{12+9+6-3}}$$

$$4\overline{)24} = 4\overline{)\overset{5+1}{20+4}} = 4\overline{)\overset{4+4-2}{16+16-8}} = 4\overline{)\overset{3+2+2-1}{12+8+8-4}}$$

$6\overline{)24} = 6\overline{)\overset{7+2}{12+12}} = 6\overline{)\overset{3-1+2}{18-6+12}} = 6\overline{)\overset{2+2+2-2}{12+12+12-12}}$

Game III: This game has the same rules as Game II, except that the quotient is to have as many odd terms as possible:

$2\overline{)36} = 2\overline{)\overset{9+9}{18+18}} = 2\overline{)\overset{7+5+6}{14+10+12}} = 2\overline{)\overset{5+5+5+3}{10+10+10+6}}$

$3\overline{)36} = 3\overline{)\overset{3+9}{9+27}} = 3\overline{)\overset{3+3+6}{9+9+18}} = 3\overline{)\overset{9+3+1-1}{27+9+3-3}}$

$4\overline{)36} = 4\overline{)\overset{6+3}{24+12}} = 4\overline{)\overset{6+6-3}{24+24-12}} = 4\overline{)\overset{3+3+6-3}{12+12+24-12}}$

$6\overline{)36} = 6\overline{)\overset{3+3}{18+18}} = 6\overline{)\overset{9-1-2}{54-6-12}} = 6\overline{)\overset{9-1-1-1}{54-6-6-6}}$

Game IV: Has no restriction.

$2\overline{)48} = 2\overline{)\overset{12+12}{24+24}} = 2\overline{)\overset{20+5-1}{40+10-2}} = 2\overline{)\overset{10+10-5+9}{20+20-10+18}}$

$3\overline{)48} = 3\overline{)\overset{12+4}{36+12}} = 3\overline{)\overset{16-8+8}{48-24+24}} = 3\overline{)\overset{20-4+4-4}{60-12+12-12}}$

$4\overline{)48} = 4\overline{)\overset{6+6}{24+24}} = 4\overline{)\overset{18-3-3}{72-12-12}} = 4\overline{)\overset{22-3-3-3}{84-12-12-12}}$

$6\overline{)48} = 6\overline{)\overset{4+4}{24+24}} = 6\overline{)12+72-36} = 6\overline{)\overset{2\ 2\ 2\ 2}{12+12+12+12}}$

In Game V, all but one of the terms in the quotient must be multiples of 10: (The teacher has a decision to make: to develop either the "subtractive" or the "traditional" algorism - or both. If the "subtractive" is selected, Game V is omitted. However, both algorisms are so basically alike that most children

will be able to handle both as they develop the general technique).

$6\overline{)72} = 6\overline{)\frac{10+2}{60+12}} = 6\overline{)\frac{20+0-8}{120+0-48}} = 6\overline{)\frac{10+0+10-8}{60+0+60-48}}$

$4\overline{)72} = 4\overline{)\frac{10+8}{40+32}} = 4\overline{)\frac{20-10+8}{80-40+32}} = 4\overline{)\frac{30-10-10+8}{120-40-40+32}}$

$3\overline{)72} = 3\overline{)\frac{20+4}{60+12}} = 3\overline{)\frac{10+10+4}{30+30+12}} = 3\overline{)\frac{20-10+10+4}{60-30+30+12}}$

$2\overline{)72} = 2\overline{)\frac{30+6}{60+12}} = 2\overline{)\frac{70-10-24}{140-20-48}} = 2\overline{)\frac{80-10-10-24}{160-20-20-48}}$

Game VI - This game has no restrictions: 13:28

$11\overline{)352}$ $12\overline{)294}$ $22\overline{)242}$ $50\overline{)550}$

What methods were employed once the restrictions were lifted? (The printed form itself is a restriction requiring the decomposition of 13:29 the dividend into two terms).

Each method used must be carefully explained by the player and thoroughly discussed by the class. Every technique deserves its day in class. Any method that always leads to the correct result is a correct method

The basis is laid for some to go on and try their techniques on such examples as those that follow. Meanwhile, time is needed to lead others over the hurdles they couldn't clear in Game IV or earlier.

Game VII: No restrictions at all! (except that

each player should be sure he understood Game IV and VI clearly).

25 $\overline{)1225}$ 81 $\overline{)5427}$ 37 $\overline{)296}$ 13 $\overline{)100}$

By this time, individual differences will have the class scattered at rough spots along the road. Some will never get to Game VII without the help of the algorism. However, the distance each has come along the way and the further advance he will make with help will contribute understanding when the algorism the teacher prefers is finally presented. Each game should be played with many examples. When children are encouraged to make their own examples, they usually vary the size of the numbers according to their interests and abilities. Some catapult themselves right into a complete mastery of a satisfactory algorism.

The games cannot fail to develop and strengthen those underlying connections between basic operations that make any division algorism possible.

Introduce the rule: All numbers written in the quotient must be odd numbers and different. 13:64

Because of this rule the following are not acceptable:

$$2\overline{)\dfrac{5+5}{10+10}}$$ (both the same)

$$2\overline{)\dfrac{6+4}{12+8}}$$ (not odd)

Division of whole numbers; remainder expressed as fraction

$2\overline{)20} = 2\overline{)\_\_ + \_\_} =$

$2\overline{)\_\_ + \_\_ - \_\_ - \_\_}$

But the following meet both tests:

$$2\overline{)14 + 6}^{\,7 + 3} \qquad 2\overline{)18 + 14 - 10 - 2}^{\,9 + 7 - 5 - 1} = 10$$

This kind of game helps develop a sense of the freedom to regroup that is essential for the division algorism. The rules help develop ingenuity and lend the atmosphere of a puzzle to the activity.

Eventually, the rules of the game are changed. Blue is reserved for a whole number and green for a number less than one.

The only acceptable response is:

A. $8\overline{)56} = 8\overline{)}^{\text{Blue}} + 8\overline{)}^{\text{Green}}$

$8\overline{)56} = 8\overline{)56} + 8\overline{)10}^{\,0 \div 8} = 7 + \frac{0}{8}$

B. $8\overline{)60} = 8\overline{)64}^{\,8} - 8\overline{)4}^{\,\frac{1}{2}} = 8 - \frac{1}{2} = 7 - \frac{1}{2}$

C. $10\overline{)78} = 10\overline{)80}^{\,8} - 10\overline{)2}^{\,\frac{2}{10}}$

$8 - 1/5 = 7\text{-}4/5$

D.

$12\overline{)154} = 12\overline{)150}^{13} - 12\overline{)2}^{\frac{2}{13} = \frac{1}{6}}$

13 - 1/6 = 12-5/6

Conventional division algorism expressing remainder as a fraction

$16\overline{)106} = 16\overline{)96}^{\text{Blue } 6} + 16\overline{)10}^{\text{Green } 10 \div 16} = 6 + 5/8$

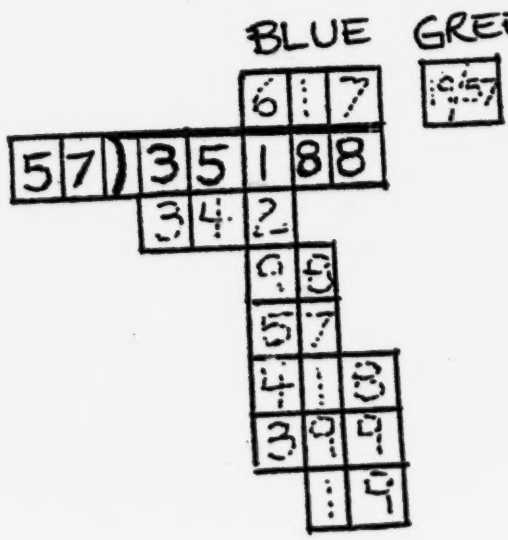
17 ÷ 10 = 1 + 7/10 or 1 7/10 or 1.7 8 ÷ 7 = 1 1/7

17 ÷ 6 = 2 5/6 28 ÷ 5 = 5 3/5

$5\overline{)3483} = 5\overline{)3480}^{\text{Blue } 696} + 5\overline{)3}^{\text{Green } 3 \div 5} = 696\ 3/5$

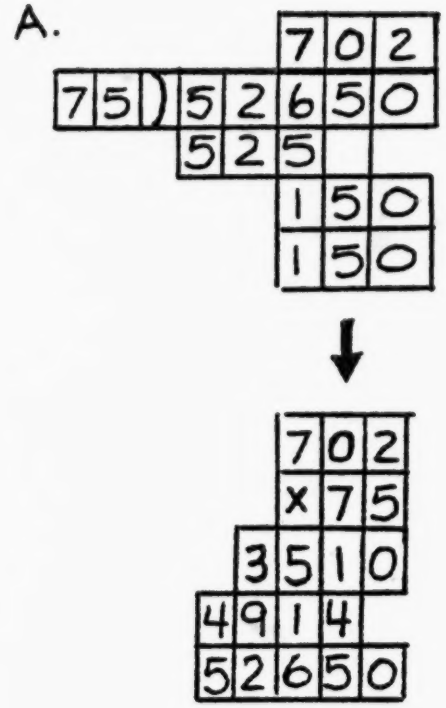
$93\overline{)6335} = 93\overline{)6324}^{\text{Blue } 68} + 93\overline{)11}^{\text{Green } 11 \div 93} = 68\ 11/43$

BLUE GREEN



Conventional algorism - division and inverse operation

A.



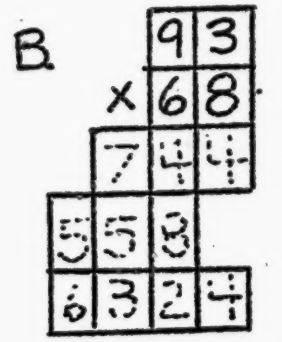
The division algorism is susceptible to errors, when people who work with it are not careful to keep everything in its proper place.

The example at the left and on the following page remind the students to be careful with neatness. It again reminds the children of the relationship between multiplication and division:

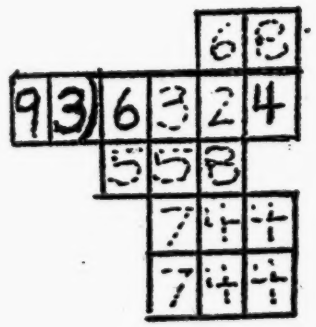
if 52650 ⋮ 75 = 702, then

702 X 75 = 52650.

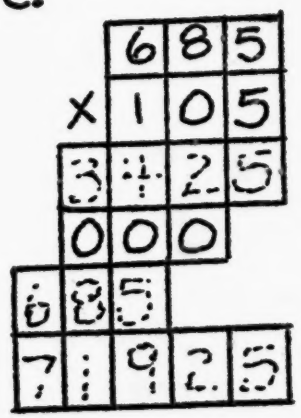
B.

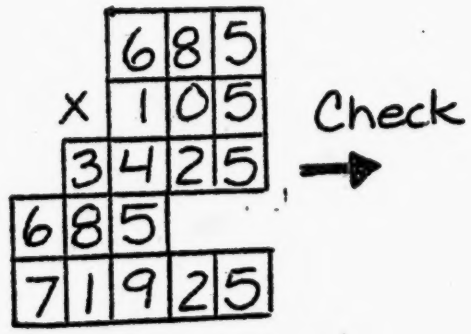


check ↓

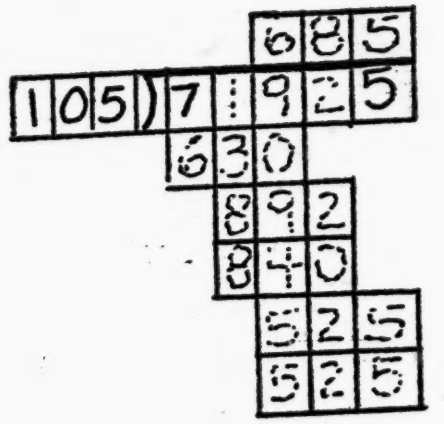


C.





Why do most people prefer this form?



Division with two-place divisors through five-place dividends with remainders using the subtractive method

$$\begin{array}{r} 5 \text{ r } 27 \\ 40\overline{)227} \\ \underline{200} \quad 5 \times 40 \\ 27 \end{array}$$

Multiplying with two digit multipliers; division with two digit divisors

10 x 26 = (260)
10 x 19 = (190)
10 x 84 = (840)

260 ÷ 10 = (26)
710 ÷ 10 = (71)
740 ÷ 10 = (74)

Extension to three digits

5400 ÷ 100 = (54)
2700 ÷ 100 = (27)
3900 ÷ 100 = (39)

100 x 7 = (700)
100 x 26 = (2600)
100 x 73=(7300)

"How many 40's are there in 227? To answer this question, find which of the following sentences are true. 6:69

a. 1 x 40 < 227 (T) b. 2 x 40 < 227 (T)
c. 3 x 40 < 227 (T) d. 4 x 40 < 227 (T)
e. 5 x 40 < 227 (T) f. 6 x 40 < 227 (F)

What is the largest number of 40's in 227? (5)"

Inequality sentences are powerful tools for finding quotients. Make sure that pupils understand that consecutively increasing multipliers are tried until a false sentence is found.

In an auditorium there are 10 rows of 12 chairs. How many chairs are there in all?" A number is multiplied by 10 by annexing a zero to the numeral for the number: 10 x 12 = 120. 4:17
"Ten boys had 120 books to carry to the music room. How many books did each boy have to carry?"

A number ending in zero is divided by 10 by dropping a zero from the numeral for the number:
120 ÷ 10 = 12

"Dean has saved 1200 pennies. For how many one-dollar bills can he exchange these pennies? 4:20

1200 ÷ 100 = □ (12)

A number ending in two zeros is divided by 100 by dropping two zeros from the numeral for the number. "A grocer sells pears in small boxes. There are 4 pears in each box. How many pears are there in 100 boxes?"

100 x 4 = □ (400)

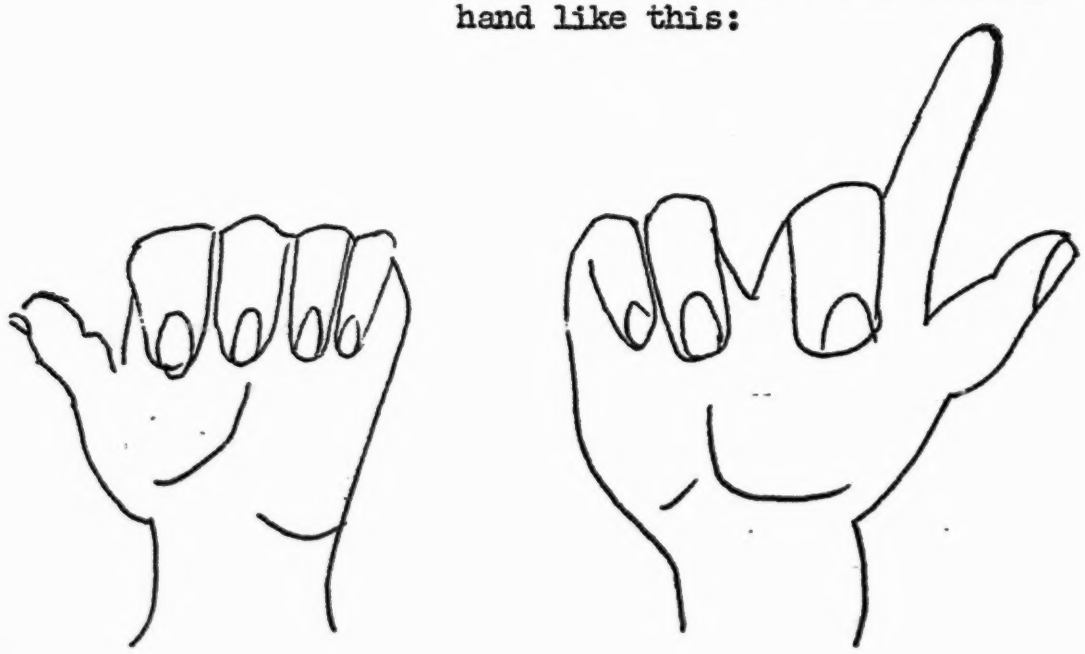
A help for finding multiplication combinations greater than 5 x 5 and less than 9 x 9

A number is multiplied by 100 by annexing two zeros to the numeral for that number.

"Finger Multiplying"
"Some of the children in France do not learn the multiplication tables beyond 5 x 5. They use their fingers to multiply two numbers that are both between 5 and 10."

9: Ways to Multiply

Suppose we ask one of the children the question, "Eight nines are how many?" To find the answer, he thinks, "Nine is 4 more than 5." Always he finds how much greater than 5 each number is. "Then he bends down 4 fingers on his left hand 3 fingers on his right hand like this:



4 + 3 = 7 (Tens)
1 x 2 = 2 (ones)

so 8 x 9 = 7 tens + 2 ones
or 72

The number of fingers bent down ways on both hands gives the tens of the product: 4 + 3 = 7 tens. So the answer is 7 tens and some ones.

How he finds how many ones there are. The product of the unbent fingers on each hand gives the ones for the answer: 1 x 2 = 2 ones. He knows then that the whole answer is 7 tens and 2 ones or 72.

To what question is Jim finding the answer:
Remember:
1. Bend down the excess over five.
2. Add the number of down fingers to get the tens answer.
3. Multiply the number of up fingers to get the one's answer.

(6 x 9 = 54)

Use your fingers to find the answers to these multiplication problems:

7 x 8 8 x 8 7 x 7 7 x 9 9 x 9

Multiplication with 2-place multipliers

"How Ethiopians Multiply"

A traveler in Ethiopia, a country in Africa, tells an interesting story. He found that people in some of the tribes had learned no more arithmetic than how to add, double, or halve numbers. Yet these people used this little knowledge to find correctly the product of any two numbers.

"Suppose one of the men of the tribe sold 13 sheep at $15 each. How did he figure out how much money he should get for the sheep? This is what he did:

13 x 15
6 30
3 60
1 120

1. He put 13 in a column at the left and 15 in a column at the right.

2. He divided 13 by 2; that is he halved 13. Of course, the answer is 6 ½. But he knew nothing about fractions; so he dropped the ½ and wrote 6 below the 13.

3. He doubled the number in the right column and wrote 30 below 15.

4. He halved 6 and doubled 30.

5. He kept this up until the last number in the left column was 1.

13 x 15
~~6 x 30~~
3 x 60
1 x 120

The traveler says that the people of these tribes do not like even numbers. They believe even numbers are evil and should be destroyed and their partners too. So the 6 was crossed out with its partner, 30. Then the numbers in the right column were added. The sum is 195, and that is the right answer.

15
x13
45
150
195

Study these multiplications:

20 x 24	12 x 23
10 --- 48	6 --- 46
5 96	3 92
2 -- 192	1 184
1 384	276
480	

Are these products correct?

Pity the poor people of Ethiopia. They cannot understand how we multiply. But of course, you see at once how they multiply, or do you?

Try it:

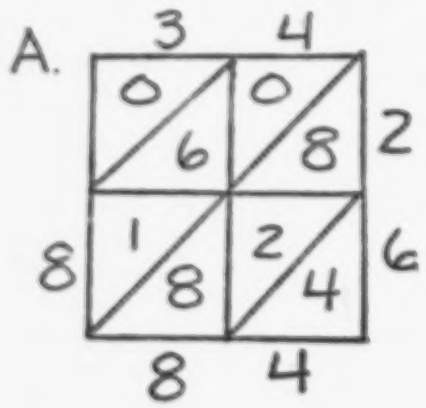
17 x 25	25
(8) -- (50)	17
(4) -- (100)	175
(2) -- (200)	250
(1) (400)	425
425	

16 x 18	18
8 --- 36	16
4 --- 72	108
2 -- 144	18
1 288	288
288	

25 x 40	25
12 --- 80	40
6 -- 160	1000
3 320	
1 640	
1000	

Multiplying with two and three place numerals

A.



$$\begin{array}{r} 34 \\ \times 26 \\ \hline 204 \\ 68 \\ \hline 884 \end{array}$$

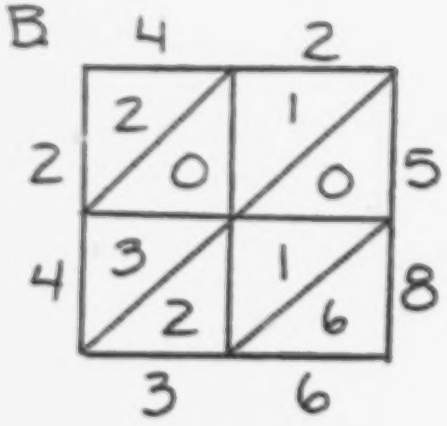
NOW

THEN

"A lattice is a framework of crossed strips. The strips may be of wood like those seen in rose gardens or they may be lines like those once used for multiplication. The picture at the left shows how a lattice was used many years ago to multiply 34 by 26.

The number to be multiplied, 34, is written at the top of the lattice. The multiplier, 26, is written at the right. Answer 884. Can you figure it out?"

B.

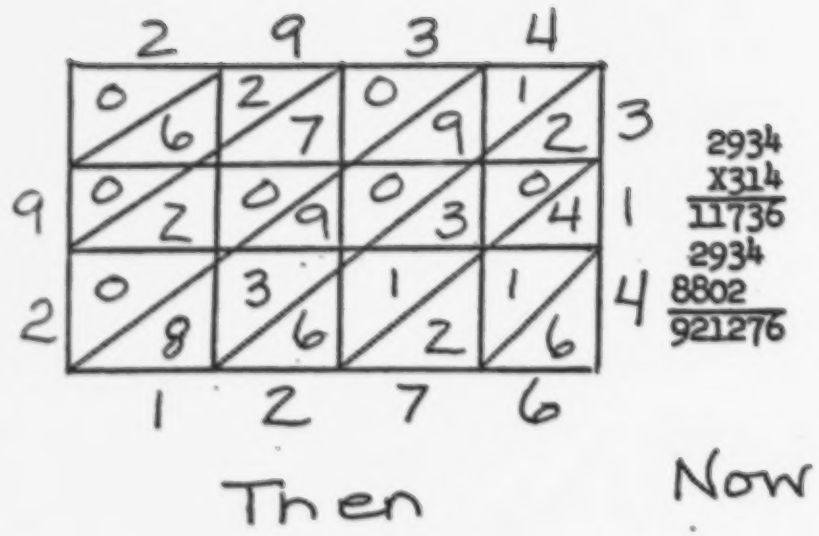


Then

$$\begin{array}{r} 42 \\ \times 58 \\ \hline 336 \\ 2100 \\ \hline 2436 \end{array}$$

Now

"Example B shows the old and new way of multiplying 42 by 58. When people multiplied in this way, they kept each part of the answer in its right place. And that is the important thing to do. We do the same thing today when we multiply. The only difference is that we do it in a simpler way."

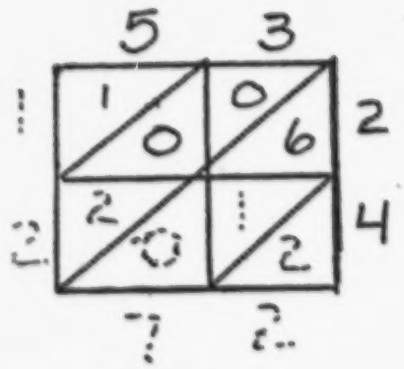


Multiplying 2-place numbers to two and three place

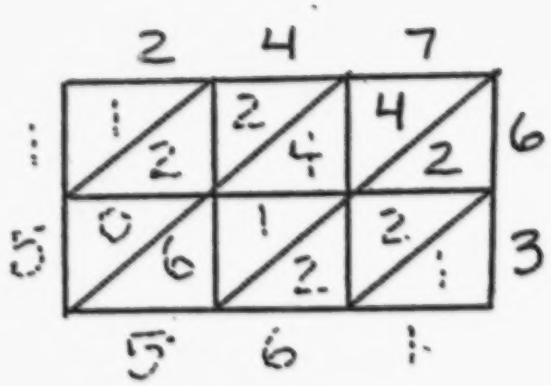
Bigger numbers are multiplied in a similar way. "See how 2934 by 314 was multiplied.

2934
X314
11736
2934
8802
921276

"Copy these lattices and complete the work. Check your answers. How can you be sure your answers are right?"



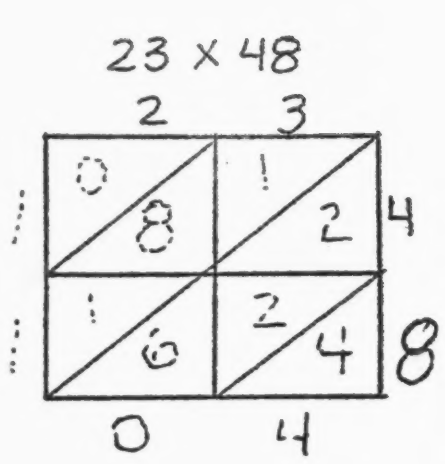
(1272)



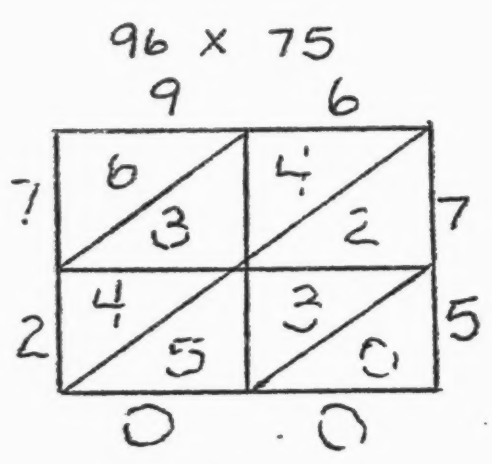
(15,561)

Multiplying two and three digit numerals to three and four place numerals

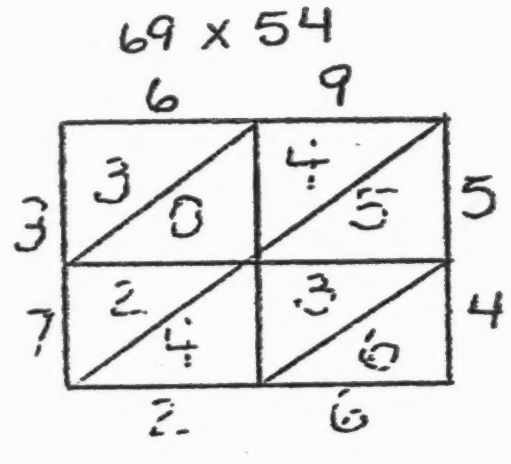
"Find the following products first by using lattices and then by the way we do at school now."



23
x48
184
920
1104



96
x75
480
6720
7200



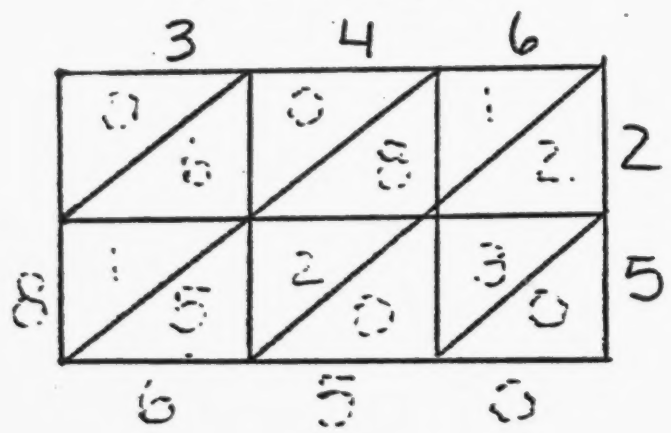
69
x54
276
3450
3726

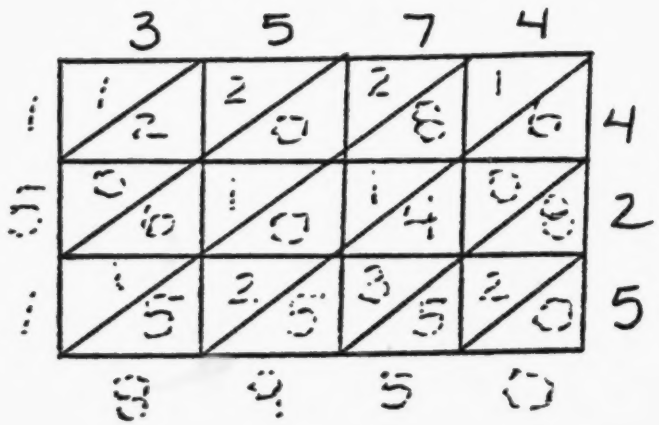
Multiplying two and three place numbers by three and four place numbers

"Now try some bigger numbers."

3574 x 425

346 x 25





346
x25
1730
6920
8650

3574
x425
17870
7148
14296
1,518,950

Rounding divisors upward and downward

A. "Think of 32 as about 30. Can you think of 1385 as 1300 + 85? (yes). What is 1385 ÷ 30? 6:77
Are there at least 40 thirties in 1385?
Then what numeral will you use to replace the first □ ? (40).

B. How many thirties are there in 105? (3). Then what numeral will you use to replace the second □ ? (3) What is 105 - 96? (9).

C. After you find the remainder, what two numbers will you add to get the quotient? (40 and 3). Above what two places in the dividend will you write the numeral for the quotient? (Tens' and ones" places).

Check your work using this sentence:

$$[(40 + 3) \times 32] + 9 = N. \quad (1,385).$$

"A committee is deciding what each member's share of the club's expenses will be. There are 29 members and the expenses are $14.15. What is each members share? How much will the club still owe?" 6:78

"Will you round 29 to 20 or 30? (30).

29 | $ .14 r 23¢
$14.15 | 40
11.60 = □ × 29
2.55 | 8
2.32 = □ × 29
23

How many cents in $14.15? (1415).

Are there at least 30 thirties in 1415? (yes). Are there at least 40 thirties? (yes). Are there at least 50 thirties? (no). What will you use to replace the first □ ? (40). What will you use to replace the second □ ? (8). Then what numeral will you write under the 255? (232).

How much should each member pay? ($.48). How much will the club still owe? ($.23).

Check using this sentence:

(48 x 29) + 23 = N. (1415).

Addition, subtraction, multiplication, and division of whole numbers

Design a scroll on the chalkboard or duplicate it on paper for the students. Predetermine both the task to be performed or numbers to be shown in the "In"and Task to be performed" columns and the instructions for finding the answer such as:

In	Task to be Performed	Out
5	(x 7) ÷ 3	11 R 2

The youngsters write the answer in the "Out" column. Deviate this process by predetermining the "In" and "Out" and have the youngsters find the "Task to be Performed." Or, give them the "Task to be Performed" and "Out" entries and have them find the "In" entry.

PROBLEM SOLVING

Grade 5

Solving problems in more than one way.

- A parking lot has 25 rows with 18 spaces for cars in each row. If 3 rows were removed for a driveway, what is the greatest number of cars that can be parked on the lot?

Suggest the students try to think of two ways in which they can solve this problem and tell what mathematical sentences would be written for each way. 11:232

A. One way might be:

What mathematical sentence can we write to express the number of cars that can be parked on the lot? (25 x 18 = P)

Then what is the sentence for the number of spaces to be removed for the driveway? (3 x 18 = D)

After the product is found for each of these sentences, a sentence can be written for the greatest number of cars that can be parked on the lot after the driveway is made.

25 x 18 = P (Before driveway)
3 x 18 = D (For driveway)
P - D = N or 450 - 54 = N

B. Another way to solve the the problem is to use (25 x 18) as the number of spaces before making the driveway and (3 x 18) as the number of spaces removed for the driveway. Then the mathematical sentence for the number of cars that can be parked after making a driveway is: (25 x 18) - (3 x 18) = N. 10:233

Ask what computations are necessary. After finding that 25 x 18 = 450 and 3 x 18 = 54, you must subtract 54 from 450.

C. With either method, you can then answer the question of the problem. There is room for 396 cars on the parking lot. You may wish to use other examples to help the students understand both way.

A coin book has 35 slots for coins on each page. If the book has 12 pages and 287 coins have been placed in the slots, how many more are needed to complete the book? 10:234

Here is a way to solve this problem using two mathematical sentences.

$12 \times 35 = P$

$$\begin{array}{r} 35 \\ \times\ 12 \\ \hline 70 \\ 350 \\ \hline 420 \end{array}$$

$420 - 287 = N$

$$\begin{array}{r} 420 \\ -\ 287 \\ \hline 133 \end{array}$$

There are 133 coins needed to complete this book. Here is a way to solve this problem using one mathematical sentence.

$(12 \times 35) - 287 = N$

$$\begin{array}{r} 35 \\ \times\ 12 \\ \hline 70 \\ 350 \\ \hline 420 \end{array} \qquad \begin{array}{r} 420 \\ -\ 287 \\ \hline 133 \end{array} \text{ coins}$$

A certain type of jet airplane has seats for 131 passengers. Another type of jet plane has seats for 165 passengers. How many passengers can be carried on 12 flights by these two planes?

Can you solve this problem by multiplying both 131 and 165 by 12 and adding the two products? Can you also solve the problem by adding 131 and 165 and multiplying the sum by 12? As a check on your answer, solve the problem both ways. 7:17

12 x 131 = 1,572;
12 x 165 = 1,980
1,572 + 1,980 = 3,552.

131 + 165 = 296;
12 x 296 = 3,552

6:49

At one place in the warehouse there are 23 stacks of canned peaches in cases with 16 cases in each stack. At another place there are 27 stacks with 16 cases of peaches in each stack. How many cases of canned peaches are there in all?

See if you can find two ways to work this problem. Which way is easier?

(23 x 16) + (27 x 16) = N
368 + 432 = N
N = 800

or

(23 + 27) x 16 = N
50 x 16 = N
N = 800

1) John paid 18¢ for 3 popsicles. How much would Mary pay for 24 popsicles for a party?

A. 18 ÷ 3 = 6
One popsicle costs 6¢.
24 x 6 = 144¢ or $1.44.

These rate problems are very good examples of open end problems.

B. $24 \div 3 = 8$

In 24 popsicles there are
8 groups of 3.
8 x 18 = 144. 8 groups
of 3 cost 144¢ or $1.44.

Which solution do you prefer?

A diagram or chart can be made that includes prices for as many items as needed.

No. of popsicles	1	2	3	4	5	6	7 . . . →
Amount of cost	6¢	12¢	18¢	24¢	30¢	36¢	42¢

No. of cans	1	2	3	4	5	6	7 →
Price in cents	5	10	15			30	

No. of cans	1	2	3	4	5	6 →
Price in cents	6	12	15	21	27	30

In grocery stores, you will find cans, cartons, bottles and packages marked this way: 3/15, 4/25, 5/20. 13:Liii

"What do you think these marks mean?"

"If a can is marked 3/15, how much would I pay for one can?" Someone will say that if 3 cans cost 15¢, then one can will cost 5¢ because $15 \div 3 = 5$. Someone else might contend that such is not the case, because then the cans would be marked 1/5. If the price is meant to indicate a bargain for purchasing 3 cans, then the price for one can must be 6¢. By purchasing 3 cans for 15¢ the customer saves 1¢ per can.

On the board draw charts to illustrate each case.

"We agree that the price is 3 cans for 15¢ in either case. How much would 6 cans cost? How much would 9 cans cost? The same in both records? Now, how about one can? We have agreed that one can might cost 5¢ or 6¢. In each case, how much would 2 cans cost?" (10¢ in the first case; and 12¢ in the second.) But someone might argue that if one can costs 6¢ and 3 cans cost 15¢, then you probably get a small bargain for buying two cans at a time. So, while one can costs 6¢, 2 cans might cost only 11¢. That chart would look like this:

		Savings
1 Can	6¢	0
2 Cans	11¢	1¢
3 Cans	15¢	3¢

So we need another chart:

No. of Cans	1	2	3	4	5	6	7 . . . →
Price in Cents	6	11	15			30	

Problems without numbers.

These problems do not give all the information necessary to find an answer. Write a sentence to explain how you would solve each problem if the information were given. 5:172

1) The school bought 18 new records for the music class. What was the average cost per record? Answer: Divide the cost of all the records by 18.

2) The combined weight of a truck and its weight in groceries is 13,670 pounds. What is the weight of the load of groceries? (Subtract the

number for the weight of the truck from 13,670)

3) The junior Red Cross members packed 243 items of clothing into boxes. What was the average number of items in each box? (Divide 243 by the number of boxes)

4) Ed weighs more than Mark. Mark weighs 72 pounds. How much did the boys weigh together? (Add the numbers of the boys weights.)

5) Jean walks 5 blocks to school each day and takes a shorter route coming home. How many blocks does she walk in going to and from school in 8 days? (Add the number of blocks in the short route to 5 and multiply this sum by 8)

6) Five girls agreed to share equally the expenses for a trip to the carnival. They paid $2.00 for bus fare, had lunch, and rode the Ferris Wheel. What was each girl's share of the expenses? (Compute the sum of 200 and the numbers for the other expenses and then divide this sum by 5.)

7) Mrs. Hector bought 3 dozen eggs. Carol used some of them to bake a cake. How many eggs were left? (Multiply 12 by 3 and subtract the number of eggs used from the product of that multiplication.)

Read each problem on the next page carefully and tell how to solve it. Then make up numbers for the problem and find the answer. 6:211

Multiplying to solve a problem

1) You know how many stamps you want and you know the cost of each stamp. How much will the stamps cost?

Subtracting to solve a problem

2) You know the cost of a badminton set, you know that you do not have that much money, but you know how much money you do have. How much money do you need?

Dividing to solve a problem

3) You know how many children are giving a gift and you know the cost of the gift. How much should each child contribute.

Have pupils write A, S, M, or D to tell what operation should be used to solve the problem. 2:14

1) John weighs ___ pounds and Jum weighs ___ pounds. One day they got on the scales together. How many pounds did they weigh? A

2) Ann has ___ cents and Sue has ___ cents. How much more does Sue have? S

3) ___ bottles of soda pop were in the shopping bag. There were ___ ounces of soda pop in each bottle. How many ounces were there in all? M or A

4) We drove ___ miles the first day and ___ miles the second day. How far did we drive all together? A

5) Ian had ___ balloons. He gave the same number to each of ___ children. How many did each child get? D

6) In our room there are ___ pupils. ____ of them are girls. How many boys are there? S

Example:

Joe wants to know the average score in his arithmetic tests. In each test, "100" was a perfect score.

He needs to know: the score for each test.

He solves the problem by: finding the sum of all the scores and dividing this total by the number of tests.

For each of these problems, tell what you need to know. Then tell what to do to solve the problem. (See example)

1) Bill wants to know how much it cost his father to fill the gasoline tank of his car.

He needs to know: How many gallons of gasoline the tank holds and how much a gallon of gasoline costs.

He solves the problem by: Multiplying the number of cents each gallon costs by the number of gallons needed.

2) Jane wants to know what part of the class is girls.

She needs to know: How many children are in the class and how many are girls.

She solves the problem by: Using a fraction to show the relation of the number of girls (numerator) to the number of children in the class (denominator).

"Encourage children to make clear statements in correct mathematical language."

Approaches to problem solving.

Purpose: To solve problems based on expenses on a trip.

Procedure: Discuss the whole situation with the class. Show that accuracy of work is necessary in solving problems. Show that problems 4 and 7 require the use of information given in earlier problems. Point out that we often must gather information from various sources to solve a problem.

1. Jimmy kept a record of expenses of his family on a three-day automobile trip.

   a. They traveled 328 miles on the first day. On the second day they traveled 163 miles, and on the third day, they traveled 421 miles. How far did they travel in the three days?

   a. Miles Traveled:

1st day	328
2nd day	163
3rd day	421
Total Miles	912

   b. Jimmy's father filled the car with gasoline before they started. He bought 16 gallons at 32 cents a gallon. How much did he pay for the gasoline?

   b. Gasoline per gallon:

16 gal.
× 32
32
48
Cost 512 = $5.12

c. 36 x 32 = $11.52
+ 5.12
$16.62 Total Cost

c. During the trip he bought 36 more gallons of gasoline at the same price. How much did he pay for the gasoline?

d. Gal. left 4 gal.
per gal. 32
$1.28

$16.64 = Total cost
- 1.28 = Gas left
$15.36 = Cost per trip.

d. When they reached home, they had four gallons left in the tank. What was the total cost of the gasoline they used on the trip?

e. 6 = qts. oil
2 = qts. added
8 = total qts. oil

280 = cost of oil
280 -- 8 = .35 per qt.

e. Jimmy's father had 6 quarts of new oil put in the car before they started the trip. He added 2 quarts of oil during the trip. If the total cost of oil was $2.80, how much did the oil cost a quart?

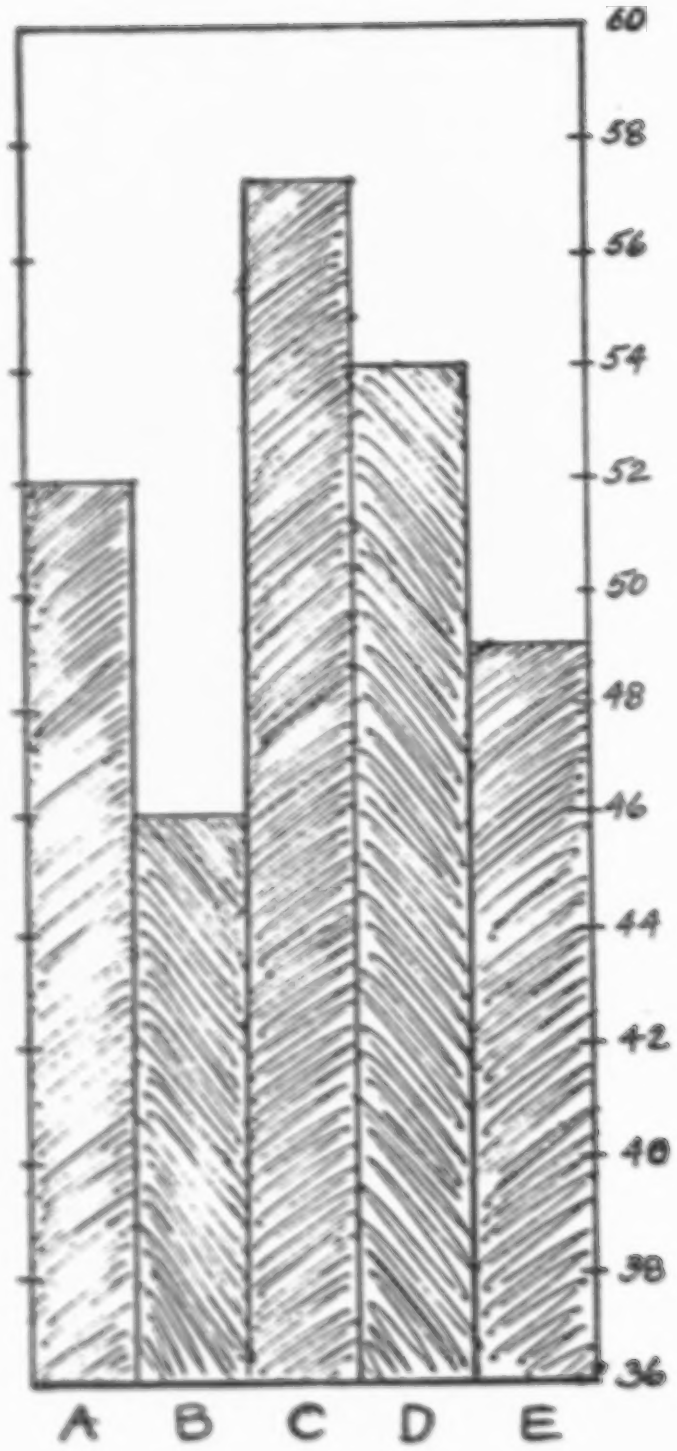
f. 28.50 = meals
18.75 = lodging
10.65 = amusement
$57.90 = total expenses for meals, lodging, amusement.

f. They spent $280.50 for meals, $18.75 for lodging, and $10.65 for amusements. How much did their meals, lodging, and amusements cost them.

g. $15.36 = gasoline
2.86 = oil
57.90 = meals, lodging, amusement
$76.12 = TOTAL COST

g. What was the total amount spent on the trip?

Purpose: to solve problems based on a graph.



Procedure: First discuss the content of the graph. Ask questions that will require the children to name items in the graph and help them see the way in which it is constructed. Show them that interpolation of the scale is required when the answer on a bar will end half way between the numbers in the scale.

Five children measured themselves and charted their heights in inches. What was the height of each child?

Encourage students to make up questions using data from the graph. Collect questions, ditto them and use for class discussion and worksheet.

Purpose: To provide mixed practice in problem solving.

Method: Discuss with the class any unfamiliar words. Explain that the problems provide a variety of applications of the four fundamental operations with dollars and cents.

1) John's father bought him a small chemistry set for $5.99. His mother bought him a microscope for $4.00. His sister bought him a set of slides for $1.75. How much did the three gifts cost? How much more did the chemistry set cost than the microscope?

2) Jane's mother bought her a doll for her collection that cost $5.25. Her aunt bought her a doll that cost $3.85. How much did the two dolls cost?

3) Jim's father bought him 3 story books and a poetry book at $.55 each. What was the total cost of the books?

4) Mary's mother bought a 12-pound ham for dinner. If the ham cost 68 cents a pound, what did the ham cost?

5) Betty earned $3.25 baby sitting for a neighbor. If she stayed 5 hours, how much did she earn an hour?

Purpose: To give the children practice in the reading an' use of the tables of measure.

Problems:

1) Jane has a piece of ribbon 32 inches long. How much less than a yard is this? Name some other things that are sold by the yard.

2) How many inches more than 2 feet is 32 inches?

3) Mary saw a package of raisins at the store that weighed 10 ounces. How much less than a pound is this? Name some things that are sold by the pound.

4) How many eggs are there in five dozen? In three dozen? Name some things that are sold by the dozen?

5) How many quart bottles can be filled from 3 gallons? How many pint bottles can be filled from 3 gallons? Name some other liquids that are sold by the gallon.

6) How many cents are in a half dollar? In a dollar?

7) How many cents are in 3 dimes, 5 nickles, and a quarter?

Adapt story problems as follows. This is primarily a reading-for-mathematical-understanding lesson.

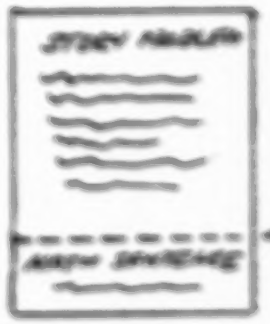
> John is 10 years old and weighs 86 pounds. Fred is 2 years older than John and weighs 100 pounds and is in the seventh grade.
>
> List the numbers needed to answer the following questions:

1. How old is Fred?
2. What is the combined weight of John and Fred?
3. How much heavier than John is Fred?

You may also continue the lesson by having the students write mathematical sentences to solve each problem.

FRONT

BACK



Mount interesting pictures from magazines on tagboard cards. On back of card glue a large manilla envelope.

Have students write a story problem to go with the picture. After writing the story problem, the student is then to write a mathematical sentence to use in solving the problem.

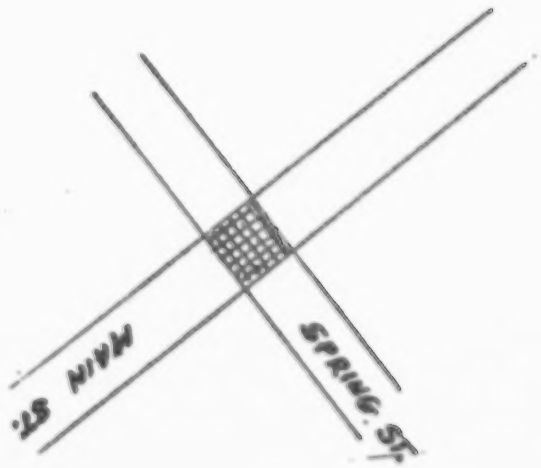
A form can be made for this work. Use a system of identification for each card.

The form can be made in such a way that the story problem may be cut off and returned to the manilla envelope. Then the students can work on the story problems written by other students.

GEOMETRY

Grade 5

Point of intersection

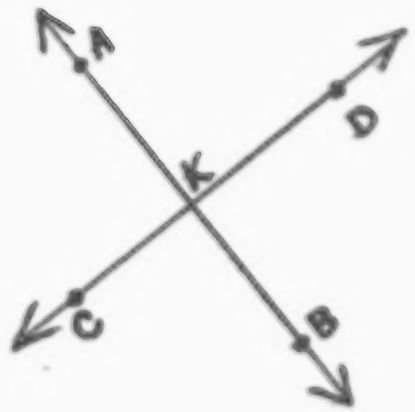


Show two intersecting streets on the chalkboard, giving them the names of a familiar intersection near the school. Ask the children to name the square region shaded in the illustration (Intersection). "Is the intersection part of Spring Street? (Yes) Is the intersection also part of Main Street?" (Yes)

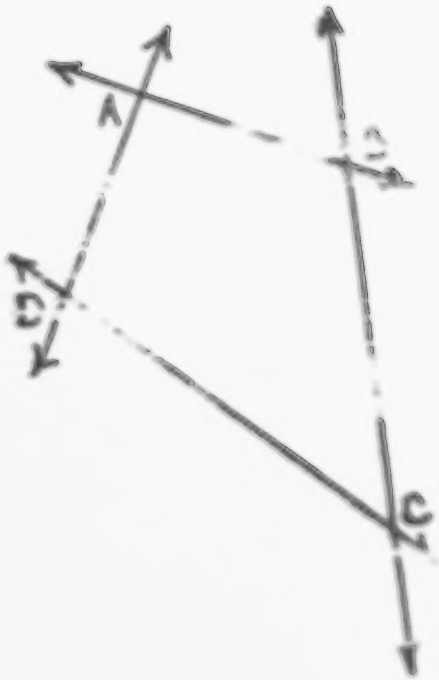
8:162

6:214

Draw alongside the first illustration a pair of intersecting lines labeled as shown. "Is point K part of line AB? (Yes) Is point K part of line CD? (Yes) Do the lines share more than one point? (No) We say that point K is a point of intersection." Ask students to draw several pairs of intersecting lines, labeling the lines and their points of intersection.

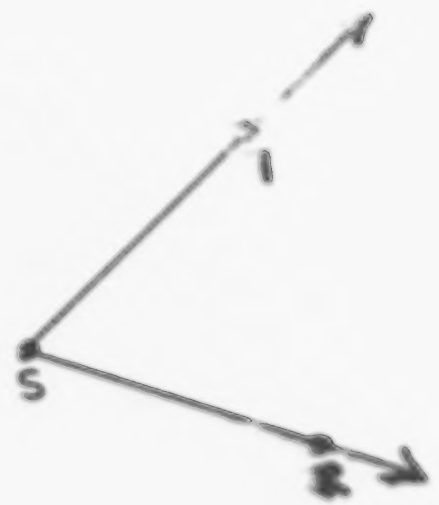


"Look for intersecting lines in our classroom. Name each line and describe their points of intersection."



Explain the symbol used for naming a line and give students opportunity to write it, i.e., $\overleftrightarrow{AB}$ means "line AB". Ask students questions about models or diagrams that would require them to write the symbol for a line:

1. Name a line that intersects point A. ($\overleftrightarrow{AB}$ or $\overleftrightarrow{AD}$)
2. Name a pair of lines that intersect at point C. ($\overleftrightarrow{DC}$ and $\overleftrightarrow{BC}$)



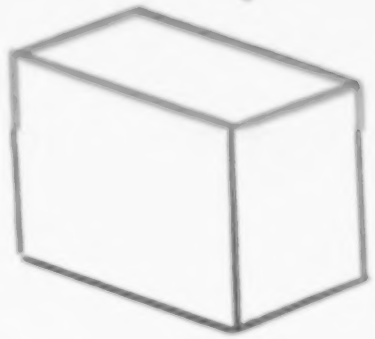
Tell students to locate two points S and T on their papers and to draw ray ST with S as its endpoint. "Choose a third point R and draw ray SR with S as its endpoint." Write the symbols $\overrightarrow{SR}$ and $\overrightarrow{ST}$ on the chalkboard. "Compare the symbols with the rays in your diagram. Does $\overrightarrow{ST}$ begin at S and pass through T? (Yes) Does $\overrightarrow{SR}$ begin at S and pass through T?" (Yes) Place symbols $\overleftarrow{TS}$, $\overrightarrow{ST}$ and $\overleftarrow{TS}$ on the chalkboard.

8:168-170

Ask students to compare these names with the rays in their diagrams. "Is there a ray which begins at T and passes through S? (No) Why is the symbol $\overrightarrow{TS}$ not a good name for the ray you have drawn on your paper?" (Discuss, accept all logical answers) Explain to class that by agreement we always name the endpoint of a ray first.

Review, if necessary, that the models they have drawn are pictures of angles. Tell students that the name given the common endpoint of two rays is <u>vertex</u>. Each ray is called a <u>side</u> of the angle. Draw several angles on the chalkboard and ask students to write the names of the sides and the vertices. P is the vertex of all three angles shown here: $\angle TPW, \angle WPA, \angle TPA$.

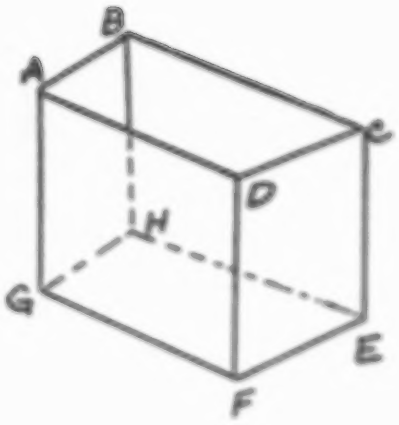
Rectangular prism



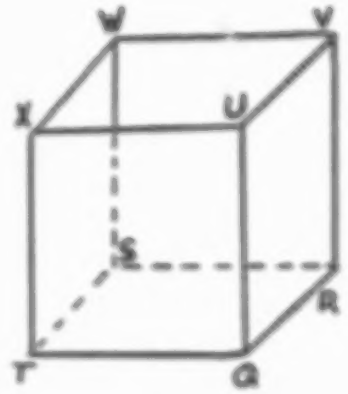
Display a large box to represent the rectangular prism. Lead the children to explore the box with their eyes while you turn the box in space. Ask them to describe the object without using the work "box." Discuss the description given. "Why is John's description good? Can you think of a better one?"

Give the students the vocabulary word "face" to use in their description. Tell them simply "the flat surfaces are called faces." Write "face" on the chalkboard. In similar fashion introduce words edge and vertex, showing what part of the box is being named.

Now give students various sized boxes of their own to explore with their fingers. Have them find and count the faces, edges, and vertices of the boxes, writing a record of their findings. Ask each student to exchange boxes with a neighbor and to find and count the edges, faces, and vertices of that box. "Compare findings. Suppose we exchange boxes again. What will we find out?"

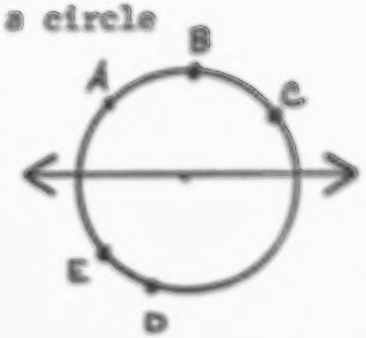


Give students dittoed pictures of rectangular prisms including the cube. Be sure that all vertices in each figure are labeled with capital letters. Working orally, ask them to name faces, edges, and vertices. Ask specific questions which encourage them to make real discriminations, i.e., name the vertex that is the intersection of line segments AB and BC.



Follow-up written work could include writing the names of the edges, vertices and faces of given figures. This is an excellent time to show the symbol for the line segment. Make a list of the names of the edges of a prism on the chalkboard, i.e., "line segment AB," line segment BC, line segment CD, etc. Alongside this list of names write the symbols $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, etc. Let students use whichever form they choose when writing their answers. After writing "line segment" several times, most students will prefer the shorter form.

Arc of a circle



Place a drawing of a circle on the chalkboard with points labeled as shown. Draw a line through the circle and its center. "Is point A above or below the line? (Above) Is point D above or below the line?" Continue by asking about B, C, E and other points. "Does the line divide the set of points into two sets? (Yes) Does the line divide the circle into two equal parts?" (Yes)

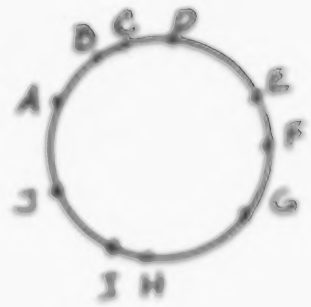
Provide students with paper and compasses. Have them:

1. Draw a circle.
2. Label the center point H.
3. Draw a radius $\overline{HK}$.
4. Draw a diameter $\overline{KJ}$.

"Does $\overline{KJ}$ divide the circle into two equal parts?" (Yes)

5. Draw a diameter $\overline{AB}$ so the circle will be divided into four equal parts.
6. "How could we describe the part of the circle between J and B? Is it a set of points? (Yes) Is it a line? (No) Is it a line segment? (No) Is it a curve? (Yes) Is it a closed curve? (No) Is it an open curve?" (Yes)

Give the students the word "arc" to name the parts of a circle. Ask them to name the arcs of the circle they have divided into four equal parts. Lead them to see that there are many more than just four arcs in their drawings. (As arc JAK and arc AKB)



$\overset{\frown}{AB}$
$\overset{\frown}{AC}$
$\overset{\frown}{AD}$
$\overset{\frown}{AE}$
$\overset{\frown}{AF}$
↓

Have students draw a circle and label ten points on it with capital letters. "How many different arcs can you count? Write their names on a sheet of paper." Show students the symbol we use for arcs. ($\overset{\frown}{AB}$ for "arc AB") Let students name the arcs for some time randomly. Then ask to see if anyone can suggest a systematic way to count all the possible different arcs named by the ten points.

Geometric constructions

For purposes of brevity and organization, geometric constructions are included here as one unit. Such constructions in reality should be closely woven into topics of geometry, measurement, science, art and the social studies. When the circle is introduced as early as kindergarten, students should be encouraged to make their own models if only by tracing around the rims of tin cans. Some time should certainly be spent showing the proper handling of compass, pencil and straightedge. 3:1-9

Detailed explanations of the correct use of these tools is often found only in junior high school and senior high school teacher commentaries.

Constructing a congruent circle.

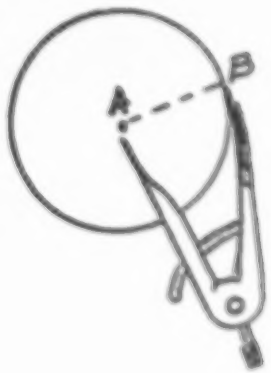


Figure I

Figure II

The constructions outlined here require only the use of compass and straightedge. These activities can be introduced as early as both academic need and student manual dexterity permits.

1. Place compass point on center point A of circle to be copied. (Figure 1) 1:186
2. Place pencil point on any point B of the circle.
3. Construct a circle at point P with a radius AB.

Constructing congruent line segments.

Figure I

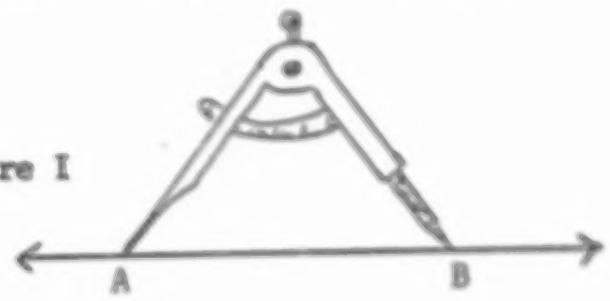
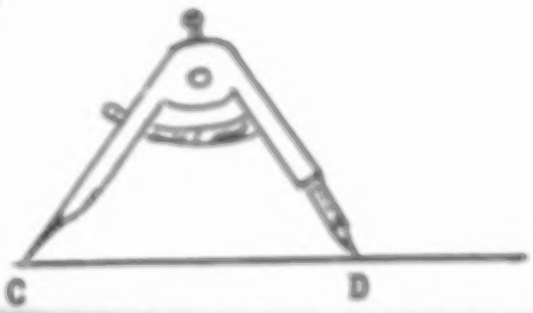


Figure II



1. Open compass points and measure given $\overline{AB}$ by placing compass point on A and pencil point on B. 10:336-347 (Figure I)
2. Use straightedge and pencil to construct a ray with endpoint C. (Figure II) 1:156
3. Place the compass point at C and mark off the given $\overline{AB}$ on the ray. $\overline{CD}$ is congruent at $\overline{AB}$. (Figure II)

Constructing congruent angles.

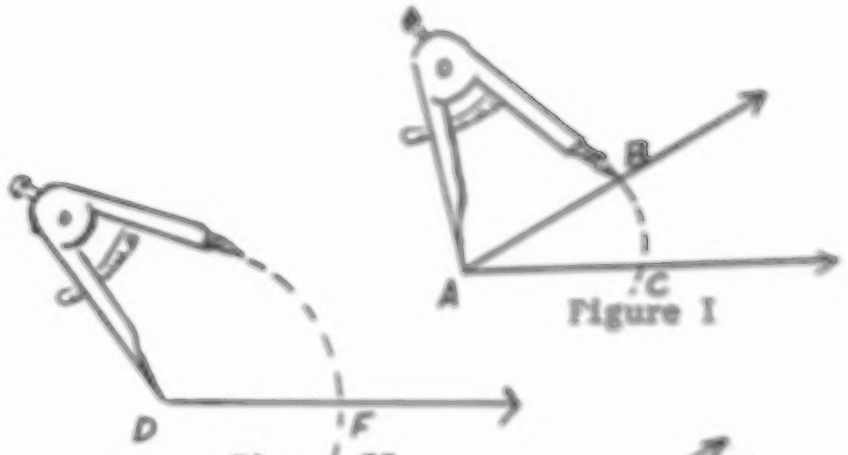


Figure I

Figure II

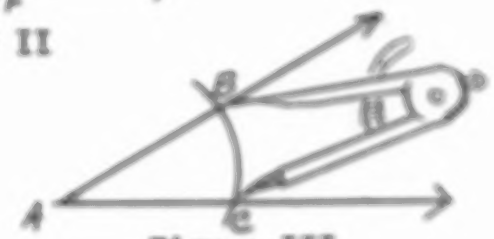


Figure III

1. Place compass point on A and mark a chord off on $\overrightarrow{AB}$ and $\overrightarrow{AC}$. (Figure I) 1:156

2. Use straightedge and pencil to construct a ray with endpoint D. (Figure II) 1:366-372

3. Place compass point on D and mark a chord off on $\overrightarrow{DF}$ so that $\overline{DF}$ is congruent to $\overline{AC}$. (Figure II) 1:198

4. Open compass points and measure given $\overline{BC}$ by placing compass point on B and pencil point on C. (Figure III)

5. Place compass point at F and mark off chord on $\overline{EF}$ so that $\overline{EF}$ is congruent to $\overline{BC}$. (Figure IV)

6. Use straightedge and pencil to construct a ray whose endpoint is D and which passes through E. (Figure V)

Figure IV

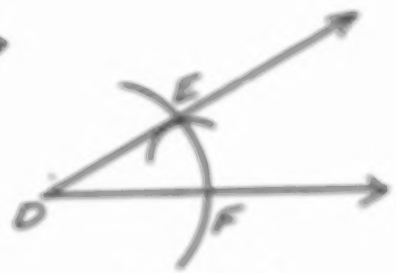


Figure V

Bisecting a line segment.

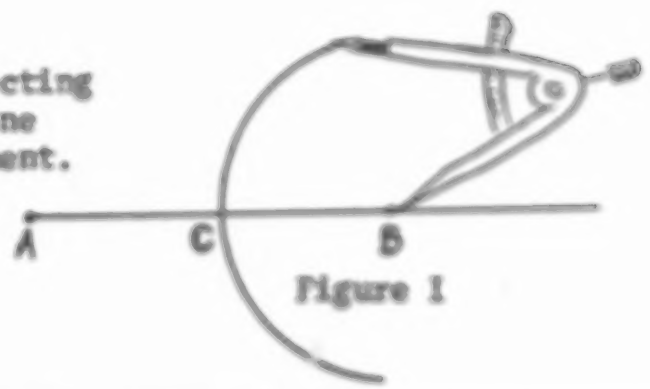


Figure I

Explain to students that to bisect means to cut in two or to divide in half. To bisect a line segment we must divide it at a point exactly half way between its endpoints. 1:157

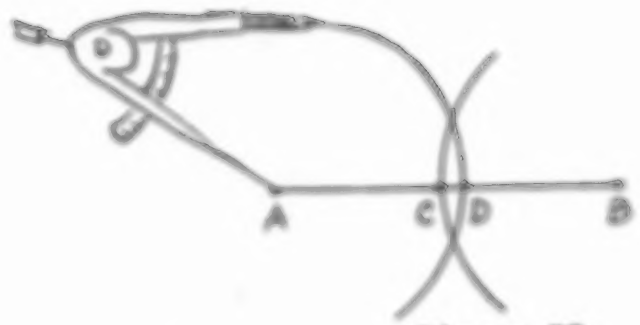


Figure II

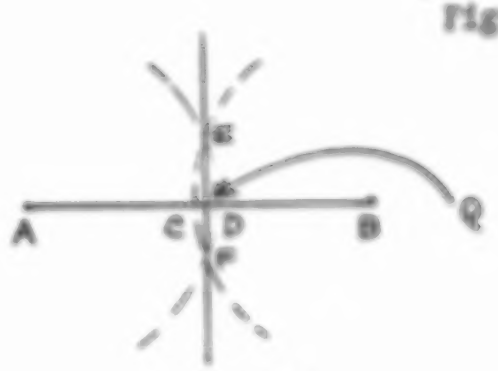


Figure III

1. Use straightedge and pencil to draw given $\overline{AB}$. (Figure I)

2. Place the compass point at B and mark off an arc on $\overline{AB}$ whose radius $\overline{BC}$ is more than half the distance B to A. (Figure I)

3. Place the point of the compass at A and mark off an arc on $\overline{AB}$ whose radius $\overline{AD}$ is congruent to $\overline{CD}$.. (Figure II).

4. Use the straightedge and pencil to construct a line that will pass through the intersections of the two arcs, namely point E and F. $\overleftrightarrow{EF}$ bisects $\overline{AB}$ at the midpoint Q so that $\overline{AQ}$ is congruent to $\overline{QB}$.

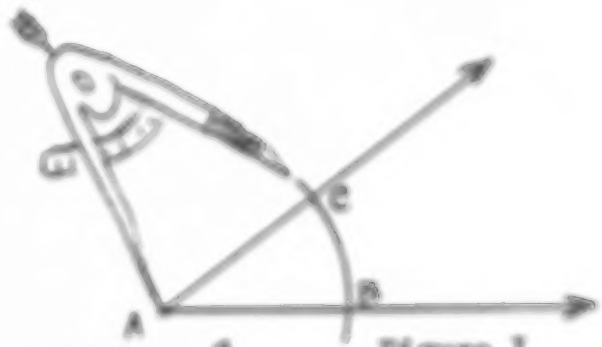


Figure I

Bisecting an angle.

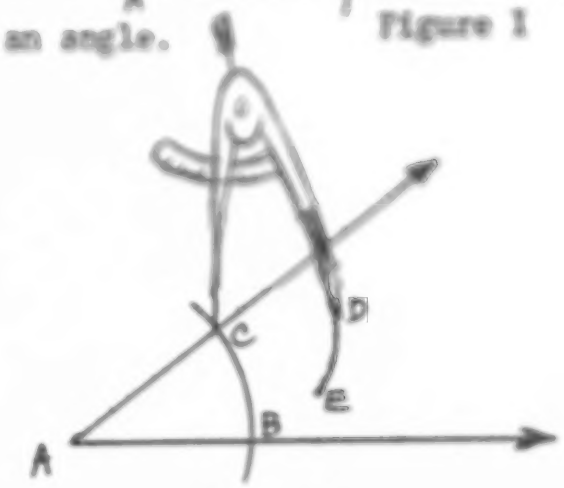


Figure II

Explain to students that to bisect an angle we divide it into two congruent angles. That is, we must construct a *ray* that divides the given angle into two equal angles. 1:157

1. Place the compass point on A and mark off any arc CB on the rays of the angle. (Figure I)

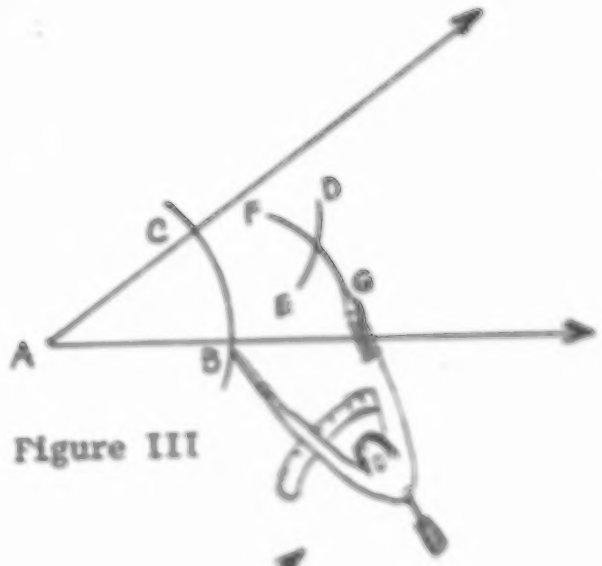


Figure III

2. Place the compass point at C and mark in the angle's interior an arc whose distance from C is greater than the distance from B to C. (Figure II)

3. Place the compass point at B and mark $\overset{\frown}{FG}$ whose distance from point B is equal in measure to the distance of $\overset{\frown}{DE}$ from point C. (Figure III)

4. Use straightedge and pencil to construct a *ray* whose endpoint is A and passes through the intersection of $\overset{\frown}{DE}$ and $\overset{\frown}{FG}$ at point Q. $\angle CAQ$ is congruent to $\angle QAB$.

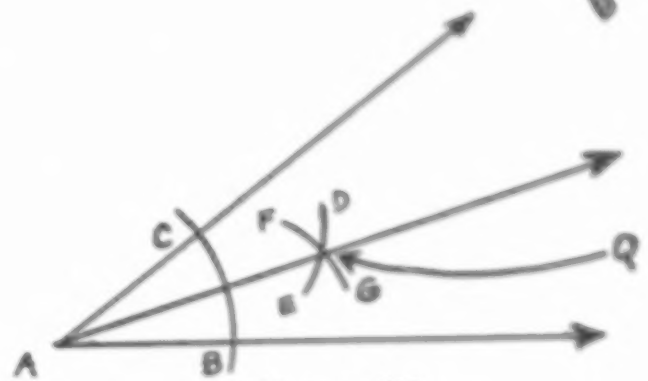


Figure IV

Constructing congruent triangles.

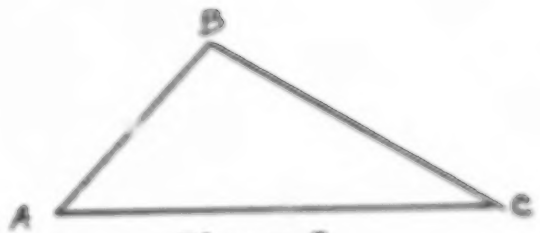


Figure I

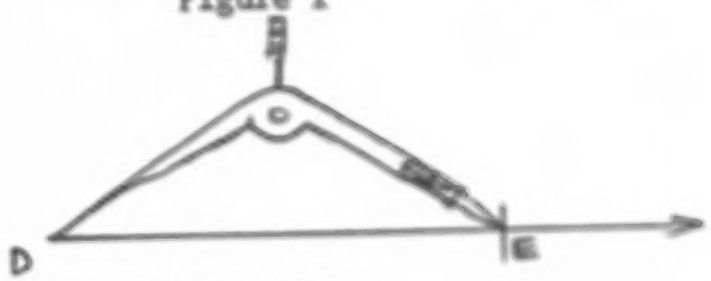


Figure II

1. Use straightedge and pencil to construct any given triangle ABC. (Figure I)

2. Use straightedge and pencil to draw a ray with endpoint D. (Figure II)

3. Measure $\overline{AC}$ in $\triangle ABC$ and, placing the compass point at D, mark off this distance intersecting the ray at point E. $\overline{AC}$ is congruent to $\overline{DE}$. (Figure II)

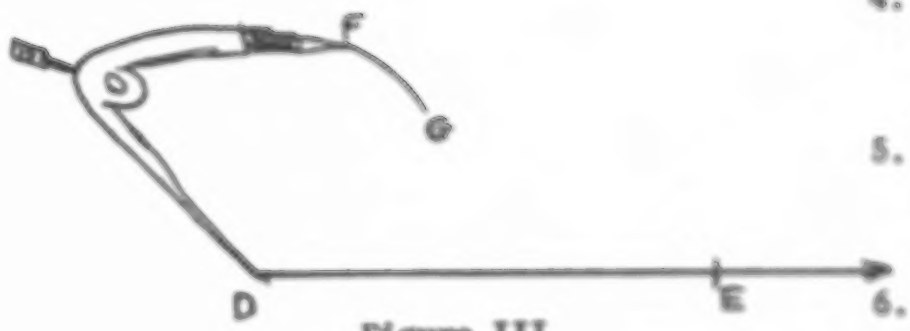


Figure III

4. Measure $\overline{AB}$ in $\triangle ABC$ and, placing the compass point at D, mark off this distance by drawing $\overset{\frown}{FG}$. (Figure III) 1:354-365

5. Measure $\overline{BC}$ in $\triangle ABC$ and, placing the compass point at E, mark off this distance by drawing $\overset{\frown}{JK}$ so that it intersects $\overset{\frown}{FG}$ at point Q. 1:160-161

6. Use straightedge and pencil to draw $\overline{DG}$ and $\overline{QE}$. $\triangle ABC$ is congruent to $\triangle QED$.

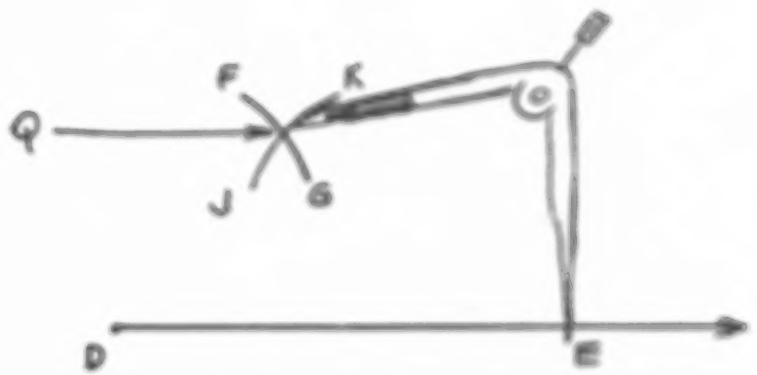


Figure IV

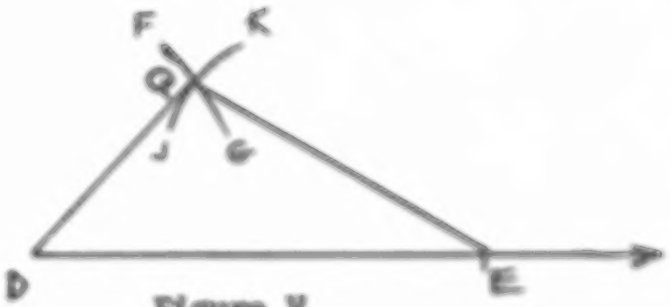


Figure V

MEASUREMENT

5th Grade

MEASUREMENT

Precision in measurement (inch and centimeter)

Provide inch (marked off in sixteenths) and centimeter rulers. 1:306-307

Cut colored straws into varying lengths and give several to each team of children.

Have them measure the straws to the nearest inch and record their findings. Then have them measure to the nearest half-inch; then the nearest quarter-inch. Have them see that if a straw is $4\frac{1}{4}$ inches long to the nearest quarter inch, it just means that it is closer to $4\frac{1}{4}$ than it is to $4\frac{1}{2}$ or 4.

Will the measurement be more accurate if we measure to the nearest eighth inch? (Yes) Have them measure the straws again--to the nearest sixteenth inch.

(a) ————————————

(b) ——————————

(c) ————————————————

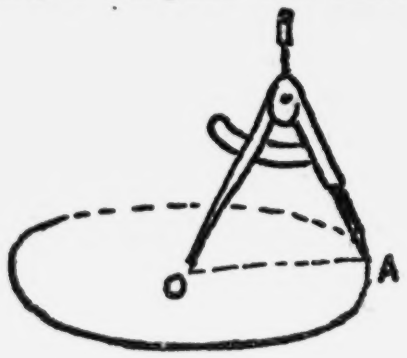
(d) ———

Have the children measure the length of line segments to the nearest $\frac{1}{2}$ centimeter. Then give the same length to the nearest inch.

Ditto a page of figures and measure the lengths of their sides in inches and centimeters.

10

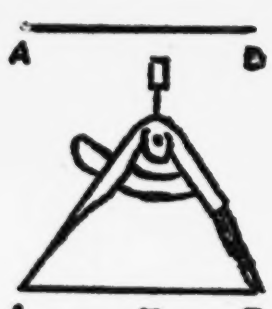
Using a compass for measuring.



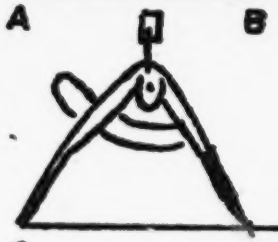
A compass is an instrument for drawing pictures of circles. It can also be used to mark off units of the same size. This can be helpful in making scales. If we are going to use a unit line segment to make a scale, we draw it (figure A), then place the points of the compass on points A and B of the unit (Figure B). Now mark off the unit on the ray (figure C). Keep the compass fixed and place the steel point on C, and touch the pencil point to the drawing of the ray. Call this point where it touches, point D. Place the steel point on D and continue to mark off the unit length on $\overline{CD}$ by moving the compass to the right. How long is $\overline{CF}$? (3 units)

8:354-355

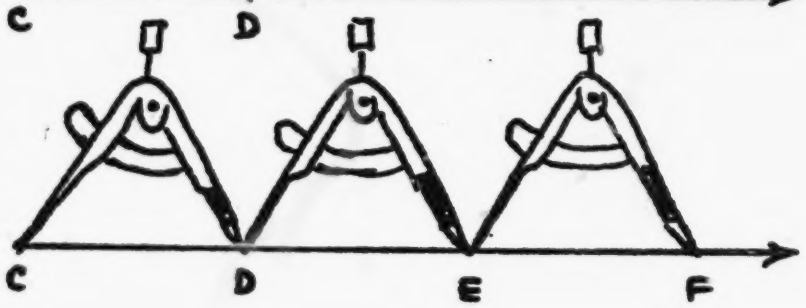
(A)



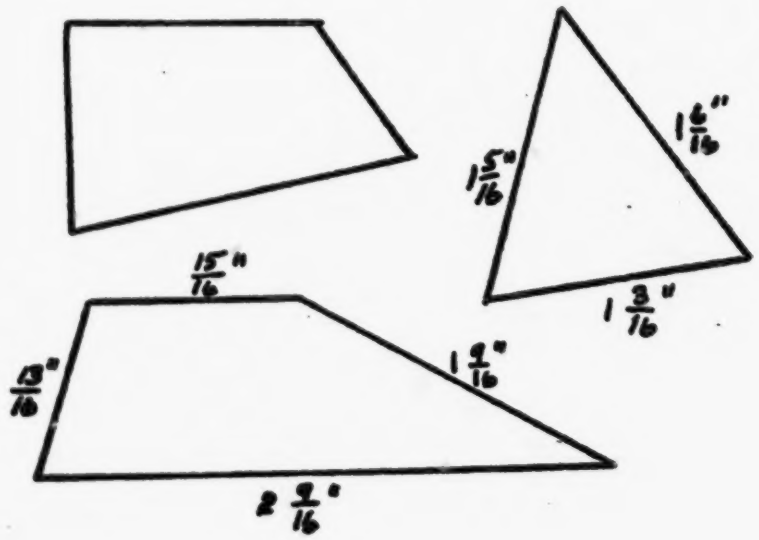
(B)



(C)



(D)



Measure each side of the figures at the left to the nearest 1/16 inch. Then find the perimeters.

Using a set of measuring cups, pour and experiment with 6 or 7 different containers. 6:157-158

Find measures to the nearest cup and record them. Then find liquid measure to the nearest 1/2 cup, 1/4 cup, 1/8 cup, etc. Record them in ounces also. The smaller the unit of measure, the more precise the results.

Using a compass to compare angles

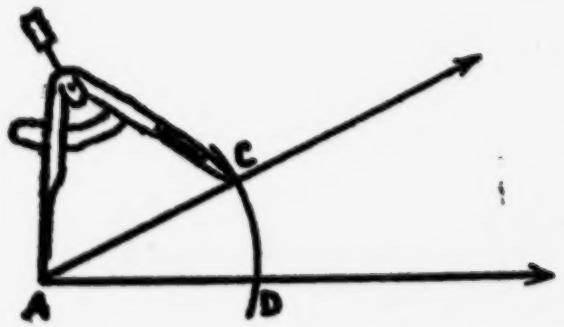
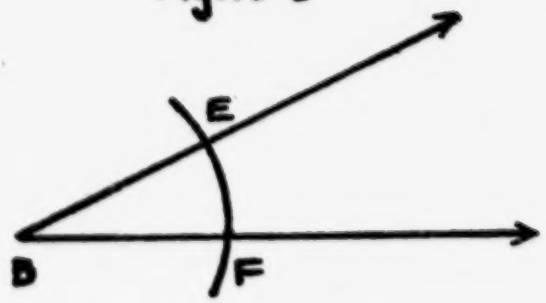


Figure 1



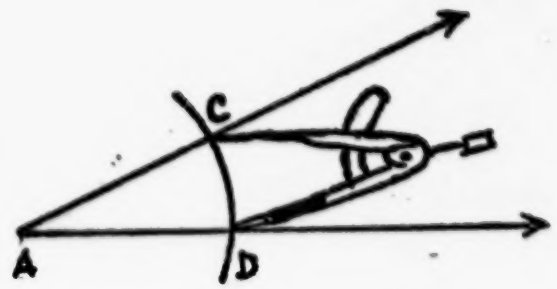


Figure 2

To compare two angles using your compass as an instrument, you would: 1:184-185

Present on the chalkboard and work through the following steps with the class.

A. Draw two angles on the board (congruent)

B. Use the compass to mark congruent segments on the rays ($\overline{AC}$, $\overline{AD}$, $\overline{BD}$, and $\overline{BF}$) are congruent to each other).

C. Open the compass to points C and D (Figure 2).

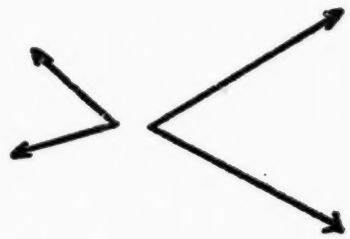
D. Place the compass (in the fixed position) on $\overline{EF}$ to see if it's congruent with $\overline{CD}$.

There are actually 2 steps involved in this exercise.

1. Keeping your compass fixed when drawing congruent segments on the rays of the angle; and

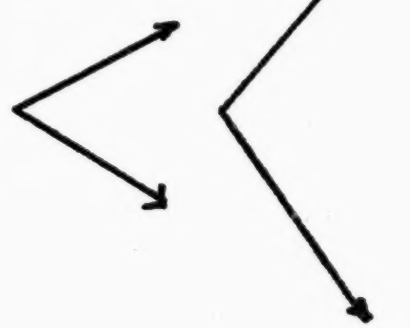
2. Using the compass as a measuring device from one point on the ray to a point on the neighboring ray.

(1) (Yes)



(2) (No)

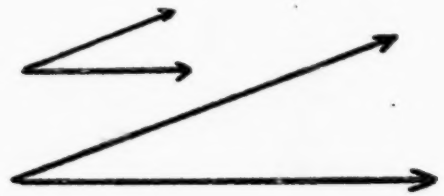
(3) (No)



Ditto a worksheet similar to the one shown here.

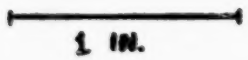
Use your compass to tell whether or not the two angles are congruent.

(4) (Yes)

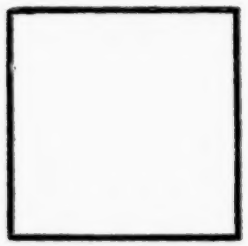


Make sure some of your examples have congruent angles but rays that vary in length considerably. (See Figure 4) Lead students to generalize that the length of each ray does not determine the size of the angle.

Finding Area



Procedure: Provide each pupil with a model of a square inch to measure many surfaces, such as sheets of paper of various sizes, books, etc. Emphasize that they need to find the number of square inches in only one row and then find the number of rows. Thus, to find the area of a rectangle that is 3 inches wide and 5 inches long, with the long dimension horizontal, the pupils find that there are five square inches in one row and that there are 3 rows. Hence, the area of this rectangle is 3 times 5 square inches. It is written thus: 3" x 5".



Objectives: To have pupils understand the distinction between perimeter and area.

Also, to develope skills for computing the perimeter of polygons and the area of rectangular regions.

Problems:

1. Using your model of a square inch, measure the size of your book page to the nearest inch. Find how many square inches are in each row. About how many rows fit on the page? About what is the area of the page?

2. A rectangle has a width of 7 inches and a length of 15 inches. How many square inches are there? How many are in each row? What is the area of the rectangle?

3. A rectangle is 5 feet wide and 9 inches long (5' x 9'). What is the area of the rectangle in square feet? (continued on next page)

4. Find the area of each rectangle:

6" x 13"	9' x 15'	67' x 128'
29 yd. x 19 yd.	14" x 32"	23 yd. x 49 yd.

5. All the measurements below are to the nearest inch. Find the area in square inches:

2'4" x 3'7"	6'3" x 3'5"	4' x 2'9"
1'10" x 11"	3' x 7"	2'6" x 1'6"
17" x 13"	3' x 4'	5'3" x 3'2"

6. Make drawings of a square 2" x 2" and a square 4" x 4". The 2" x 2" area is what part of the 4" by 4".

7. A room that is 24 feet long and 18 feet wide is to be covered with square shaped floor tiles. Each piece of tile is 6 inches on a side.

   a. What is the floor area of the room?

   b. How many pieces of tile are needed to cover 1 square foot of area?

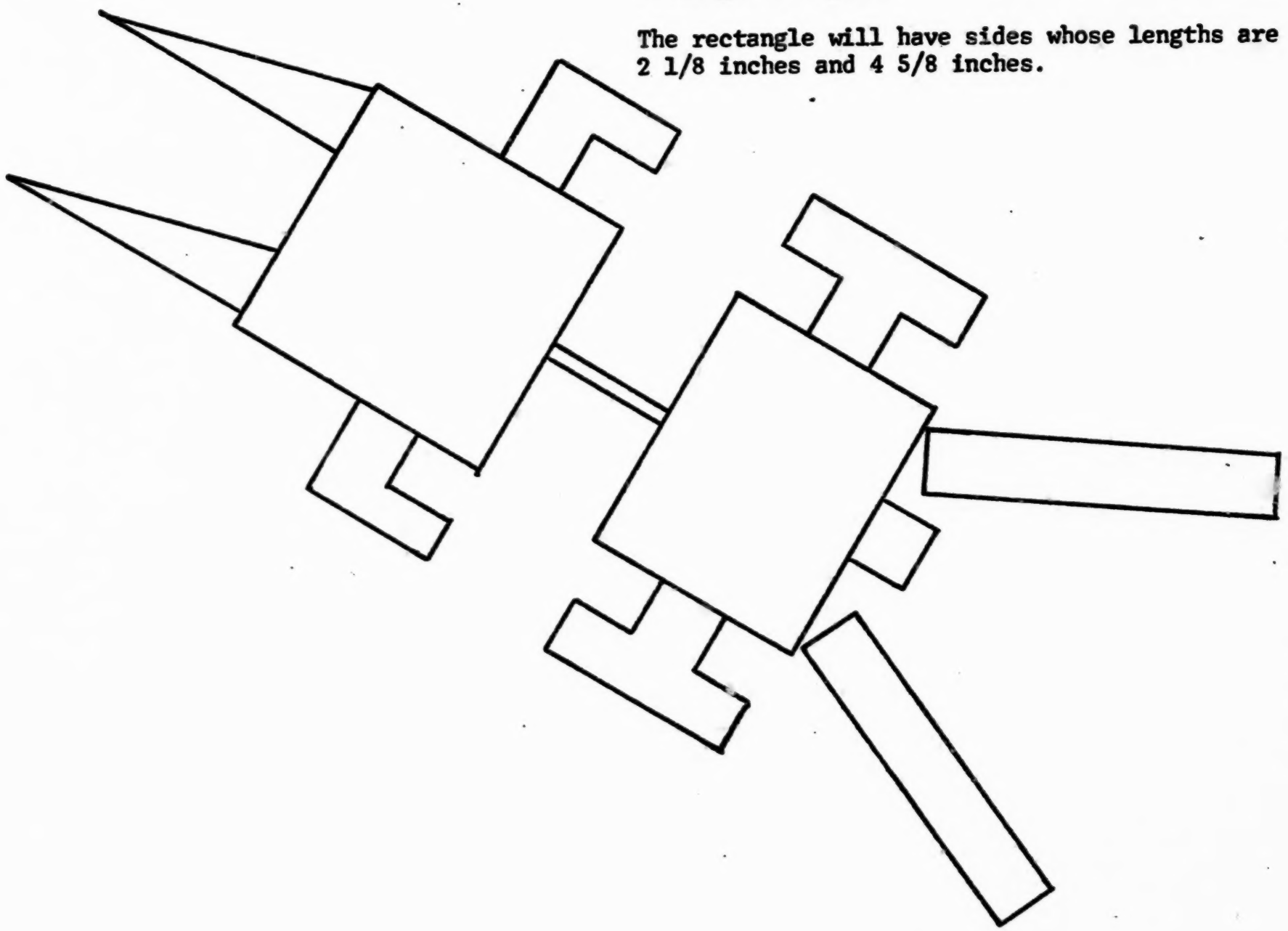
   c. How many pieces of tile are needed to cover the floor of the room?

8. A room has the shape and measurement shown at the left. Find the total area of the room in square feet.

Braintwister Game

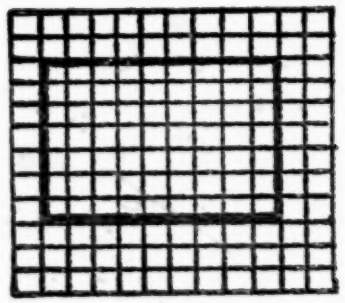
Trace "Robert Robot." Can you arrange the parts of the "robot" in such a way that they form a rectangular region?

The rectangle will have sides whose lengths are 2 1/8 inches and 4 5/8 inches.



Estimating area of a circle.

1



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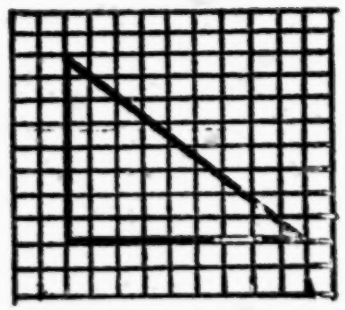
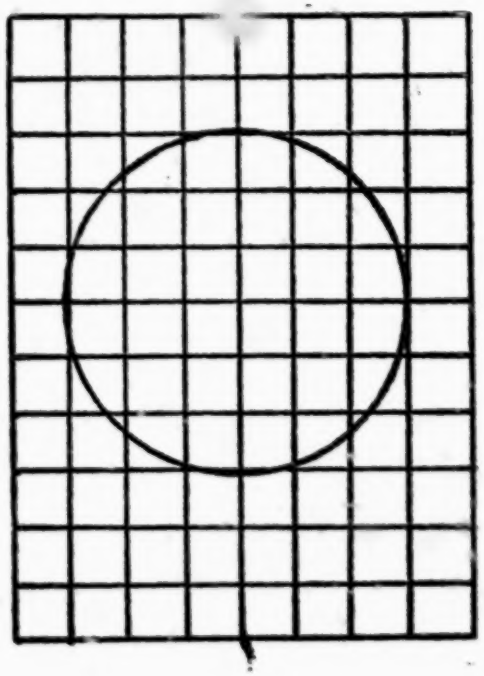


3



Provide sheets of paper with one-centimeter squares. On the paper draw a circular region, a right triangle region, and a rectangular region.

Estimate the regions in square centimeters. For example, in Figure 3:

(1) The area of the circular region is at least _____ square centimeters and at most _____ square centimeters.

(2) What would you estimate the area would be? ( ) (Answers will vary)

In Figure 1:

(1) How many square regions of the grid are contained entirely in the rectangular region? What does this tell about the area of the region? (It is at least 45 square centimeters.)

10:804-809

(2) How many square regions of the grid are needed to cover this region completely? (63) What does this tell about the area of the region? (It is at most 63 sq. cm.)

(3) Can you look at the region and tell <u>about</u> what the area is? (Answers will vary)

On the picture of the right triangle region (Figure 2) we can say:

(1) The area is at least _?_ centimeters and at most _?_ square centimeters.

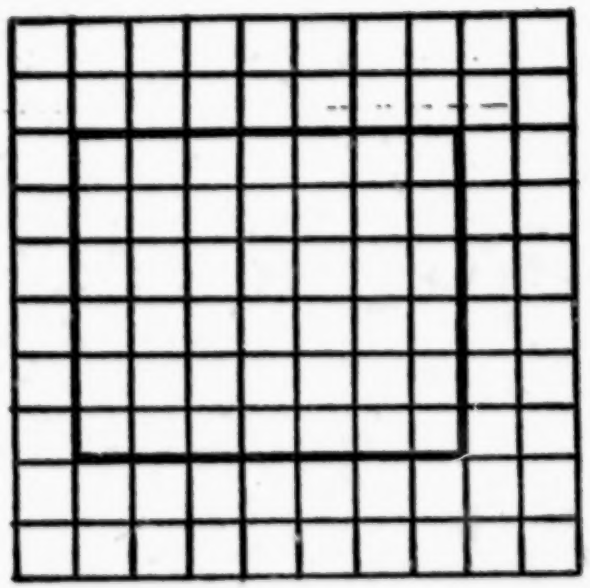
Scale drawing

10:404-410

SCALE 1:18

The toy rabbit at the left bears the scale of 1 to 18. It means that the picture of the rabbit represents 18 lengths of the real toy rabbit. The height of the rabbit is two inches in the drawing. What is the height of the toy it represents? (2 x 18 = 36") or 3 feet

Provide the children with square-ruled paper. At the left is a room plan. How large is the room? (70' x 60') What is the scale length of the room? (7 scale units) What is its real length? (70 feet) What is its area? (6 x 7 = 42 square scale units) What is its real floor area? (4,200 sq. ft.)

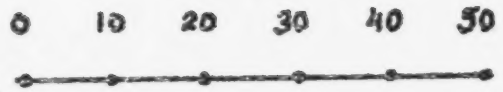


SCALE 1 UNIT = 10 FEET

Have the children pace off, plan, and do a scale drawing of their classroom on square-ruled paper. They should do this in teams and decide on their own scales. Each team will doubtless have a different scale, but all drawings will represent the same room.

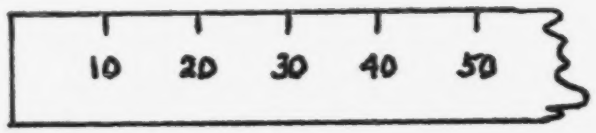
Have a team map the playground area choosing their own scale. For this activity they should have a good steel measuring tape and square-ruled paper.

Scale of yards.



Have another team read their map and compute the actual measurements of the playground. The team may wish to make their map without square-ruled paper. The scale should be at the bottom with a "scale ruler" available. What would you estimate the area would be? (Answers will vary)

Scale Ruler for map.

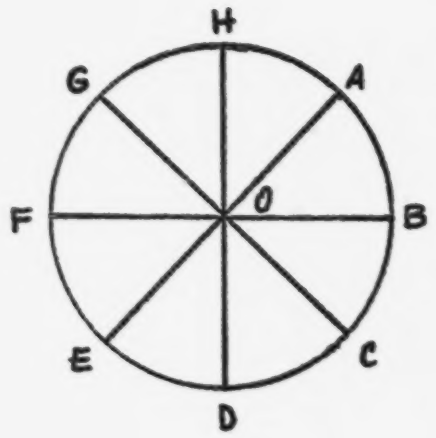


Unit of measure for angles

The protractor is not formally introduced till Grade 6, unless the teacher feels the class is ready for its introduction. Here we try and emphasize the fact that to measure angles we think of laying off a unit angle.

To measure length we use a ruler. How can we measure an angle? If a wheel has 8 spokes, what is the measure of the angle formed between any two neighboring spokes? If we have no instrument to measure angles, how can we do this? Cut a pie-shaped form (as the shaded area in the sheel) and use it as a unit of measure.

How many of these angle units make a complete Circle? (8)



What is the measure of angle AOC? (2 units)

What is the measure of HOC? (3 units)

Can we be more precise in these measures? (yes)
How? By making a smaller unit angle

Now that the children see that to measure an angle, they must lay off a given unit angle, point out that this has been done thousands of years ago by the Sumerians. We still use their angle unit today and we call it a degree which is written thusly (^{o}). They divided the circle into 360 of these equal degrees. Each arc degree is 1/360, as a unit of measure.

6:224-225

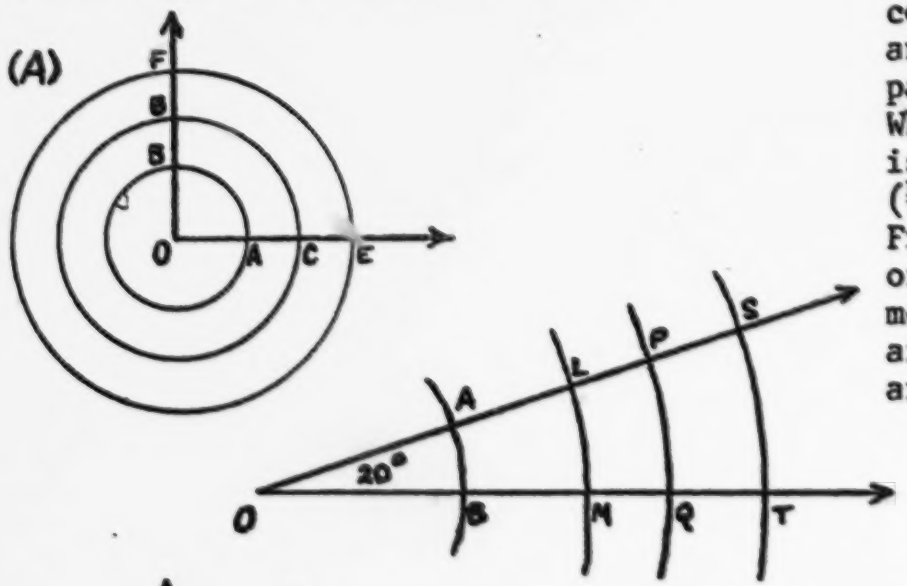
The degree ⟶

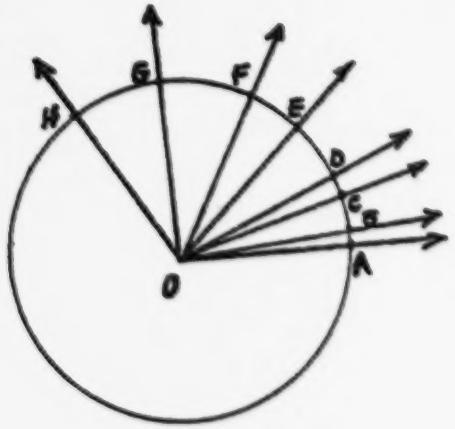
A Sumerian degree

Parts of circles called "arcs"

In discussing arcs we are not discussing their length, because different lengths may have the same number of arc degrees. (See Geometry, grade 5, of this syllabus)

Measuring arcs of circles





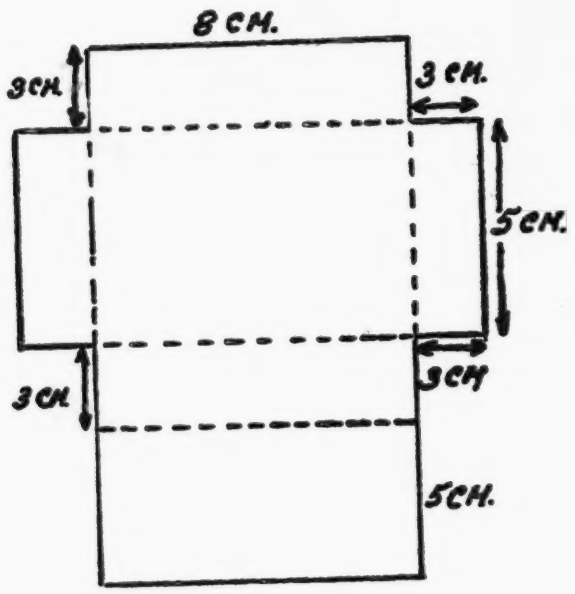
Dividing a circle into 360 arcs having the same measure gives us 360 central angles each measuring 1°.

Put 3 concentric circles on the board. The center of the 3 circles is labeled 0 and the angle at 0 is a right angle. What fractional part of the smaller circle is arc AB? ($\frac{1}{4}$) What fractional part of the medium-sized circle is arc CD? ($\frac{1}{4}$) Is each arc 90°? (yes) Why? ($\frac{1}{4}$ of a circle is always a 90-degree arc) In Figure B (as well as figure A) are the lengths of the arcs the same? (No) What is the angle measure? (20°) Is it the same for all the arcs? (yes) When we measure arc degrees we are <u>not</u> measuring lengths.

If we take any circle and divide it with 360 points so that we get 360 arcs that all have the same measure, we would have how many central angles? (360) What would each angle measure? (1°) Figure C is a picture of points, arcs, and rays. If we start at A, point B is the fifth point of the 360 points. So, BOA measures 5 degrees. Point C is the 16th point. So $\angle$COA measures 16° and $\angle$COB measures 11°. Point D is the 23rd point. Find the measure of $\angle$DOA, $\angle$DOB, and $\angle$DOC.

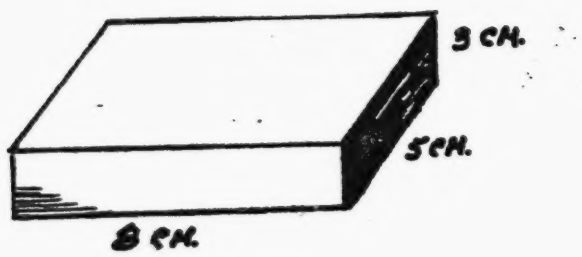
If you construct a paper model of a circle divided into 360 minor arcs which are equal in measure, you can measure angles with it. We call this instrument a protractor. (We will construct our own protractors in Grade 6.)

Total surface area of rectangular prisms



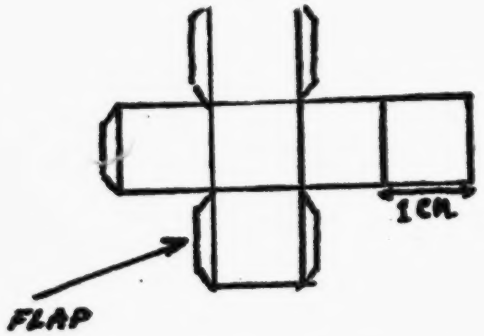
Provide the children with scotch tape, rulers, and scissors. Give each student a copy of the figure at the left (dittoed on heavy construction paper). The students may construct their own if time permits. What is the area of this plane figure? (158 cm.) Have them cut out the figure and fold it on the dotted lines. Use scotch tape to fasten the sides together. 1:110-111

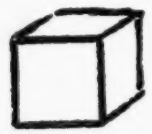
Now, what is the surface area of this space figure? (158 cm.)



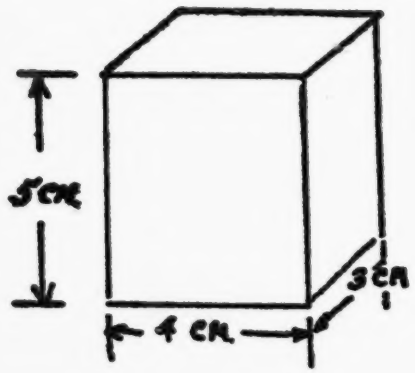
Using a plane figure to construct a space figure

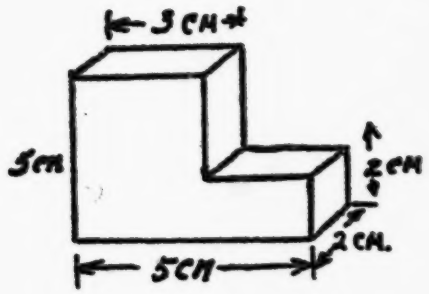
Have the students construct a cube. Since each square on this cube is the same, and since there are 6 sides (surfaces), six times the area of one surface is the total surface area.

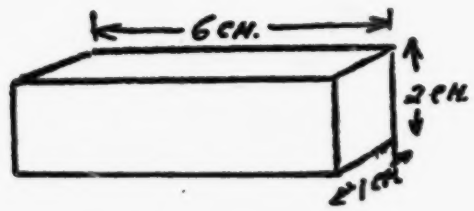




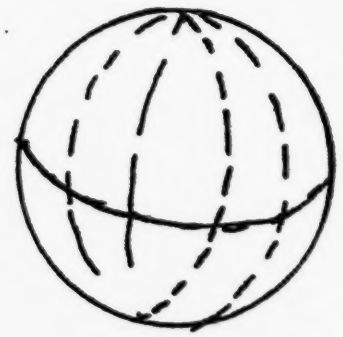
Ditto figures similar to those at the left and ask the children to find the surface area of each.







Time - standard time zones.



Obtain a world globe with time-zone boundaries indicated by broken lines, etc. Discuss the meaning and need for these zones.

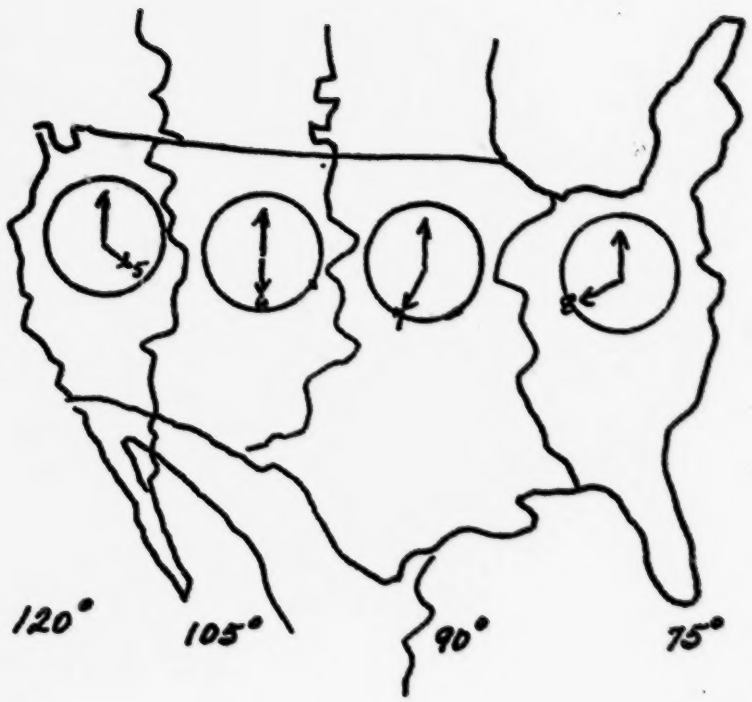
In learning about standard time, children should know that clock time is different in different parts of the country, by time zones. If it weren't for the time zones, two people on opposite sides of the globe would use the same time names, although it might be the middle of the day for one man and the middle of the night for the other. The surface

No mathematical concepts are involved in time zone patterns.

area of the globe, banded by 2 half-meridians 15 degrees apart, is theoretically equivalent to a time zone. Many reasons (political and topographic) have changed the time-zone boundaries from arcs of great circles to odd zig-zag lines. 5:126-127

Have a team of students make several paper clocks to demonstrate times in the time-zones of the world.

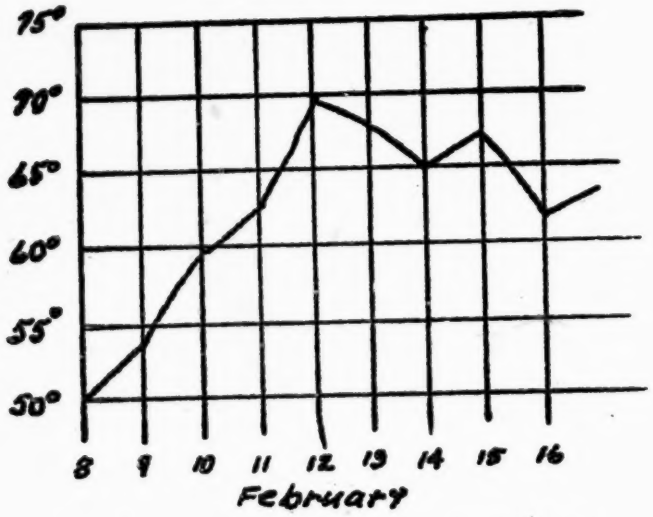
Combine the study of time-zones with the social studies unit.



Temperature

The centigrade thermometer, which is calibrated simply and is easier to read than the Fahrenheit scale, is used in laboratory work as well as widely outside the U.S.A. Don't have the children memorize the three steps for changing from one scale to another. This conversion table is just to give the students arithmetic practice as they are studying conversion. 1:291

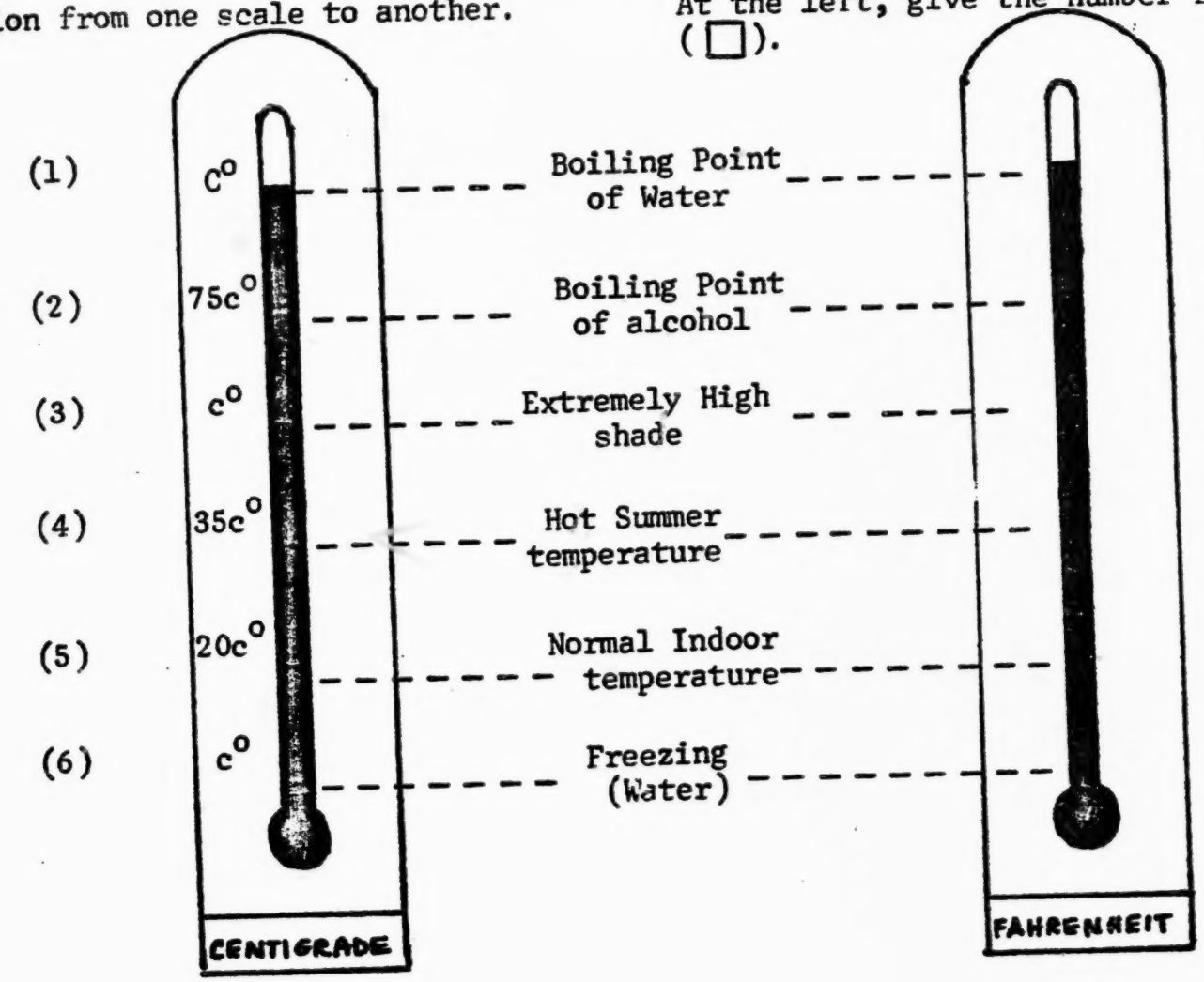
(A)



Have the class record temperatures over a period of a week or two and keep a graph on the changes. They can also devise a line graph showing the temperature at each hour of the day (see Figure A). Then they can find the average of the hourly temperatures.

Conversion from one scale to another.

At the left, give the number for each blank square (□).



(1) $100^{o}C$

(2) $167^{o}F$

(3) $55^{o}C$

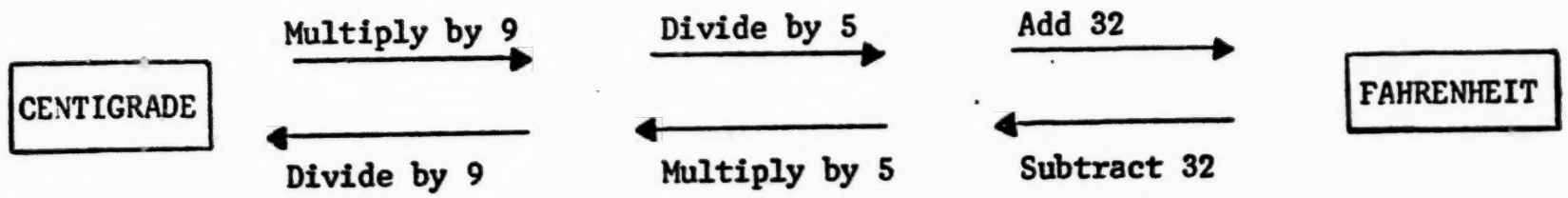
(4) $95^{o}F$

(5) $68^{o}F$

(6) $0^{o}C$

Have both types of thermometers available in the classroom and compare daily temperatures.

To convert from Centigrade to Fahrenheit, the following rules apply:



The children need not understand this, but only use it in their computations.

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ED026269

O.C.S.E.I.P. SYLLABUS

Grade 6

SE006 069

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OFFICE OF EDUCATION

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NUMBERS AND NUMERATION

Grade 6

PREFACE

The Orange County Science Education Improvement Program (O.C.S.E.I.P.) is sponsored by the National Science Foundation and hosted by U.C. Irvine. It is a cooperative venture undertaken by the University of California, Irvine, California State College at Fullerton, the Orange County Schools Office and local school districts throughout Orange County. This syllabus was written by O.C.S.E.I.P. to help teachers teach the best aspects of recent mathematics programs. It is not meant to be another textbook for a new program. Instead, it is meant to be a sharing and synthesis of effective teaching methods. The outline of topics is a minimum coverage which is common to all schools in Orange County. Topics adequately covered in the majority of texts in use are given a minimum treatment in the syllabus.

The first draft of this syllabus was written during an 8 week session at University of California, Irvine during the summer of 1966 by:

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We wish to thank all the participants in this program for their hard work and fine cooperation.

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Grade 6

NUMBERS AND NUMERATION

Reading and writing numerals through 9,999,999,999.999999

Give students duplicated reference tables such as the one following to use as an aid in reading and writing large numbers. Such tables can be placed in student notebooks to use throughout the school year as needed.

Have students write numerals on chalkboard for the class to:

1. Add commas in appropriate places
2. Write out in words (one hundred twenty-one).
3. Read aloud
4. Give place value names of each digit in the numeral
5. Give period or family names of each group of digits (thousands, millions, billions, etc.).

Have students dictate numbers for class members to write at their desks. Discuss how individual students decide where the commas are inserted when writing from dictation. (Most successful students write the comma upon hearing any family or period name.)

The importance of family names can be easily pointed out by having students read or write large numbers made of identical numerals such as: 333,333,333; 777,777,777,777. Point out that in 333,333,333, the first name of each group of three units is the same (three-hundred thirty-three). To show that they have different values, we must give them their family names.

Insure that the class has mastered the reading and writing of whole numbers before going on to decimals. The exponential form for each place value need not be discussed at this point though many are able to understand that the value of any given place is ten times greater than the value of the place to its right. e.g. millions is 10 x hundred-thousands, ten thousands is 10 x thousands, hundreds is 10 x tens, etc.

10^9	$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	→	billions
		,	comma
10^8	$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	→o	hundred-millions
10^7	$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	→oo	ten-millions
10^6	$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	→ooo	millions
		, , ,	comma
10^5	$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	→oooo	hundred-thousands
10^4	$10 \cdot 10 \cdot 10 \cdot 10$	→ooooo	ten-thousands
10^3	$10 \cdot 10 \cdot 10$	→oooooo	thousands
		, , , , , ,	comma
10^2	$10 \cdot 10$	→ooooooo	hundreds
10^1	10	→oooooooo	tens
1	1	ooooo→ooooooooo	ones
		. . . . .	decimal point
$(\frac{1}{10})^1$	$\frac{1}{10}$	ooooo→ooooooooo	tenths
$(\frac{1}{10})^2$	$\frac{1}{10} \cdot \frac{1}{10}$	oooo→oooooooooo	hundredths
$(\frac{1}{10})^3$	$\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$	ooo→ooooooooooo	thousandths
$(\frac{1}{10})^4$	$\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$	oo→oooooooooooo	ten-thousandths
$(\frac{1}{10})^5$	$\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$	o→ooooooooooooo	hundred-thousandths
$(\frac{1}{10})^6$	$\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$	→oooooooooooooo	millionths

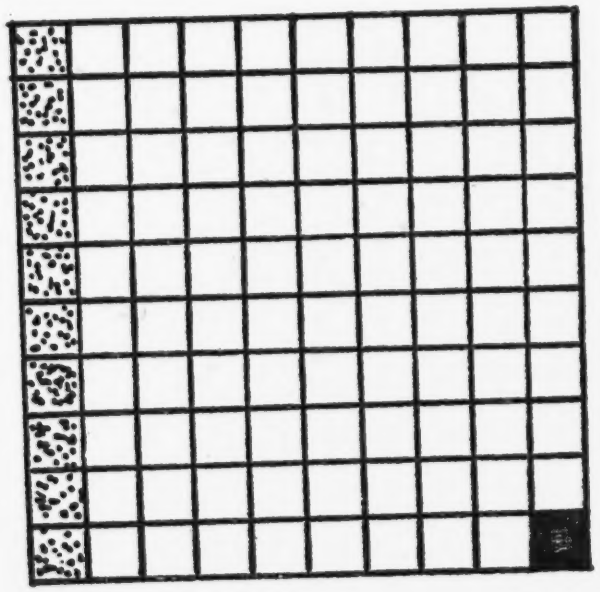
Reading and writing decimal numerals through .999999.

Pre-requisite - an understanding of place value of whole numbers and common fractions. 1:46-49

Materials: a grid, decimal place value chart and a number line to give clarity and meaning to reading and writing decimals.

Read decimals as if whole numbers, and give the decimal place value name of the digit farthest to the right; thus .204 (204--two hundred four). The 4 being in thousandths place, is farthest to the right. So read two hundred four thousandths.

Caution: The decimal point (.) is read "and" if there are digits to the left of the decimal point; thus 3.04 is read three and four hundredths. If the students are allowed to use "and" for any other reason than the decimal point, confusion results.



■ = 1 of 100 = $\frac{1}{100}$ = .01

▦ = 10 of 100 = $\frac{1}{10}$ = .1

$$\left(\frac{1}{10}\right)^1 = \frac{1}{10}$$

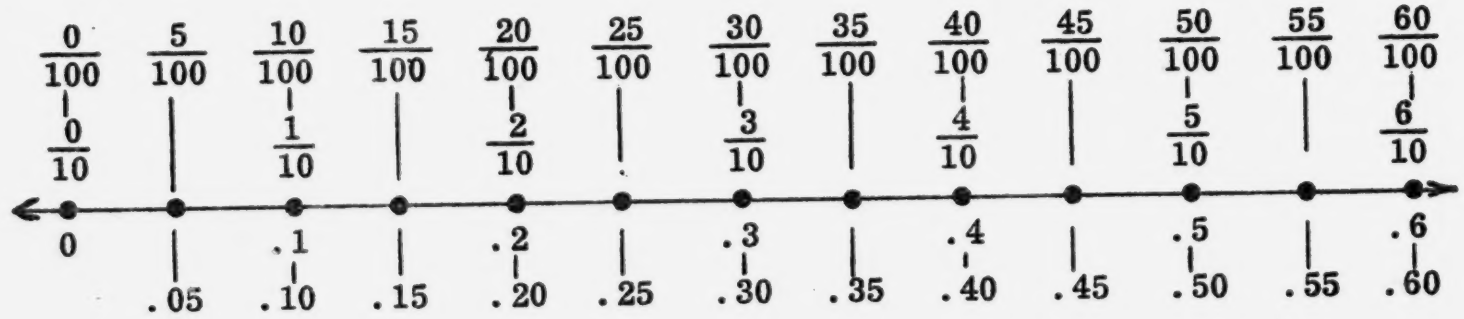
$$\left(\frac{1}{10}\right)^2 = \frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$$

$$\left(\frac{1}{10}\right)^3 = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{1000}$$

$$\left(\frac{1}{10}\right)^4 = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{10,000}$$

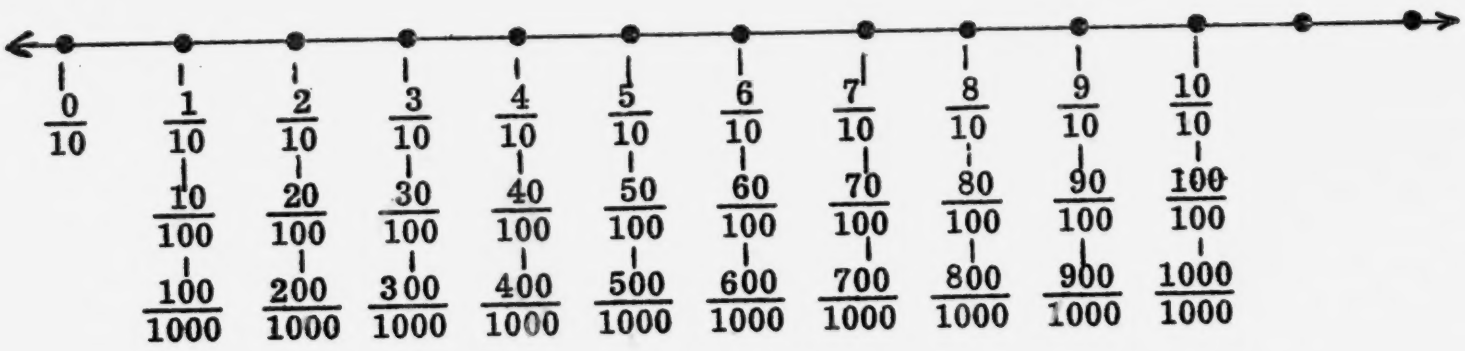
$$\left(\frac{1}{10}\right)^5 = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{100,000}$$

$$\left(\frac{1}{10}\right)^6 = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{1,000,000}$$



What decimal has the value of $60 + 3 + \frac{7}{10} + \frac{8}{100}$? We read 63.78 as "sixty-three and seventy-eight hundredths". Look at the number line.

$\frac{7}{10} = \frac{?}{100}$ $\quad \frac{70}{100} + \frac{8}{100} = \frac{?}{100}$.



$63.78 = 60 + 3 + \frac{7}{10} + \frac{6}{100}$

$= 60 + 3 + \frac{70}{100} + \frac{8}{100}$

$= 60 + 3 + \frac{78}{100}$

$= 63\frac{78}{100}$

$1.275 = 1 + \frac{2}{10} + \frac{7}{100} + \frac{5}{1000}$

$= 1 + \frac{200}{1000} + \frac{70}{1000} + \frac{5}{1000}$

$= 1 + \frac{275}{1000}$

$= 1\frac{275}{1000}$

The function of zero in decimal numerals.

$. 05	$.60	$1.45
no tenths 5 hundredths	6 tenths 0 hundreths	one 4 tenths 5 hundredths

.5
.50
.500
.5000

.05
.005
.0005
.0500
.0050
.0055

"Write the numerals that show five cents as a part of a dollar."

"What does the 5 stand for in the numerals $.05?"

"Yes, it stands for 5 hundredths of a dollar (or five pennies)."

"What does the zero stand for?" (no dimes) "We shall let "dimes" stand for the tenths and the pennies or cents stand for hundredths."

Give students a worksheet you have prepared that requires students to name the place value of various numerals including zero.

Be sure the examples on the first part of the lesson are less than one dollar.

Now, explain that dollars could be thought of as whole numbers. In the example, $1.45, the $1 means one dollar, the 4 means 4 dimes (tenths),and the 5 means 5 cents (hundredths).

When the class understands the meaning of the zero as it applies to money, extend concepts to decimal fractions with three and four places. Ask students to write as many decimal fractions as they can using zero and the numeral five. Establish rules for the game such as:

1. Only 1, 2, 3, and 4-place fractions can be used.
2. Many names for the same fraction can be used (.5, .50, .500, etc.).
3. Each fraction must contain at least one zero.

Rules can be varied. Results can be read aloud or placed on chalkboard. Students can be asked to group fractions that name the same number (equivalent fractions) on the chalkboard for discussion. Interesting discussions arise when students are asked to write a number dictated to them by the teacher or a student.

Reading and writing numerals in exponential notation (powers of ten).

BASE TEN

ten thousands	thousands	hundreds	tens	ones
10,000	1000	100	10	1
10x10x10x10	10x10x10	10x10	10	1
10^4	10^3	10^2	10	1
$(base)^4$	$(base)^3$	$(base)^2$	base	ones
base x base x base x base	base x base x base	base x base	base	ones

BASE SIX

6^4	6^3	6^2	6	1

Prerequisite: A thorough understanding of place value in base ten.

With proper understanding of place value, grouping, and expanded notations, the use of exponential notation becomes a practical value. The understanding of exponential notation in base ten leads to a more complete understanding of place value in all bases.

Writing the numeral 32.215 in expanded notation is cumbersome, but with the use of exponential notation, it becomes simplified. Exponents save us time and space in reading and writing of very small and very large numerals (the speed of light is 5.88×10^{12}).

To demonstrate for the pupils, write on the chalkboard:

$30,000 + 2000 + 200 + 10 + 5$

$(3 \times 10,000) + (2 \times 1000) + (2 \times 100) + (1 \times 10) + (5 \times 1)$

$3 \times (10 \times 10 \times 10 \times 10) + 2 \times (10 \times 10 \times 10) + 2 \times (10 \times 10) + 1 \times (10) + 5 \times (1)$

$3 \times (10)^4 + 2 \times (10)^3 + 2 \times (10)^2 + 1 \times (10) + 5 \times (1)$

Base six numerals can provide an interesting way to view

place value and exponential notation. Write on the chalkboard:

$32215_{(6)}$ "Three two two one five, base six"

$30{,}000_{(6)} + 2000_{(6)} + 200_{(6)} + 10_{(6)} + 5_{(6)}$

$3\times(10{,}000)_{(6)} + 2\times(1000)_{(6)} + 2\times(100)_{(6)} + 1\times(10)_{(6)} + 5\times(1)_{(6)}$

$3\times(10\times10\times10\times10)_{(6)} + 2\times(10\times10\times10)_{(6)} + 2\times(10\times10)_{(6)} + 1\times(10)_{(6)} + 5\times(1)_{(6)}$

Note that $10_{(6)} = 6_{(10)}$ therefore:

$3\times(6\times6\times6\times6) + 2\times(6\times6\times6) + 2\times(6\times6) + 1\times(6) + 1\times(1)$

$3\times(6^4) + 2\times(6^3) + 2\times(6^2) + 1\times(6) + 1\times(1)$

Students should be encouraged to look for comparisons between the use of exponents in base ten and base six. Have students study the example in base ten and see if they can explain the development of the pattern. Lead class to generalize that the exponent refers to the number of times that ten is used as a factor. e.g. $10{,}000 = 10 \times 10 \times 10 \times 10 = 10^4$. Sharp students will often see that the exponent also tells the number of zeros in the original number. Thus, 100,000 has five zeros and in exponential form is expressed 10^5.

Give students many opportunities to both read and write exponents.

10^2 ten to the second power

10^3 ten to the third power

10^4 ten to the fourth power

Counting and writing in base 6.

The place value of a base can be shown by having 3 or 4 students stand at the front of the class and "count" using only the fingers of one hand on each

student. In base 6, the first student (on the right) would show 1, 2, 3, 4, and 5 respectively. At 6, the sixes place student would show 1, and the ones-place student, zero (nothing). After counting to 100 (2 thirty-sixes, 4 sixes, and two ones). Have students in their seats try to give them numbers that are hard to do. Exchange pupils in the "counting crew," and try still more numbers.

Reverse the game having the counting crew portray a number, and let members of the class name the number or convert it back to base 10.

When students can easily show base six numbers on their fingers, continue the game by having a third crew write the number on the chalkboard or say the number aloud. Students should easily learn to write and count aloud to 100 base six.

Follow-up material might include:

SET	WRITE NUMERAL AS:	READ NUMERAL AS:
●	1	one
●●	2	two
●●●	3	three
●●●●	---	four
----	5	----
●●●●●●	---	one zero
	11	one one
	---	one two
	13	
	14	
	15	
	20	
	20	

Problem Answers

7¢ = 1 sixels, 1 El or 11 L
12¢ = 2 sixels, or 20 L
15¢ = 2 sixels, 3 Els, or 23 L
$2.95 = 1 buckniks
2 thirty-sixels
1 sixels +
1 Els or $1.211

Note: "20 " is read "Two-Zero Els."
"$5.023" is read "Five buckniks, and Two-sixel Three-Els.

Students may grasp the idea of place value in base six more readily if it is handled as a monetary system, as follows:

"In the land of Sixia, their money is different from ours. The El is like our penny, worth one cent. However, they use a six-cent coin (called a sixel); a thirtysixel (six sixels); a Bucknik (six thirty-sixels) and a sixbucknik, (six buckniks). Using the smallest number of bills and coins, change the following amounts from our money to that used in Sixia." (Use dittoed sheets, and perhaps dittoed coins and bills. Use construction paper or shirt stiffening cardboard upon which to ditto the money)

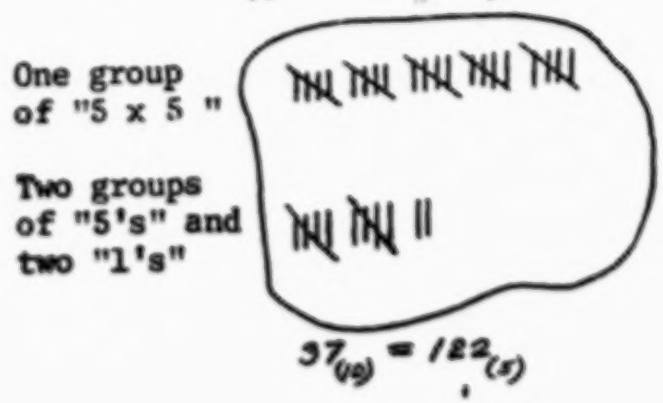
A "siximal point" may be invented to separate buckniks from thirty-sixes.

Computing in base 5 and base 6.

counting by tally marks

tens	ones	fives	ones
2	1	4	1

$_{10}$ = $_{5}$



Lead up to operations on bases 5 and 6 by having pupils count to 100

The concepts you may wish to review are:

a. Each thing counted is represented by one mark.
b. The size of the "groups" is based upon the number chosen as the "base."
c. The base number is written 10 in any base.
d. The place value is found by using one for the right-hand place; the base number for the second place to the left; base times base for third place, etc.

Addition Table
Base Five:

+	0	1	2	3	4
0	0	1	2	3	4
1	1	3	3	4	10
2	2	4	4	10	11
3	3	10	10	11	12
4	4	11	11	12	13

Multiplication Table
Base Five:

x	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	11	13
3	0	3	11	14	22
4	0	4	13	22	31

Work in bases other than 10 is primarily a technique for teaching the principles of the decimal system. Therefore, we need not try to develop computational efficiency. Pupils should develop and use tables of addition and multiplication facts.

$$\begin{array}{r} 2 \text{ five} \\ +\ 1 \text{ five} \\ \hline (3 \text{ five}) \end{array} \qquad \begin{array}{r} 2 \text{ five} \\ 3 \text{ five} \\ \hline (10 \text{ five}) \end{array}$$

$$\begin{array}{r} 22 \text{ five} \\ +\ 24 \text{ five} \\ \hline (101 \text{ five}) \end{array} \qquad \begin{array}{r} 23 \text{ five} \\ \times\ 14 \text{ five} \\ \hline 202 \\ 23 \\ \hline (432 \text{ five}) \end{array}$$

$$\begin{array}{r} 3\ 10 \\ \cancel{4}13 \\ -\ 24 \\ \hline 334 \text{ five} \end{array} \qquad \begin{array}{r} (3 \text{ r } 2_5) \\ 13_5 \overline{)\,101_5} \end{array}$$

Use a dittoed worksheet to give pupils practice in base 5 work. Any number operation can be performed and many parallels to base 10 can be drawn. Develop from simple to more difficult--from adding without regrouping to multiplication and division. Most pupils should use their "tables," and not try to memorize base 5.

Rounding to the nearer thousand, ten-thousand, and hundred-thousand.

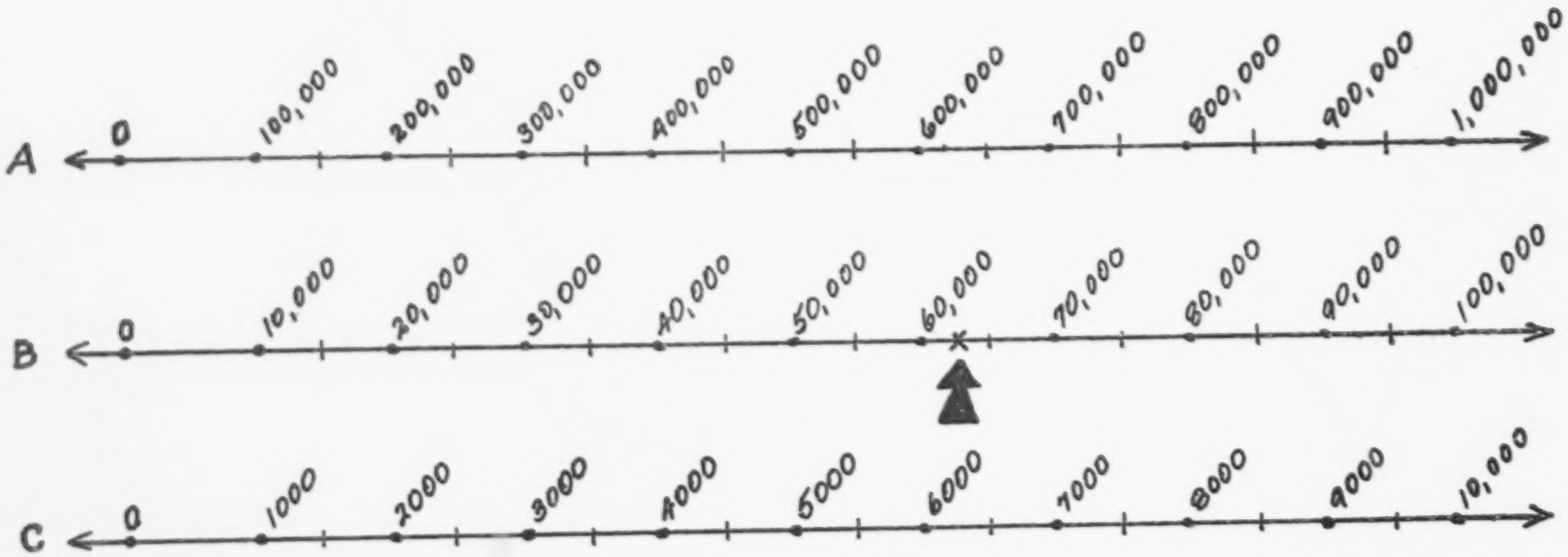
Pre-requisite: an understanding of place value whole numbers.

Materials: numberlines and a rounding chart.

Take a numeral such as 63,000. Refer to the number line "B," point x, and note that it is nearer 60,000; therefore, 63,000 rounded to the nearer ten thousand is 60,000. After ample practice such as this, complete the rounding chart. Discover the pattern in rounding which is--if the digit to the right of the place being rounded is 5, 6, 7, 8, or 9, "round up." If the digit to the right is 0, 1, 2, 3, or 4, "round down." Discover that all digits to the right of the place being rounded becomes zeros.

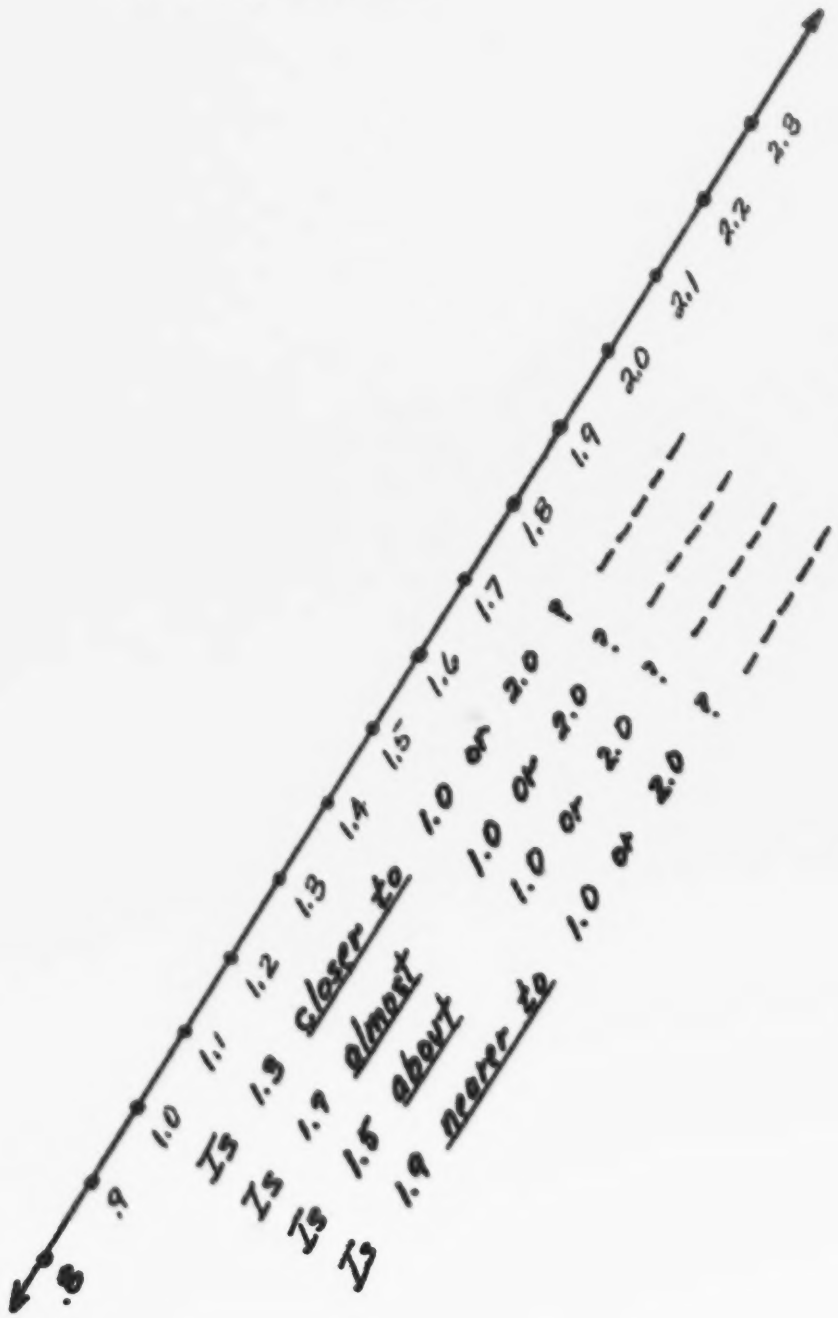
Examples: to nearest thousands.

72,560 - 73,000 "up"
72,430 - 72,000 "down"



	NUMERALS TO BE ROUNDED	TO NEARER THOUSAND	TO NEARER TEN-THOUSAND	TO NEARER HUNDRED THOUSAND
a.	92,331	92,000	------	------
b.	106,212	------	110,000	------
c.	390,514	------	------	400,000
d.	57,304	------	------	------
e.	726,999	------	------	------
f.	880,517	------	------	------

Rounding decimals



Provide students with sheets of dittoed number lines. Have each mark a number line from 1.0 to 2.0 showing only tenths (1.1, 1.2, 1.3, etc.). Ask students to "round off" dictated numbers to the nearer whole number. "Find 1.3 on your number line. Is 1.3 closer to 1.0 or 2.0? 1.3 rounded to the nearer whole number is ....? (one) Continue with other number lines showing tenths only (17.0, 17.1, 17.2, 17.3, etc.)

Extend activity to include number lines showing hundredths (.01, .02, .03, .04, etc.). Ask students to round given numbers to the nearer tenth. "Find .07 on your number line. Is .07 nearer to .00 or .10? Rounded to the nearer tenth, .07 is closer to .10 (one tenth).

After rounding off to the nearer thousandth by inspection of number lines, lead students to discuss how they might round off a four place decimal to the nearer tenth. Let the class explore all suggested methods and attempt to prove them on chalkboard. Students should eventually see that only the right hand neighbor can affect the rounding of the place value position.

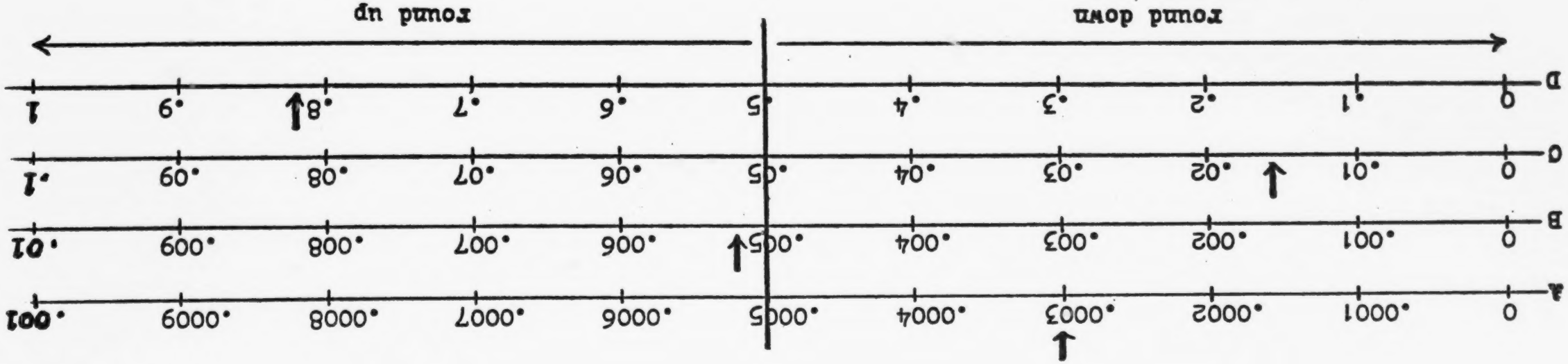
Provide students with rounding chart and number lines.

Given a number such as 0.8153 to be rounded to the nearer thousandth, the first step is to look at the digit in the ten-thousandths place. Find the 3 ten-thousandths place (.0003) on number line A (see arrow). Since the 3 ten-thousandths falls below the mid-point of the number line (.0005) and is nearer to 0 than to .001, the digit in the thousandths column will remain 5 and the digit in the ten-thousandths column will be changed.

If the number 0.8153 is to be rounded to the nearer hundredth, the first step is to look at the digit to the right of the hundredths place. Find the 53 ten-thousandths (.0053) on number line B (see arrow). Since 53 ten-thousandths (.0053) falls above the mid-point of the number line (.0005) and is nearer to 1 than to 0, the digit in the hundredths place will be increased by 1 and the digit to the right will be changed to zeros.

If .08153 is to be rounded to the nearer tenth then all the digits to the right should be used and found on the number line C. Find 153 ten-thousandths (.0153) on the number line. Since it is closer to zero than to 0.1 (one tenth) the digit in the tenths place will remain the same and all digits to the right will be changed to zero.

If the number 0.8153 is to be rounded to the nearer whole number, then find 8153 ten-thousandths (.8153) on number line D. Since the number falls on the number lin e closer to the 1 than to zero the numeral in the ones column should be increased by one and all the digits to the right of it will be changed to zeros.



ten-thousandths	thousandths	hundredths	tenths	decimal point	ones		
	3	5	1	8	.	0	original number to be rounded off ← A
	0	5	1	8	.	0	rounded to the nearest thousandth ← B
		0	2	8	.	0	rounded to the nearest hundredth ← C
			0	8	.	0	rounded to the nearest tenth ← D
				0	.	1	rounded to the nearest one

Exponential notation involving "greater than," "less than," and "equal to."

When writing larger numbers, it is often simpler to use the exponent than to write out the whole numeral. It is easy to see that one numeral is greater than or less than another one when the exponents are compared.

The signs (>), (<), and (=) can be used with effectiveness when comparing numbers. We know the sign (=) means "is equal to" and we have a special sign $<$ that means "is less than." The point of the sign is aimed at the smaller number.

$$300 = 3 \times 100$$
$$= 3 \times (10 \times 10)$$
$$= 3 \times 10^2$$

The example following is read "five to the second power is greater than two to the fifth power." "Is the sentence true?" (no) $5^2 > 2^5$

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2$$
$$= 32$$

$$5^2 = 5 \times 5$$
$$= 25$$

Which of these sentences is true?:

(a) $7_5 \times 30 < 242$ (b) $8_3 \times 30_4 = 242$

(c) $3^5 < 15^2$ (d) $6^3 > 4^4$

Prime and composite numbers

Familiarity with primes and com posite numbers will help the pupil in reduction of fractions, quick mental division, and finding least common denominators. A prime number is any whole number that has exactly two different factors (namely itself and one). Mathematicians agree that one is neither prime nor composite. To "discover" the primes less than 100, have students construct Eratosthene's Sieve. Give each pupil one sheet of graph paper with one-inch squares. Have the pupils fill the squares with the counting numbers to 100.

Two is the first prime number.

To start the "Sieve" have the pupils cross out every multiple of two beginning with four. Three is the next prime number, so have the pupils cross out each

multiple of three starting with six. Continue in a similar manner taking each unmarked number in turn and crossing out its multiples. Continue until these prime numbers are left: 2, 3, 4, 7, 11, 13, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 91, 97. Pupils should memorize the primes less than 50.

The numerals crossed out are called composite numbers. Eratosthene actually wrote the numerals on a piece of parchment and instead of crossing out the composite numeral, he cut them out, leaving holes in the paper. Hence, the name Eratosthene's Sieve.

Roman numerals.

If pupils and teacher are of this type wit, cartoons may be shown using Roman numerals. Two such examples are described below:

1. A golfer, wearing clerical robes, is about to tee-off on a golf ball. In the balloon is written this exclamation: IV!

2. A company of Roman soldiers marches along a road chanting, "I, II, III, IV, etc."

```
  MCMLVI   (year)
+    VI   (since)
--------
MCMLXII   (now)
```

Write a date on the baord. "What year was this motion picture filmed? "How many years since then have passed? How do you write that? Add, "What is the year now?"

```
   XVI
 (x) X
 -----
   CLX
 ↗  ↑  ↖
(X·X) (X·V) (X·1)
```

Write a multiplication example on the chalk board. Have a pupil call out the steps "(ten times one is ten," and write "X." "Ten times five is fifty." and write "L," etc.

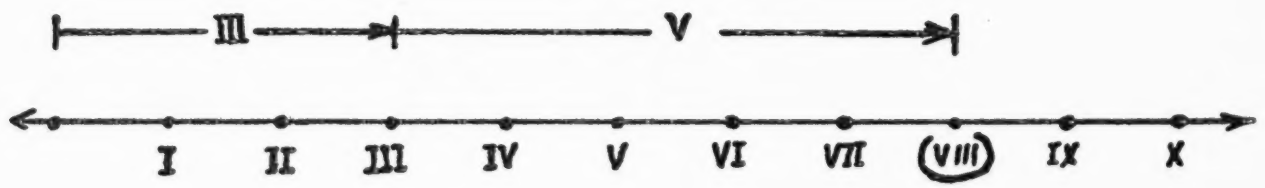
"Fill in the missing numerals:

(1) I, II, III, ___, ___, VI, ___, ____, ____, ____.
(2) LVI, LVII, ____, ____, LX, ____.
(3) XC, XCI, ____, ____, ____, ____.
(4) XCVII, ____, ____, ____, . . . (etc)."

Make a counting chart of Roman numerals. What patterns do you see? (Make one 5 squares wide and another 10 squares wide).

Use a number line if helpful to add and subtract.

I	II	III	IV	V
VI	VII	VIII	IX	X
XI	XII	XIII	XIV	XV
XVI	XVII	XVIII	XIX	XX
XXI	XXII	XXIII	XXIV	XXV
				XXX
				XV
				XL



III + V = ___?___

For pupils who want the information, another device used in the Roman system is a short bar above the numeral to show multiplication by one thousand. This bar is called a vinculum.

$\overline{V}$ = 5,000
$\overline{X}$ = 10,000

Comparison of Roman and Decimal Systems.

The Roman system depends on four principles of combining numerals:

1. Addition: XVI = X + V + I = 16
2. Subtraction: fours and nines only
3. Repetition: "like symbols" are grouped (when they appear together) XXX = 30, CCC = 300.
4. Multiplication = the vinculum indicates multiplication by 1,000. V = 5; $\overline{V}$ = 5,000.

The Hindu-Arabic System uses only two principles:

1. Place value: Digits have both absolute value ("face value") and positional value ("place value"). Because of this, zero must be used to "hold" a place.

The Roman system is not base 10, but quinary (base five) and binary (base two). That is, five ones equal one five, or V; two V's equal one X; five X's equal one L, two L's equal one C; five C's equal one D; two D's equal one M.

The system's symbols alternate between two and five.

ADDITION AND SUBTRACTION

Grade 6

Addition of six-place numerals.

Teacher - Refer back to the place value chart page 2, Chap. 1 and the students' copy of this page.

Example I: Addition without carrying plus level I expanded notation.

$$\begin{array}{rcl} 625,374 & = & 600,000 + 20,000 + 5,000 + 300 + 70 + 4 \\ +172,613 & = & +100,000 + 70,000 + 2,000 + 600 + 10 + 3 \\ \hline 797,987 & & 700,000 + 90,000 + 7,000 + 900 + 80 + 7 \end{array}$$

Example II: Addition with carrying plus level I expanded notation.

$$\begin{array}{rcl} 496,847 & = & 400,000 + 90,000 + 6,000 + 800 + 40 + 7 \\ +328,364 & = & +300,000 + 20,000 + 8,000 + 300 + 60 + 4 \end{array}$$

When adding the conventional or standard way, you are not carrying a 1 to the next column to the left. Therefore, when carrying to the left column you are actually carrying one times the value of that column. For example, let's follow the addition of Example II in expanded notation. When adding in the one's column we find a sum of 11 or 10 + 1. We write the 1 in the one's column and carry the 10 to the ten's column which is to the immediate left of the one's column.

$$\begin{array}{rcl} {}^{1}47 & = & {}^{10}40 + 7 \\ +64 & = & +60 + 4 \\ \hline 1 & & \quad\quad 1 \end{array}$$

Next add the figures in the ten's column, 40 + 60 + 10, our sum is 110 or 100 + 10.

$$\begin{array}{rcl} {}^{11}847 & = & {}^{100}800 + {}^{10}40 + 7 \\ +364 & = & +300 + 60 + 4 \\ \hline 11 & & \quad\quad 10 + 1 \end{array}$$

We record the 10 in the ten's column and carry the 100 to the hundred's column, which is to the immediate left of the ten's column.

Now add the digits in the hundred's column, 800 + 300 + 100, we find the sum to be 1,200 or 1,000 + 200.

$$\begin{array}{rcl} \overset{11}{6,847} & = & \overset{1,000}{6,000} + \overset{100}{800} + \overset{10}{40} + 7 \\ +8,364 & = & +8,000 + 300 + 60 + 4 \\ \hline 211 & = & 200 + 10 + 1 \end{array}$$

To proceed as before, we record the 200 in the hundred's column and carry the 1,000 to the left in the thousand's column. To continue, add the digits in the thousand's column, 6,000 + 8,000 + 1,000, our sum is 15,000 or 10,000 + 5,000.

$$\begin{array}{rcl} \overset{1\ 11}{96,847} & = & \overset{10,000}{90,000} + \overset{1,000}{6,000} + \overset{100}{800} + \overset{10}{40} + 7 \\ +28,364 & = & +20,000 + 8,000 + 300 + 60 + 4 \\ \hline 5,364 & = & 5,000 + 200 + 10 + 1 \end{array}$$

Write the 5,000 in the thousand's column and carry the 10,000 to the ten-thousand's column. Next, add the digits in the ten-thousand's column, 90,000 + 20,000 + 10,000, the sum is 120,000 or 100,000 + 20,000. Write the 20,000 in the ten-thousand's column and carry the 100,000 to the next column to the left, the hundred-thousands column.

$$\begin{array}{rcl} \overset{11\ 11}{496,847} & = & \overset{100,000}{400,000} + \overset{10,000}{90,000} + \overset{1,000}{6,000} + \overset{100}{800} + \overset{10}{40} + 7 \\ +328,364 & = & 300,000 + 20,000 + 8,000 + 300 + 60 + 4 \\ \hline 25,211 & & 20,000 + 5,000 + 200 + 10 + 1 \end{array}$$

To complete the problem, add the digits in the hundred-thousand's column, 400,000 + 300,000 + 100,000, the sum is 800,000, which is recorded in that column since there isn't any million to carry to the next column.

$$\begin{array}{rcl} 496,847 & = & \overset{100,000}{400,000} + \overset{10,000}{90,000} + \overset{1,000}{6,000} + \overset{100}{800} + \overset{10}{40} + 7 \\ +328,364 & = & +300,000 + 20,000 + 8,000 + 300 + 60 + 4 \\ \hline 825,211 & = & 800,000 + 20,000 + 5,000 + 200 + 10 + 1 \end{array}$$

The idea involved in this expanded notation is to illustrate to the students that in "carrying" (or regrouping), he isn't

always carrying one (1), but he is carrying 1 times the place value of the column to which he is carrying.

Subtraction of six-place numerals.

Teacher - Refer back to the place value chart page 2, Chap. 1 and the students' copy of this page.

Example I:

$$\begin{array}{rcl} 749,568 & = & 700,000 + 40,000 + 9,000 + 500 + 60 + 8 \\ -526,143 & = & -500,000 + 20,000 + 6,000 + 100 + 40 + 3 \\ \hline 223,425 & & 200,000 + 20,000 + 3,000 + 400 + 20 + 5 \end{array}$$

Example II:

$$\begin{array}{rcl} 934,750 & = & 900,000 + 30,000 + 4,000 + 700 + 50 + 0 \\ -757,986 & = & -700,000 + 50,000 + 7,000 + 900 + 80 + 6 \\ \hline 176,764 & & 100,000 + 70,000 + 6,000 + 700 + 60 + 4 \end{array}$$

To complete the operation of subtraction on the above six-place problem, one would start with the one's column. You soon discover that 6 can not be subtracted from 0; therefore, you need to "borrow" from the ten's place. You are not borrowing a 1 from the ten's place, but rather 10, which decreases the value of the ten's column from 50 or five tens to 40 or four tens.

$$\begin{array}{rcl} 50 & = & \overset{40}{\cancel{50}} + \overset{10}{\cancel{0}} \\ -86 & = & -80 + 6 \\ \hline & & 4 \end{array}$$

Moving next to the ten's column, 80 cannot be subtracted from 40; therefore, there is a need to "borrow" from the hundred's column. Regrouping from the hundred's column, 700 is decreased to 600

and the borrowed 100 is added to the 40 in the ten's column to give a value of 140.

$$\begin{array}{rl} & \overset{600}{\cancel{700}} + \overset{140}{\cancel{50}} + \overset{10}{\cancel{8}} \\ 750 = & \\ \underline{-986} = & \underline{-900 + 80 + 6} \\ & 60 + 4 \end{array}$$

Now the 80 can be subtracted from the 140 to leave a remainder of 60 in the ten's column.

Turning our attention next to the subtraction of the hundred's column, we find 900 cannot be taken from 600. To complete this operation, we need to "borrow" from the thousand's column.

$$\begin{array}{rl} & \overset{3,000}{\cancel{4,000}} + \overset{1,600}{\cancel{700}} + \overset{140}{\cancel{50}} + \overset{10}{\cancel{8}} \\ 4,750 = & \\ \underline{-7,986} = & \underline{-7,000 + 900 + 80 + 6} \\ & 700 + 60 + 4 \end{array}$$

Regrouping from the thousand's column, the 4,000 is decreased by 1,000 to 3,000. The 1,000 that was borrowed is added to the hundred's column to give it the value of 1,600. Now 900 can be subtracted from the 1,600 to leave a remainder of 700 in the hundred's column.

To subtract in the thousand's column, we find 7,000 cannot be taken from 3,000. In order to complete this operation we need to "borrow" from the ten-thousand's column.

$$\begin{array}{rl} & \overset{20,000}{\cancel{30,000}} + \overset{13,000}{\cancel{4,000}} + \overset{1,600}{\cancel{700}} + \overset{140}{\cancel{50}} + \overset{10}{\cancel{8}} \\ 34,750 = & \\ \underline{-57,986} = & \underline{-50,000 + 7,000 + 900 + 80 + 6} \\ & 6,000 + 700 + 60 + 4 \end{array}$$

Regrouping from the ten-thousand's column, the 30,000 is reduced by 10,000 to 20,000. The 10,000 that was borrowed is added to the 4,000 in the

thousand's column to give it a value of 13,000. Now the 7,000 can be subtracted from the 13,000 to give a remainder of 6,000 in the thousand's column.

To continue, next we have to subtract in the ten-thousand's column. However, we find the 50,000 cannot be taken from the 20,000. We need to "borrow" from the hundred-thousand's column.

$$\begin{array}{rcl} & & \scriptstyle 800{,}000 \quad 120{,}000 \quad 13{,}000 \quad 1{,}600 \quad 140 \quad 10 \\ 934{,}750 & = & \cancel{900{,}000} + \cancel{30{,}000} + \cancel{4{,}000} + \cancel{700} + \cancel{50} + \cancel{0} \\ \underline{-757{,}986} & = & \underline{-700{,}000 + 50{,}000 + 7{,}000 + 900 + 80 + 6} \\ & & 70{,}000 + 6{,}000 + 700 + 60 + 4 \end{array}$$

To regroup, we borrow 100,000 which reduces the value of the hundred-thousand's column to 800,000. Add the borrowed 100,000 to the value of the ten-thousand's column. The value of the ten-thousand's column is now 120,000, from which we can subtract the 50,000 leaving a remainder of 70,000.

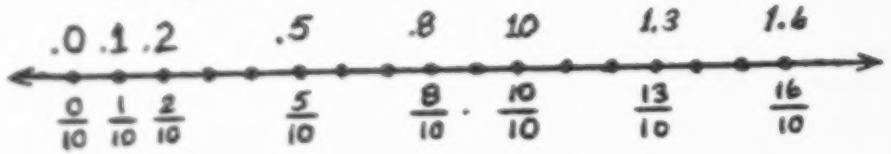
To complete the problem, we need to subtract in the hundred-thousand's column. Since the minuend is 800,000 and larger than the subtrahend 700,000, the operation of subtraction can thus be carried out directly, leaving a remainder of 100,000.

$$\begin{array}{rcl} & & \scriptstyle 800{,}000 \quad 120{,}000 \quad 13{,}000 \quad 1{,}600 \quad 140 \quad 10 \\ 934{,}750 & = & \cancel{900{,}000} + \cancel{30{,}000} + \cancel{4{,}000} + \cancel{700} + \cancel{50} + \cancel{0} \\ \underline{-757{,}986} & = & \underline{-700{,}000 + 50{,}000 + 7{,}000 + 900 + 80 + 6} \\ & & 100{,}000 + 70{,}000 + 6{,}000 + 700 + 60 + 4 \end{array}$$

Addition and subtraction with decimal fractions.

Example A: The number line is very good for demonstrating the renaming of common fractions as decimal fractions. Give the pupils many opportunities to practice with like

Example A:



decimals and common fractions in denominations of tenths, hundredths, and thousandths. (See example B)

Example B:

$$\frac{23}{100} = 0.23$$

$$+\frac{14}{100} = 0.14$$

$$\frac{37}{100} = 0.37$$

Example C:

With an understanding of addition of like fractions, the pupils can be given practice in addition of unlike fractions with denominators of tenths, hundredths, and/or thousandths (see Example C).

$$\frac{234}{1000} = \frac{2340}{10000} = 0.2340$$

$$\frac{1742}{10000} = \frac{1742}{10000} = 0.1742$$

$$\frac{11}{100} = \frac{1100}{10000} = 0.1100$$

Example D:

$$\begin{array}{r} \frac{2}{100} = \frac{200}{10000} = 0.0200 \\ -\ \frac{11}{10000} = \frac{11}{10000} = 0.0011 \\ \hline \frac{189}{10000} = 0.0189 \end{array}$$

With plenty of practice in subtraction of like common and decimal fractions such as $\frac{4}{10} - \frac{2}{10} = \frac{2}{10}$ and .04 - 0.2 = 0.2, students move to those of Example D. Here the pupil should discover the use of the zero to keep the decimal points in vertical line.

Mixed numerals.

Example A:

1. $2\frac{1}{4} + 3\frac{3}{4} = \square$
2. $(2 + \frac{1}{4}) + (3 + \frac{3}{4}) = \square$
3. $(2 + \frac{1}{4} + 3) + \frac{3}{4} = \square$

Example A: Step 1 is the statement of the problem. Step 2 is renaming each mixed numeral. Step 3 uses the associative property of addition . Step 4 uses the commutative property of addition, that is changing the order of the $\frac{1}{4}$ and the 3. Step 5 then is use of the commutative property of addition again to accomplish the addition of whole numbers to whole numbers and fractions to fractions. Step 6 is the completion of the problem. We see in Example B that the vertical notation does not offer any problem of what to add to what.

4. $(2 + 3 + \frac{1}{4}) + \frac{3}{4} = \square$

5. $(2 + 3) + (\frac{1}{4} + \frac{3}{4}) = \square$

6. $5 + \frac{4}{4} = 5 + 1 = 6$

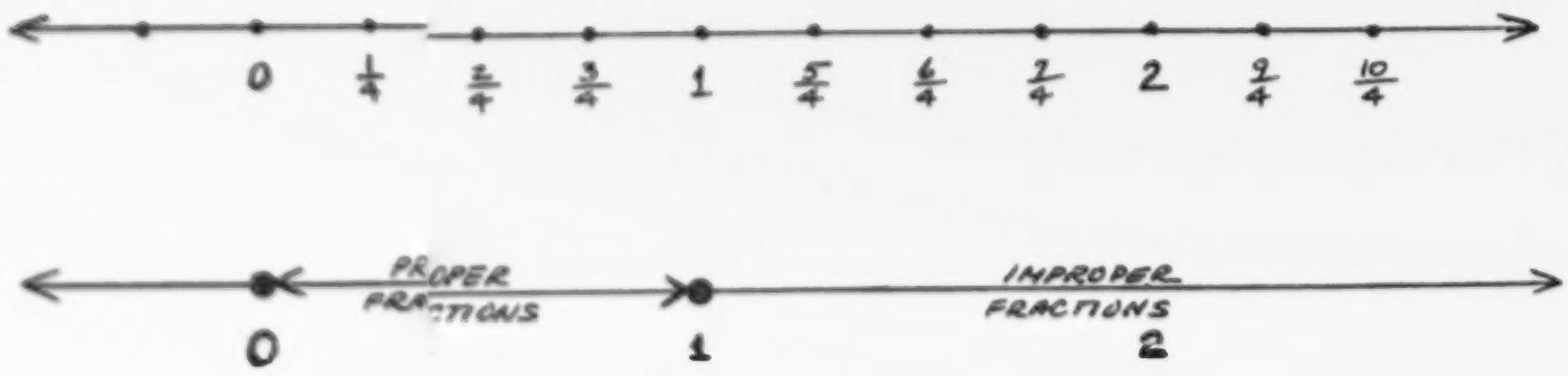
Example B:

$$\begin{array}{r} 2\ \frac{1}{4} \\ +\ 3\ \frac{3}{4} \\ \hline 5\ \frac{4}{4} = 5 + 1 = 6 \end{array}$$

Proper and Improper fractions.

Example C demonstrates with the use of the number line that the segment designated for proper fractions is from zero (but not including zero) to one (but not including 1). Improper fractions start with one (or its fractional equivalent and go on indefinitely to the number line.

Example C:



Example D:

$\frac{9}{4} + \frac{3}{4} = \square$

$(\frac{4}{4} + \frac{4}{4} + \frac{1}{4}) + \frac{3}{4} = \square$

$(\frac{4}{4} + \frac{4}{4}) + (\frac{1}{4} + \frac{3}{4}) = \square$

$1 + 1 + 1 = 3$

Example D: Renaming an improper fraction will frequently clarify that the child is to regroup and at the same time see the whole number equivalents.

Subtraction of Like Fractions.

Example E:

1. $\frac{3}{4} - \frac{1}{4} = \square$

   sum - addend = addend

2. $\frac{1}{4} + \frac{\square}{4} = \frac{3}{4}$

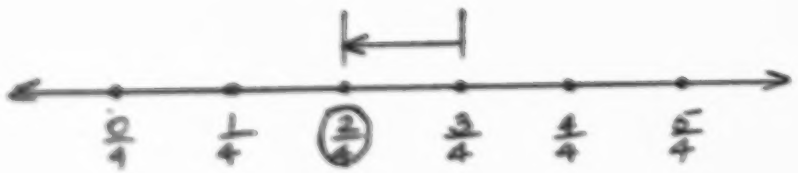
Using facts learned in addition and subtraction of whole numbers, Example E is one approach to solving the problem.

3. $\frac{1}{4} + \frac{\square}{4} = \frac{3}{4}$

4. $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$ ∴

$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$ or $\frac{1}{2}$

Example F:

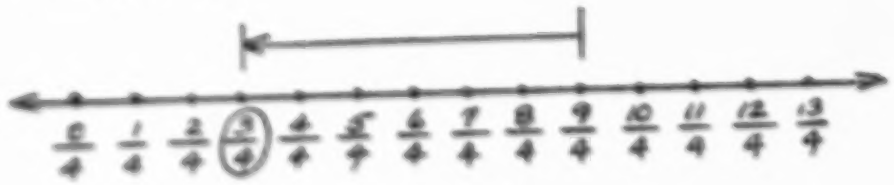


$\frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

Example F, the number line, may be clearer for many pupils. Starting at the point marked 3/4 and moving to the left $\frac{1}{4}$, we arrive at the point marked 2/4 which is our answer. Since moving to the right on a numberline is adding, moving to the left is subtracting. Another, and equally simple, demonstration is to have the pupils cut a piece of paper into fourths. Now have them put just three of these fourths on their desks. Now ask them to take away one of the fourths. Now ask for the answer (2/4). In order to get this answer in its simplest form, have the pupils put the four fourths on their desks and see if they can find another name for the 3/4, ($\frac{1}{2}$).

Subtraction of Improper Fractions.

Example G:



$$\frac{9}{4} - \frac{6}{4} = \frac{3}{4}$$

Example G: Any or all of the above methods may be used to demonstrate this type of problem.

Renaming in Simplest Form.

Example H:

$$\frac{2}{2} = \frac{2 \cdot 1}{2 \cdot 1} = \left(\frac{2}{2}\right) \cdot \frac{1}{1} = \square$$

$$1 \cdot 1 = 1$$

Example I:

$$\frac{15}{5} = \frac{3 \cdot 5}{1 \cdot 5} = \frac{3}{1} \cdot \frac{5}{5} = \square$$

$$3 \cdot 1 = \square$$

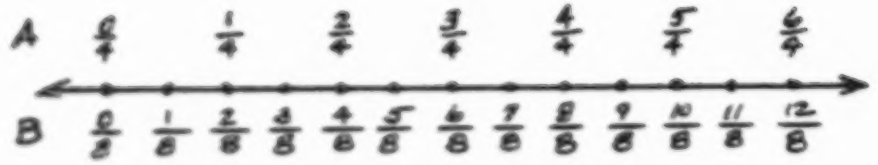
$$3 = 3$$

Any numeral can be renamed by multiplying or dividing it by some name for one. The goal for the pupil in this concept is to quickly recognize the greatest common factor found in both numerator and denominator. This is a good place to put the knowledge of the "prime factored form" of a number to good use (see Examples I and J). Demonstrate to the class on a flannel board with several halves of circles. Show what happens when they are grouped in twos (two halves make a whole circle). Then do the same with thirds, then with fourths, and then with fifths. Put each step on the board in the form shown in Example I. Use two number lines for Example J. Caution pupils that they must use common sense. When one way fails, try another. Example K shows that use of the prime factored form fails, but the use of renaming works as does the division of the numerator by the denominator. A number line would also work

with this example.

Example J:

$$\frac{6}{8} = \frac{2 \cdot 3}{2 \cdot 4} = \left(\frac{2}{2}\right) \cdot \frac{3}{4} = \square$$



Example K:

$$1\frac{7}{5} = 1 + \left(\frac{5}{5} + \frac{2}{5}\right) = \square$$

$$1 + 1 + \frac{2}{5} = 2\frac{2}{5}$$

Properties of the operations of fractional numerals.

Demonstrate for the class, with a flannel board, with four fourths of a square and five fifths of a rectangle and eight eights of a circle each of the following properties of the operations of fractional numerals. Then have the pupils construct their own fractional parts and prove the properties at their desks. They may also be proven on number lines.

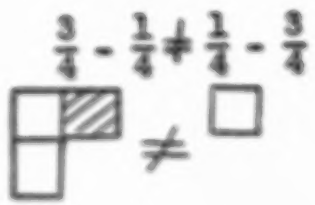
Example A: Addition of fractions is commutative. That is, we may add the fractions in any order without changing the value.

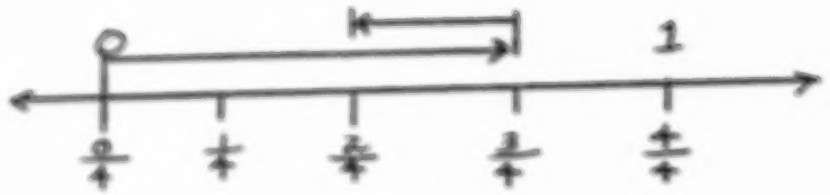
Example A:

$$\frac{3}{4} + \frac{1}{4} = \frac{1}{4} + \frac{3}{4}$$

Example B:

$\frac{3}{4} - \frac{1}{4} \neq \frac{1}{4} - \frac{3}{4}$

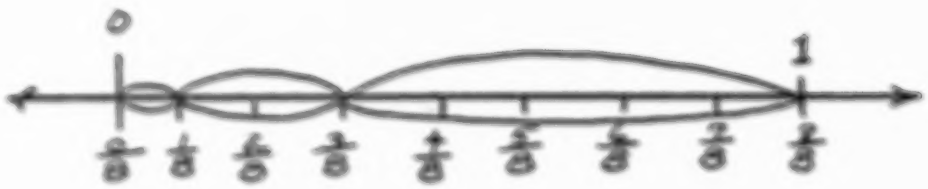




Example B: Subtraction of fractions is not commutative. That is, we may not add the fractions in any order without changing the value.

Example C:

$(\frac{1}{8} + \frac{2}{8}) + \frac{5}{8} = \frac{1}{8} + (\frac{2}{8} + \frac{5}{8})$

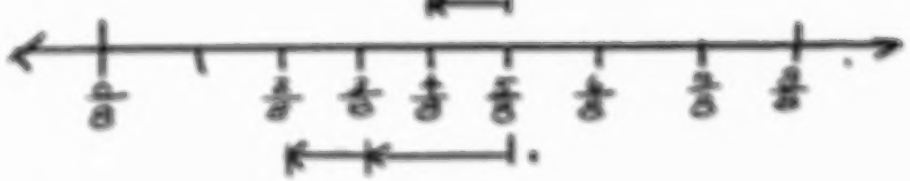


Example C: Addition of fractions is associative. That is, we may group the fractions in any manner and not change the value.

Example D:

$\frac{5}{8} - (\frac{2}{8} - \frac{1}{8}) \neq (\frac{5}{8} - \frac{2}{8}) - \frac{1}{8}$

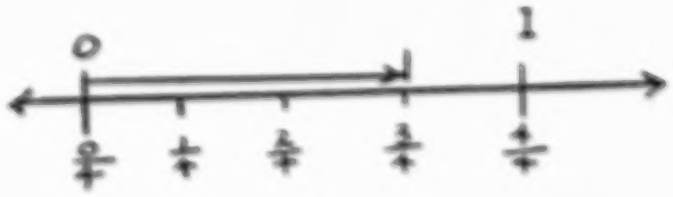
$\frac{4}{8} \neq \frac{2}{8}$



Example D: Subtraction of fractions is not associative. That is, we may not group in any manner without changing the value.

Example E:

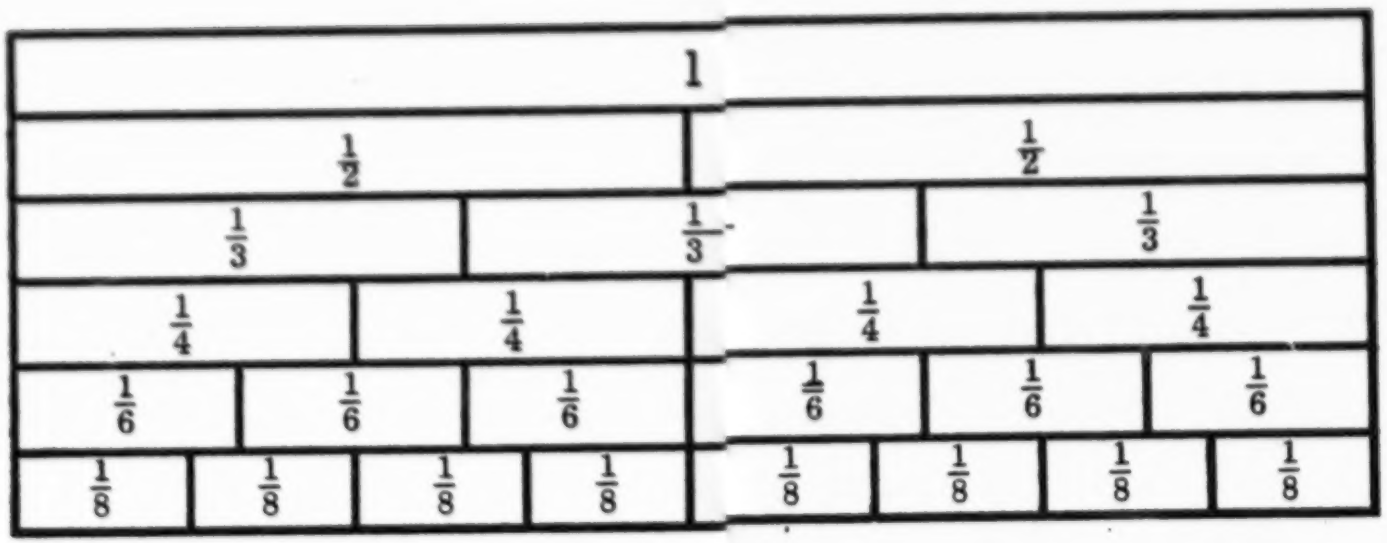
$\frac{3}{4} + 0 = \frac{3}{4} - 0 = \frac{3}{4}$



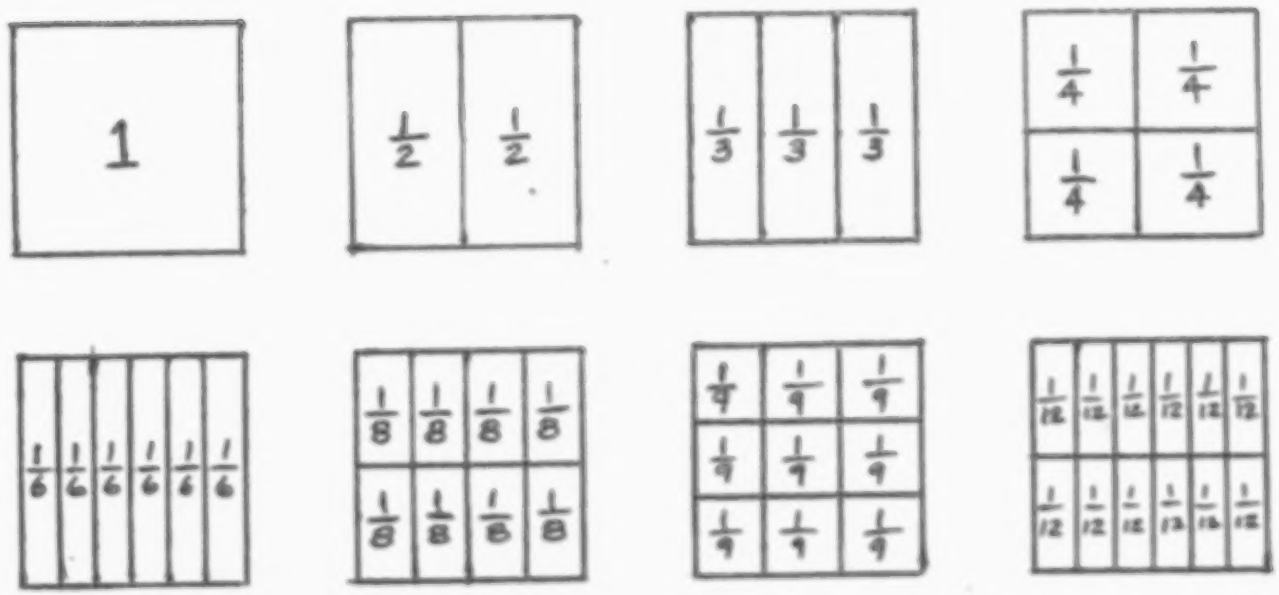
Example E: Zero is the identity element of addition of fractions. That is, we may add zero to a number or subtract zero from a number without changing its value.

Finding the common denominator.

A chart may be used to illustrate equivalent fractions. Pupils must realize that "changing to a common denominator" involves finding an equivalent fraction. Acetate overlays clearly show how one fraction "equals" another, as 3/4 = 6/8.



Also, graph paper "fraction kits" are helpful. Each student makes a "kit," using 7 squares, each six spaces on a side (Inch or half-inch ruled paper). Six of the square "cards" are cut into halves, thirds, fourths, sixths, ninths, and twelfths respectively. One card is labeled "1", and is not cut into parts.



Give problems with these denominators: 2, 4, 6, 9, 12. Ask pupils to add (or subtract), using the fraction kits. In the example 1/4 + 2/3, ask, "What group of other pieces could you substitute for these, using only one denominator? (12's)

How many 12ths would be needed for each ¼? (3 twelfths) How many 12ths would you substitute for each 1/3? (4 twelfths) Using replacements of 3/12 for each ¼, and 4/12 for each 1/3, make a new arrangement of parts to find the answer for 1/4 + 2/3 = □ . Is there any other answer for this problem, using a smaller denominator? (No)

Develop more problems, gradually moving away from the manipulative aids as you see that pupils grasp the meaning of "finding a common denominator." If pupils show any confusion in the conventional form of changing denominators, bring them back to the use of manipulatives. Such words as "substitute," "replacement," and "same amount" may need to be repeated.

$\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$

$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$

Other methods of finding common denominators may be more useful in computation.

1. Multiplication by one.

   Any fraction keeps the same value when both of its terms are multiplied by one (or by any fraction that equals one, such as $\frac{3}{3}, \frac{4}{4}, \frac{8}{8}$,)

   Examples: find the missing numerals.

$$\frac{3}{4} = \frac{\square}{8} = \frac{12}{\square} = \frac{\square}{32}$$

$$\begin{array}{r} \frac{1}{4} \\ +\ \frac{2}{3} \\ \hline \end{array} \quad 3 \times 4 = \underline{12}$$

C. D. = 12

2. Multiplication of the denominators together to find a common denominator. This is not always the Least Common Denominator, but pupils can "reduce" or simplify answers later.

$$\begin{array}{r} \frac{1}{4} \\ +\ \frac{2}{3} \\ \hline \end{array} \quad \begin{array}{l} (4,\ 8,\ 12,\ 16,\ 20,\ 24,\ --) \\ (3,\ 6,\ 9,\ 12,\ 15,\ 18,\ 21,\ 24,\ 27,\ --) \end{array}$$

3. Sets of multiples.

Pupils may put several multiples of each denominator in parentheses after each of them, and select the smallest common multiple as a denominator.

x	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36
7	7	14	21	28	35	42
8	8	16	24	32	40	48
9	9	18	27	36	45	54

4. Multiplication table.

Some students may need a handy "crutch" for a few days. Their multiplication tables give a set of equivalent fractions when used as follows: Find the numerator in the top line. Run down that column to the denominator. Move across that row to the new denominator, and the new numerator will be the top numeral in that column.

$$\frac{1}{8} = \frac{?}{40}$$

When a "top numeral" other than 1 is used, run down the 1 column to the numerator. Its new numerator will be in that row, and in the column containing the new denominator. (In the example, change 3/8 to □/40) The numerators for <u>three</u> will all be in the row that begins with 3. Continue down to the denominator, then move to the right to the new denominator. The new numerator will be in that column, where it crosses the "3" row.

$$\begin{array}{r} \frac{13}{21} \\ +\ \frac{9}{28} \\ \hline \end{array}$$

(3, 7)

(2 x 2 x 7)

Prime factors of Denominators.

"Use the common factor" only once.

L.C.D. = 84 (not 588)

5. Prime factors: When working with examples such as this (13/21 + 9/28), students do not need to multiply 21 x 28 to find the common denominator. The L.C.D. can be found by multiplying together only the prime factors of both numbers without repeating a common factor. Thus, the factors 3 x 2 x 2 x 7 = 84 may be used instead of (21 x 28 = 588)

Find the L.C.D's using prime factors:

a. $\begin{array}{r} \frac{13}{30} \\ +\ \frac{7}{12} \\ \hline \end{array}$ (2, 3, 5) (2, 2, 3)

Answer: (2 x 3 x 5) x (2) = 60

b. $\begin{array}{r} \frac{3}{20} \\ +\ \frac{3}{50} \\ \hline \end{array}$ (2, 2, 5) (2, 5, 5)

Answer: (2 x 2 x 5) x (5) = 100

Fractional number sentences
(equalities and inequalities)

A.

1							
$\frac{1}{2}$				$\frac{1}{2}$			
$\frac{1}{3}$			$\frac{1}{3}$			$\frac{1}{3}$	
$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$	
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$		
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

Procedure:

1. Ditto diagram "A" and give to students to cut out. Paper discs or paper plates may be used, too.

2. Give oral drill in comparing fractions. Pupils use their paper strips to discover the relationships.

   Example: $\frac{1}{2} = \frac{2}{4}$, $\frac{1}{3} = \frac{2}{6}$, $\frac{1}{4} = \frac{2}{8}$ etc.

3. Give students worksheets to use independently. They will discover the relationship by using the paper strips.

   Example A: Place $<$, $=$, or $>$ in each number sentence to make it true.

1. $\frac{1}{2} \bigcirc \frac{2}{4}$
2. $\frac{2}{3} \bigcirc \frac{3}{4}$
3. $\frac{7}{8} \bigcirc \frac{1}{2}$
4. $\frac{5}{6} \bigcirc \frac{2}{3}$
5. $\frac{1}{4} \bigcirc \frac{1}{6}$
6. $\frac{2}{2} \bigcirc \frac{4}{4}$
7. $\frac{6}{6} \bigcirc \frac{3}{4}$
8. $\frac{3}{8} \bigcirc \frac{3}{4}$

Example B: Place $<$, $=$, or $>$ in each circle to make the sentence true.

1. $\frac{1}{3} + \frac{1}{3} \bigcirc \frac{1}{4} + \frac{2}{4}$ (<)
2. $\frac{1}{6} + \frac{2}{6} \bigcirc \frac{1}{8} + \frac{3}{8}$ (=)
3. $\frac{3}{4} + \frac{3}{4} \bigcirc \frac{2}{8} + \frac{1}{8}$ (>)
4. $\frac{3}{6} + \frac{3}{6} \bigcirc \frac{4}{8} + \frac{4}{8}$ (=)
5. $\frac{5}{2} + \frac{7}{2} + \frac{1}{2} \bigcirc 6$ (>)
6. $\frac{5}{6} + \frac{6}{6} + \frac{2}{6} \bigcirc 1\frac{1}{2}$ (<)
7. $\frac{3}{9} + \frac{1}{9} + \frac{2}{9} \bigcirc \frac{6}{9}$ (=)
8. $\frac{2}{8} + \frac{8}{8} + \frac{1}{8} \bigcirc 2$ (<)

Example C: After each sentence write true or false.

1. $\frac{3}{6} + \frac{1}{3} > \frac{5}{6}$ false
2. $\frac{8}{12} + \frac{1}{4} = \frac{1}{4} + \frac{2}{4}$ false
3. $\frac{1}{3} + \frac{2}{6} = \frac{2}{3}$ true
4. $4\frac{1}{3} + 3\frac{1}{3} > 6\frac{1}{3}$ true
5. $7\frac{1}{8} + 5\frac{5}{8} < 3\frac{7}{12} + 5\frac{7}{12}$ false
6. $(\frac{19}{16} + \frac{5}{4}) + \frac{3}{2} > 3$ true

Example D: After each sentence write true or false.
(The symbol $\neq$ means "is not equal to")

1. $\frac{1}{2} + \frac{1}{3} \neq \frac{4}{6} + \frac{1}{6}$ _ false _

2. $\frac{3}{6} + \frac{2}{6} \neq \frac{3}{4} + \frac{1}{4}$ _ true _ _

3. $2\frac{1}{2} + \frac{1}{2} \neq 3\frac{3}{4} + \frac{1}{4}$ _ true _ _

4. $4 + \frac{5}{8} \neq 3 + \frac{1}{2} + 1\frac{1}{3}$ _ false _

5. $\frac{(4 + 2)}{2} \neq \frac{(4 \times 2)}{2}$ _ true _ _

6. $\frac{2}{5} + \frac{3}{5} \neq \frac{2}{6} + \frac{1}{6} + \frac{1}{6}$ _ false _ _

7. $\frac{3}{8} + \frac{3}{8} + 4 \neq 4 + \frac{1}{4}$ _ true _ _

8. $\frac{(8 - 2)}{2} + \frac{5}{8} \neq \frac{(8 \times 8)}{2}$ _ true _ _

Solution

$\frac{2}{3} + \frac{3}{3} \neq \frac{3}{4} + \frac{2}{4}$

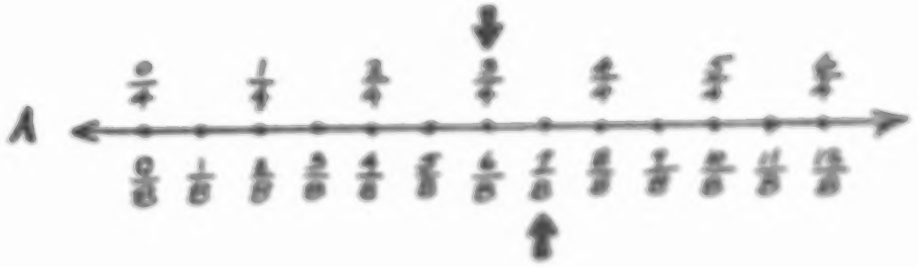
$\frac{5}{3} \neq \frac{5}{4}$

$1\frac{2}{3} \neq 1\frac{1}{4}$ _ true _ _

Procedures:

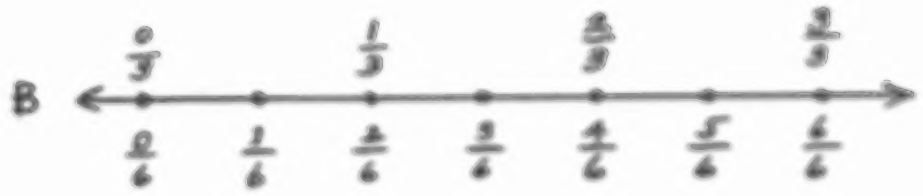
1. Give ample practice in renaming fractions.
   example: $\frac{4}{3} = \frac{8}{6} = 1\frac{1}{3}$

2. Re-enforce the meaning of these symbols: ($<$, $>$, $=$).
   (Remember the point in the inequality symbols is directed toward the smaller number)



3. Refer to number line "A" and note the location of $\frac{3}{4}$ and $\frac{7}{8}$ for the example below.

$\frac{3}{4} < \frac{7}{8}$



Refer to number line "B" and note the location of

$\frac{2}{3} > \frac{2}{6}$

ADDITION AND SUBTRACTION GAMES AND DRILL

GRADE 6

6th Grade

Addition and subtraction through six-place numerals

Addition and Subtraction Games and Drill

16:1 Puzzles give lots of practice with the benefit of letting a child feel he is going somewhere. The "chain reaction" puzzles may be given with or without explanation of rules. In the example each number is the sum of the two that precede it. The pupil "checks" his work when he comes to the numbers indicated by the "given" numerals.

12 21 33 ___ ___ ___ ___

___ 987 ___ 193 216 ___ ___

___ 1659

16:8 "What's my rule", or "Pattern Hunt", is played either at the chalkboard or on paper. The notation "3 ⟶ 8" may be read, "Three suggests Eight", "Three produces Eight", or "Three gives Eight."

A.		B.	
3 ⟶ 8		10 ⟶ 107	
0 ⟶ 5		0 ⟶ 7	
10 ⟶ 15		5 ⟶ 57	
5 ⟶ --		½ ⟶ 12	
7 ⟶ --		20 ⟶ 207	
		8 ⟶ --	
		2 ⟶ --	

In the triangular-shaped sketch, the number indicated in each box is the sum of the two numbers above it (one on the left, one on the right). The numeral 1 sits on each step of the pyramid and are addends in the puzzle. Thus, 6 is the sum of the number 5 and number 1.

In the following "cross number" puzzles you add each row across and each column down. When the sums are then added across and down the answer is identical and is written in the lower right-hand box. Puzzles can be made more complex by omitting various numerals. Practice in subtraction is provided by omitting addends. The last four examples require the student to subtract to complete the puzzle.

1	9	10
7	12	19
8	21	29

8	7	15
9	1	10
17	8	25

7	3	10
1	5	6
8	8	16

2	8	10
6	4	10
8	12	20

12	4	16
10	14	24
22	18	40

17	13	30
9	15	24
26	28	54

14	8	22
12	24	36
26	32	58

28	16	44
12	34	46
40	50	90

16	26	42
28	12	40
44	38	82

19	27	46
33	21	54
52	48	100

15	31	46
37	13	50
52	44	96

32	22	54
12	44	56
44	66	110

Magic squares provide opportunities to practice both addition and subtraction skills. The sums of the rows, columns, and diagonals are identical and are written in the lower right hand box. Magic squares of sixteen, twenty-five, and thirty-six cells can be constructed. Instructions for their construction can be found in mathematical puzzle books.

MAGIC SQUARES

9	4	5
2	6	10
7	8	3

18

7	6	11
12	8	4
5	10	9

24

11	10	5
16	12	8
9	14	13

36

3	8	7
10	6	2
5	4	9

18

12	17	10
11	13	15
16	9	14

39

9	1	8
5	6	7
4	11	3

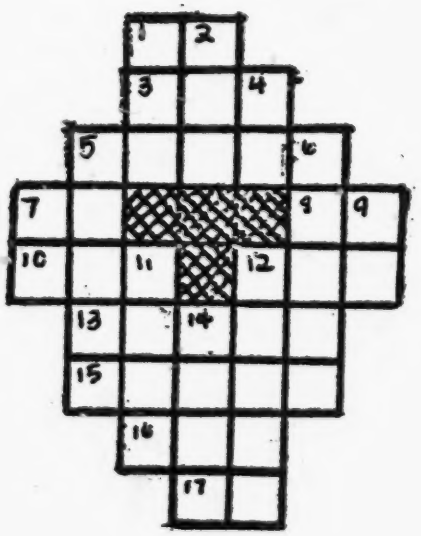
18

In this "cross number" puzzle, subtraction is used. Appropriate difficulty level should be suited to the pupils. 16:31

216	512	
	406	216

"The number in the second row is always subtracted from the one above it."

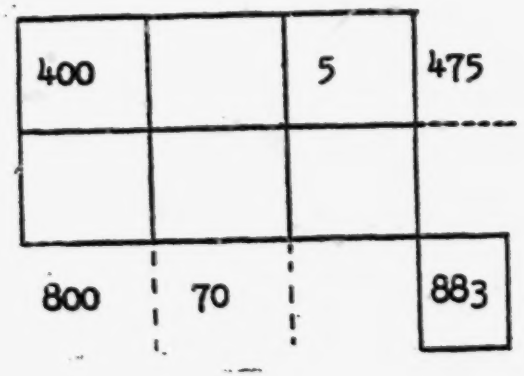
Cross number puzzles can assume many forms. The "cross word puzzle" form gives a different type of reinforcement than the four-block pattern, and so requires more pre-planning than does the four-block type. An occasional "extra sheet" of puzzles is usually well-received by the students as a game. 12:8



(SRA Puzzle +15 a.)

Across		Down	
1	29 + 19	1	249 + 249
3	666 + 257	2	578 + 246
5	29,284 + 49,194	4	19 + 18
7	15 + 38	5	41,991 + 31,798
8	15 + 19	6	64,822 + 19,159
10	196 + 178	7	36 + 17
12	399 + 199	9	29 + 19
13	19,297 + 62,331	11	1,978 + 2,159
15	29,997 + 63,134	12	28,797 + 23,567
16	458 + 258	14	1,666 + 4,452
17	39 + 45		

The four-block puzzle can be expanded into six or more squares. Suggest that pupils design their own puzzles and see if they work. 16:31



Routine problems can become more interesting if "missing numerals" are used. Any desired level of complexity can be used. 5:12

A.
$$\begin{array}{r} 435 \\ 6x2 \\ 851 \\ \underline{x7x} \\ x,934 \end{array}$$

B.
$$\begin{array}{r} 31,965 \\ \underline{-xx,xxx} \\ 20,506 \end{array}$$

Addition and subtraction through six-place numerals.

The student is given the dittoed pattern with two digits in each vertical column. They are told that the circle (◯) and the box (□) have a different value in each vertical column. When given the value of the circle (◯) and box (□) the student is expected to find the sum (◯ + □) and the difference (◯ − □). However, when given the sum (◯+□) and one of the addends, circle or box, the student is expected to work by the inverse operation (subtraction) to determine the other addend. Once both addends are known, the difference can be calculated. If given the difference (◯−□) and either the minuend or subtrahend, the student is expected to calculate by the inverse operation the unknown.

Again, once both addends are known, the sum can be calculated. If the sum and difference are both given, the student is encouraged to find both unknowns so that their sum equals the given and the difference also equals the given.

California Mathematical Practice Sets - Grade 5

	A	B	C	D	E	F	G
○ + □	109,167	119,998	190,941	631,016	493,862	999,943	987,848
○	62,304	82,713	128,394	537,841	365,289	723,592	658,563
□	46,863	37,285	62,547	93,175	128,573	276,351	329,285
○ - □	15,441	45,428	65,847	444,666	236,716	447,241	329,278

Solutions to the problems are handwritten.

Addition and subtraction with decimals

Pupils often do not develop a confident attitude about their knowledge of decimals. This may be because decimals are not used in the same context as "common" fractions. Exercises that use both types should be selected with a mind toward uniting the pupil's experience.

A. $63.78 = 60 + 3 + \frac{7}{10} + \frac{8}{100}$

$= 60 + 3 + \frac{70}{100} + \frac{8}{100}$

$= 60 + 3 + \frac{78}{100}$

$= 63\frac{78}{100}$

B. $5.36 = 5 + \frac{3}{10} + \frac{6}{100}$

$= 5 + \frac{\Delta}{100} + \frac{6}{100}$

$= 5\ \frac{(\Delta + 6)}{100}$

These "interchange" problems should produce a solid understanding of the base ten numeration system and its place values.

$.8367 = \frac{3}{10} + \frac{3}{100} + \frac{6}{1000} + \frac{7}{10,000}$

$= \frac{\triangle}{10,000} + \frac{\bigcirc}{10,000} + \frac{\square}{10,000} + \frac{7}{10,000}$

$= \frac{\diamondsuit}{10,000}$

Writing and Reading Decimals--
Complete this chart.

Common Fraction	We Write the Decimal as:	We Read the Decimal as:
$3\frac{4}{10}$	3.4	three and four-tenths
$19\frac{6}{100}$	19.06	nineteen and six hundredths
$7\frac{56}{100}$	_____	seven and fifty-six hundredths
$5\frac{9}{1000}$	_____	_____
$21\frac{544}{1000}$	_____	_____
$72\frac{3}{10,000}$	72.0003	seventy-two and three ten-thousandths
$67\frac{9}{10}$	_____	_____
_____	7.0001	_____

The grid pattern is useful again in giving practice with decimals. Any blanks are to be filled in by performing the indicated operation.

+	4	.1	3	.4	2.5	.03	.9
4							
.3							
.9			3.9				
3.7							
.16							
16							

Magic squares give good addition practice. Blanks may be left for pupils to fill in. "See if you get a new magic square if you multiply each number by the number given below it."

6:413

1.3	.4		1.7
.9		1.1	.6
1.3	.7		
.2		1.5	.5

(X 3)

FRONT VIEW—

.002	.3%	$\frac{5}{1000}$	.007
$\frac{1}{100}$	$\frac{11}{1000}$	.015	1.6%
.02	$\frac{23}{1000}$	1.01	1.211
175%	1.8	2	203%

BACK VIEW—

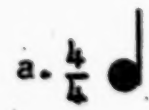
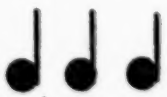
D	C	B	A
H	G	F	E
L	K	J	I
P	O	N	M

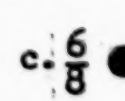
Duplicate a sheet that looks like the one in the illustration. Letters indicated by dotted lines are a key to the correct order. Students may letter the back in crayon (or duplicate this also). Cut along solid lines, shuffle, and pass them out in sets. Each student is to arrange the squares in ascending order by rows. To check, students turn the cards over and discover that they have the alphabet in proper order.

5:15

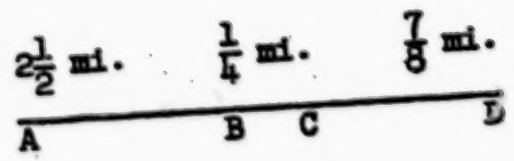
Addition and subtraction with fractional numbers

"What note to each measure would complete the measure? Check some familiar songs."

a. $\frac{4}{4}$  b. $\frac{3}{4}$ 

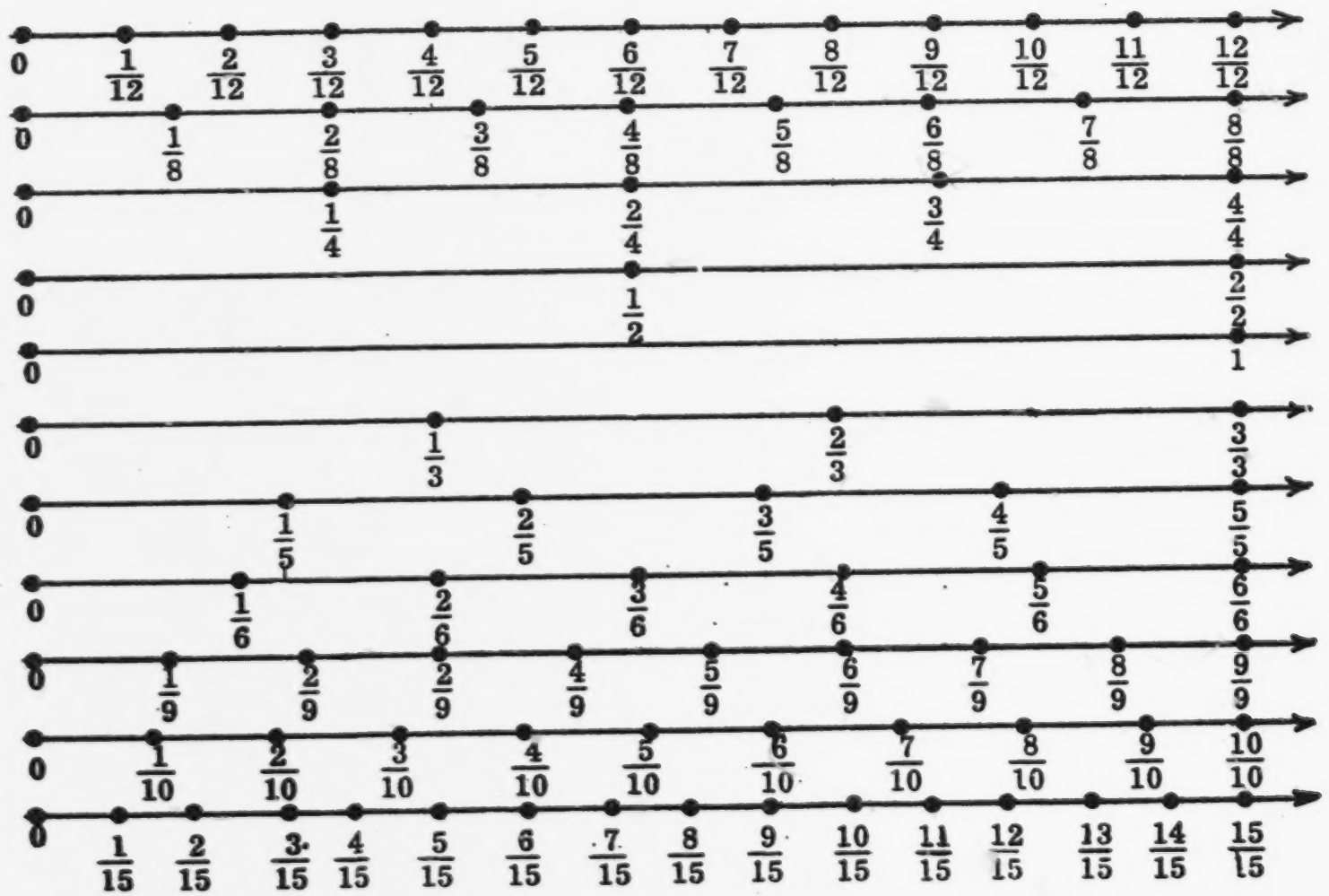
c. $\frac{6}{8}$  "Musical Fractions"

"One morning Jack rode from A to C. Then he rode back to B to find an arrow he had lost. Then he rode on to D. How far did he ride in all?" (4 1/8)



Until pupils become quite proficient in working with common fractions, they should have available some visual or tangible means of comparing the relationships of commonly used denominators. It is difficult to build meaningful abstract principles. It is still more difficult to do when the pupils have never seen the concrete examples.

Comparing Fractions

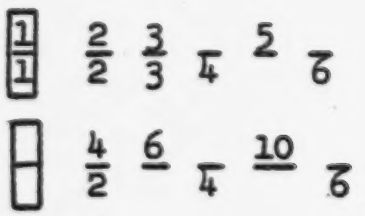


Write T after each true sentence and F after each sentence that is false. Use the number-lines above ("Comparing Fractions") if you need help.

$\frac{1}{2} = \frac{3}{6}$	T	$\frac{1}{5} \neq \frac{3}{15}$		$1 = \frac{8}{8}$	
$\frac{3}{5} = \frac{6}{10}$		$\frac{15}{15} = \frac{10}{10}$		$\frac{2}{3} < \frac{3}{4}$	
$\frac{5}{6} \neq \frac{7}{8}$		$\frac{1}{5} > \frac{1}{4}$		$\frac{2}{3} > \frac{6}{9}$	

Developing a set of equivalent fractions.

"Reserve the first box in each group for the simplest name you can find."



MULTIPLICATION AND DIVISION

GRADE 6

I. Multiplication without carrying

A. Example I - in conventional form plus level I expanded notation

$$
\begin{array}{rrl}
3121 & = & 3{,}000 + 100 + 20 + 1 \\
\times\ 213 & = \times & \quad 200 + 10 + 3 \\
\hline
9363 & & \qquad\qquad\quad 3 \\
 & & \qquad\qquad 60 \\
 & & \qquad\quad 300 \\
 & & \quad 9000 \qquad\quad 9{,}363
\end{array}
$$

Multiplication with 3 place multipliers and 4 place multiplicands.

Teacher - The algorism of multiplication in expanded notation is used only as a chalkboard lesson to illustrate to the student the concept of the algorism itself plus the operation involved in carrying.

1. **When carrying to the next column to the left, do you always carry a 1?**

2. **When the multiplier is a multiple number, why do you indent the written product to the left one space each time you change the digit multiplier?**

In the above example using the conventional form of multiplication we multiply 3 times 1 and record the product of 3 under the multiplier. To proceed we say 3 times 2 and record the product of 6 to the left of the previous product of 3. Next, we take 3 times 1 and again the product 3 is recorded to the left of the preceeding product of 6. To conclude the multiplication by the 3, we take 3 times the 3 and again record the product 9 to the left of the previous product.

However, in the preceeding example are we really multiplying $3 \cdot 1$, $3 \cdot 2$, $3 \cdot 1$, and finally $3 \cdot 3$? Let's turn our attention to the same example in expanded notation. We can readily see our multiplier has a unit value of one; therefore, we are actually multiplying by 3 ones. To complete the algorism for the ones we would start by multiplying the 3 ones times the 1, which also has a unit value of one. The product of 3 ones is written below the problem. Next, multiply the 3 ones times the 2. Or is it a 2? To look more closely, the 2 has a unit value of 10 or $2 \cdot 10 = 20$. Actually we are multiplying 3 ones times 20. The product 60 is written directly below the previous product of 3. Be sure to line the ones columns over each other. It is suggested that vertical column lines be drawn on the chalkboard or graph paper used

if it is a seat exercise to help illustrate to the student the importance of lining up the columns vertically. To proceed with the algorism, we next multiply the 3 ones times the 1 which has a column value of 100 or 1 · 100. Therefore, we are multiplying 3 ones times 100. Our product 300 is written under the previous product being careful to keep the place values of the product in the correct vertical column. Notice how all the ones are lined up vertically, all the tens are lined up vertically, etc. To conclude multiplication by the ones column, next multiply the 3 ones by the 3 which has the place value of thousands 3 · 1,000 or 3,000. The product 9,000 is written under the previous product as before. Adding the four separate products together, we find the sum to be 9,363, which is equal to the same product we received when we multiplied by the 3 the conventional way.

To continue with the conventional problem, we next multiply the 1 times the 1 and the product of 1 is written directly below the 6. The problem is then completed for the multiplier of 1 in the same manner as was the multiplier of 3.

Why write the product of 1 below the 6 and not the 1, the students ask? The usual response to the students' query is "If you don't indent to the left each time you change multipliers, you won't get the right answer."

Do we really indent or does the explanation lie in the true value of the digit multiplier? To look at the same problem in expanded notation, we soon discover that we are <u>not</u> really multiplying by one at all. However, we are multiplying by 10, since the 1 is in the tens place column. Therefore, we multiply 10 times the 1 and the product of 10 is again written in the vertical column as before, being careful to line up the column values. To proceed, we next multiply the 10 times the 20 and the product 200 is written in the vertical column under the problem. The next step is to take 10 times the 100 and the product of 1,000 is also written in the vertical column, by lining up the place values of the

3121	=	3,000 + 100 + 20 + 1	
x 213	=	x 200 + 10 + 3	
9363		3	
3121		60	
		300	
		9000	
		10	31,210
		200	
		1000	
		30000	

numeral with the others. The final step in the multiplication by the tens place, we multiply the 10 times the 3,000 and again the product is recorded in the usual manner.

Summing up the four products of the multiplication by the tens place, we find the sum of the products to be 31,210. However, when we look at the conventional way we find a product of 3,121. Where does the discrepency lie? Could it be the fact that in the conventional problem the product was indented? Was the product really 1 or was it 10? By looking at the problem in expanded notation we see that the product was 10. Therefore, instead of writing 10 we recorded a 1 in the tens place and left the ones column blank. If we fill the ones column with a zero (place holder) we soon see that the product for the tens place for both problems is 31,210.

To complete the problem with the multiplication by the 2 or 200 in expanded notation, we proceed exactly as before. In the conventional form we multiply $2 \cdot 1$, $2 \cdot 2$, $2 \cdot 1$, $2 \cdot 3$ and record the products in a right to left progression as before, after first indenting one space to the left from the last digit on the right of the previous product.

Looking at the problem in expanded notation, we understand that we are really <u>not</u> multiplying by 2; however, our multiplier is 200. Therefore, instead of indenting the product, fill in the place holders of zero and we find our product to be 624,200 the same as the sum of the products in the expanded notation problem.

By adding the vertical column in both problems we find the final sum of the products to be 664,773.

Why do all this extra work in expanded notation if we can get the product a shorter and quicker way? Expanded notation is used here only to illustrate to the student that each time we multiply two numerals together we are <u>not</u> always

3121	=	3,000 + 100 + 20 + 1	
x 213	=	x 200 + 10 + 3	
9363		3	
3121		60	
+ 6242		300	
664773		9000	
		10	
		200	
		1000	
		30000	
		200	624,200
		4000	
		20000	
		+ 600000	
		664773	

multiplying ones times ones. However, we are multiplying numerals that have the value of the digit times its column value.

II. Multiplication with carrying

A. Example II - in conventional form plus level I expanded notation

```
   8475 =  8,000 + 400 + 70 + 5
 x 693 = x         600 + 90 + 3
 -----                     ----
  25425                      15
 76275                      210   25,425
+ 50850                     1200
 -------                   24000
 5873175                     450
                            6300   762,750
                           36000
                          720000
                            3000
                           42000   5,085,000
                          240000
                        + 4800000
                        ---------
                         5873175
```

In the example to the left, when multiplying by the 3 ones in the conventional manner, 3 · 5 equals 15 ones or 10 + 5. We record the 5 ones below the problem and carry the 1 ten to above the 7 in the multiplicand. Next multiply the 3 ones times the 7 or 7 tens and the product of 21 tens plus the 1 ten carried, equals 22 tens or 200 + 20. The 2 tens is recorded to the immediate left of the previous product of 5 and the 2 one-hundreds is carried to above the 4. To continue, multiply the 3 ones times the 4 or 4 one-hundreds and the product of 12 one-hundreds plus the 2 one-hundreds that was carried, equals 14 one-hundreds or 1,000 + 400. The 4 one-hundreds is written beneath the problem, again to the immediate left of the previous product of 2 and the 1 one-thousand is carried to above the digit 8 of the multiplicand. To conclude multiplication by the ones place, next multiply the 3 ones times the 8 or 8 one-thousands. The product of 24 one-thousands plus the 1 one-thousands carried, equals 25 one-thousands or 20,000 + 5,000. This product is also recorded to the left of the previous product of 4. Since there aren't any more digits to be multiplied, there is no need to carry the 2 ten-thousands to the next column. Therefore the complete product of 25 one-thousands is recorded, to give a final product for the ones multiplier of 25,425.

Multiplying by the tens place, begin by finding the product of 9 or 9 tens and 5 ones. The product of 45 tens or 400 + 50 is recorded below the previous product, by writing the 5 tens under the 2 tens. There are <u>no</u> ones; therefore, that column is left blank which makes the product look like it is indented. However, if the student prefers he may use a place holder of zero in the ones column. The problem is then completed in the usual manner.

To complete the same example in expanded notation, each product is recorded in the vertical columns below the problem, which eliminates the need for carrying. For example, the product of 3 ones and 5 ones equals 15 ones. This product is written directly below the problem and nothing is carried to the next digit in the multiplican. Next the product of 3 ones and 7 tens is found to be 21 tens or 210. This product is also written in the vertical columns below the problem, being careful to keep the place value of the digits lined up correctly. This example is then completed in the same manner as the previous expanded notation example.

We find that the sum of the products in the conventional example to be 5,873,175 and the sum of the products of the expanded notation problem to be exactly the same.

B. Division through 6 place dividend and 3 place divisors

Teacher - When utilizing the long division algorism the process is renaming the dividend to be: 963 + 2568 + 1284 + 321

```
      3
321 | 123456
    - 963
      271
```

The algorism of division in the conventional form leads us through the following steps: Look at the dividend and moving from the left to the right we try to find the smallest numeral that 321 can be divided into. We decide upon the number 1,234 or is it really 2,234? However, we say the quotient is 3 and write it above the 4 in the dividend. The product of 3 times the divisor 321 is 963 and this is written below the dividend of 1234 and the difference of 271 is calculated. Since our divisor of 321 is unable to be divided into the remainder of 271, the 5 from the original dividend is brought down to form a new dividend of 2715. We decide our divisor of 321 will go into 2715 about 8 times. The product of 8 and 321 equals 2568 and is written directly under the new dividend of 2568. The difference of the two numerals is found to be 147. To complete this example, the 6 in the original divider is brought down to the remainder to form a new dividend of 1476. We find that our divisor will go into the new dividend about 4 times. The 4 is written above the 6 in the

```
      38
321 | 123456
    - 963
      2715
    - 2568
      147
```

```
      384
321 | 123456
    - 963
    ------
      2715
    - 2568
    ------
       1476
     - 1284
     ------
        192
```

original dividend. The product of 4 and 321 equals 1284 and is written directly under the new dividend of 1476. The difference between the two numerals is found to be 192, which constitutes our remainder in the example.

C. Division the "new" way (the scaffold or subtraction method)

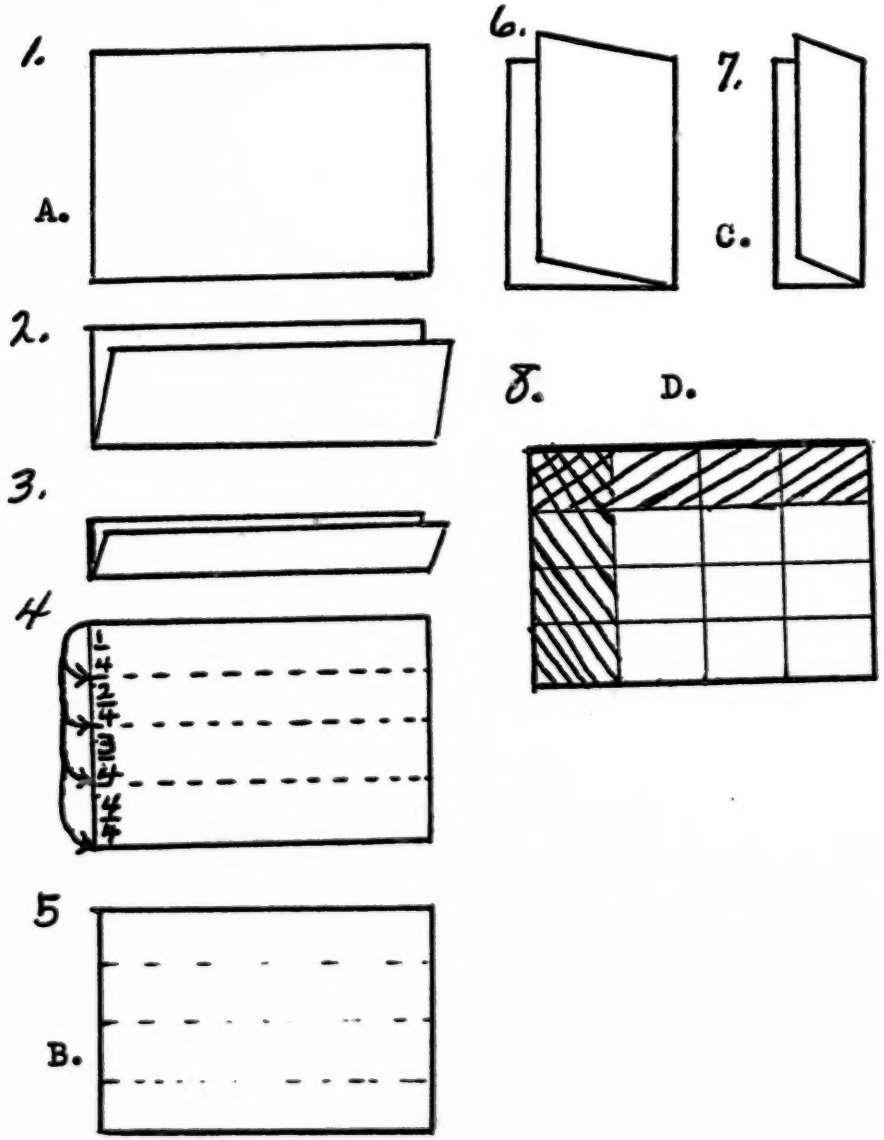
The more modern way of division allows the student to estimate his quotient and does not hold him to having to know it exactly as in the conventional form. The student divides the divisor into the whole dividend, rather than doing it a few numerals at a time. He is able to under estimate his quotient; however, at the end of the problem he is able to compensate for the under estimation and still come up with the correct quotient. Observe the following examples of the same problem and see how they are divided differently, but the end quotient is always the same and correct.

```
321 | 123456
    - 32100 | 100
    -------
      91356
    - 32100 | 100
    -------
      59256
    - 32100 | 100
    -------
      27156
    - 16050 |  50
    -------
      11106
    -  6420 |  20
    -------
       4686
    -  3210 |  10
    -------
       1476
    -   642 |   2
    -------
        834
    -   642 |   2
    -------
        192 | 384
```

```
321 | 123456
    - 96300 | 300
    -------
      27156
    - 16050 |  50
    -------
      11106
    -  9630 |  30
    -------
       1476
    -  1284 |   4
    -------
        192 | 384
```

Multiplication with like, unlike and mixed fractional numbers including finding common factors before multiplying

A. Development of a multiplication table for the fourths



Give each child a sheet of newsprint (9" x 12"). Have them fold it in half, the long way and then in half again (see steps 1, 2, 3). Now have them discuss what fractional part each section is of the whole sheet. Can they see the parts as labeled in step #4? The teacher should make a drawing on the chalkboard to demonstrate. Now have the pupils fold the paper in half the short way twice (see steps 5, 6, 7). Have a discussion as before to arrive at labels for new folds. The students paper should look like the following page. Help the pupils determine that each small space is $\frac{1}{16}$ of the whole sheet. When they do understand, have them label the upper left hand space with a large $\frac{1}{16}$. Do not at this time label any other spaces. Let's now review the construction of a multiplication table. This may be done on the back side of the students paper. There are four rows and four columns of spaces. The teacher can again demonstrate on the chalkboard what the students will do on their paper. When completed, it will look like sheet #2 which follows. Now, have the students turn their paper back to the fraction side and discuss the possibility of making this a multiplication table. Start by helping them to see that since each space is $\frac{1}{16}$ of the whole we can show this by putting $\frac{}{16}$ in each space. To show how this is true, ask the pupils to reason that $\frac{1}{4}$ of $\frac{1}{4}$ is equal to $\frac{1}{16}$ (see step #8). The space which is double cross hatched is one fourth of one fourth of the whole sheet and is equal to one sixteenth of the whole sheet. Now put this reasoning together with the idea of sheet #1 being a multiplication table we can conclude that $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$? (yes) Do the students see where the numerators and denominators in the products come from? (They should conclude that the numerator times the numerator equals the numerator of the product and that the denominator times the denominator equals the denominator of the product.) The students are now ready to complete sheet #1 by multiplying the fractions as they did the whole numbers.

$\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$

$\frac{1}{4}$ $\frac{1}{16}$

Folds

$\frac{2}{4}$

SHEET 1

$\frac{3}{4}$ $\frac{9}{16}$

$\frac{4}{4}$ $\frac{16}{16}$

1	2	3	4	
1	2	3	4	1
2	4	6	8	2
3	6	9	12	3
4	8	12	16	4

Folds

SHEET 2

63 (b)

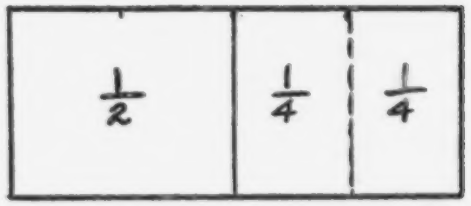
Division of fractions

"$\frac{1}{2} \div \frac{1}{4}$ asks the question, 'how many fourths can I get from a half?'" (2)

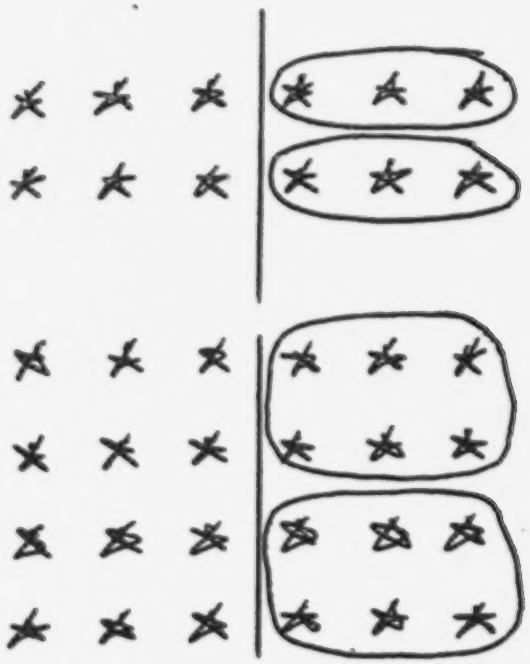
There are several ways to divide fractions. Pupils should be reminded that division asks the question. "How many of these are there in that number? Build a common-sense understanding by developing several methods:

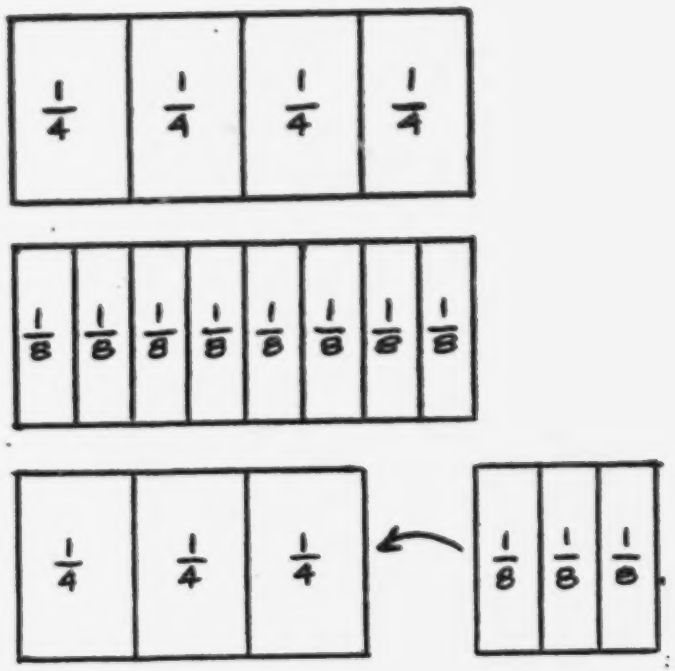
1. Picturing the pieces - Use real objects (candy bars, paper, sticks, etc., or drawings to show simple problems). Cut the whole into fourths. How many are needed to make a half?

$\frac{1}{2} \div \frac{1}{4} = ?$



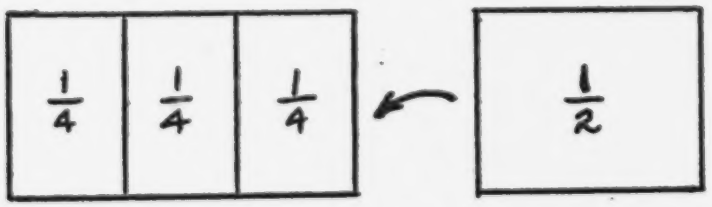
"How many fourths are in one half?





"How many $\frac{3}{8}$'s will it take to equal $\frac{3}{4}$?"

At first, the problem ($\frac{3}{4} \div \frac{3}{8}$) should be read, "How many $\frac{3}{8}$'s are there in $\frac{3}{4}$?" Use 2 sheets of paper as a model. Divide one sheet into fourths, and tear off the amount of the dividend ($\frac{3}{4}$). Divide the other sheet into eighths, and tear off the amount of the divisor ($\frac{3}{8}$). Ask, "How many $\frac{3}{8}$'s will exactly equal (or "cover") $\frac{3}{4}$? (2)



"How many halves are there in $\frac{3}{4}$?"

Another difficulty arises when the problem has an answer that is not a whole number. For example, if you divide $\frac{3}{4}$ by $\frac{1}{2}$, you want to know how many halves you can get from $\frac{3}{4}$. The models will still work, but judgement as to the size of the pieces is needed. "There is more than one half, but less than two halves, in $\frac{3}{4}$. How many halves are there?" ($1\frac{1}{2}$) (Avoid inconvenient denominators!)

A. $\frac{3}{8} \div \frac{1}{4} = \frac{3 \div 1}{8 \div 4} = \frac{3}{2} = 1\frac{1}{2}$

B. $\frac{9}{16} \div \frac{3}{4} = \frac{9 \div 3}{16 \div 4} = \frac{3}{4}$

2. Straight - across division: (Examples A and B)
In some examples (which you will need to devise), simple division of both terms will work. (Numerator divided by numerator - denominator divided by denominator)

C. $\frac{3}{5} \div \frac{1}{4} = \left(\frac{3}{5} \times \frac{4}{4}\right) \div \left(\frac{1}{4} \times \frac{5}{5}\right) =$

$\frac{12}{20} \div \frac{5}{20} = \frac{12 \div 5}{20 \div 20} = \frac{12 \div 5}{1}$

$12 \div 5 = 2\frac{2}{5}$

D. $\frac{5}{8} \div \frac{1}{4} = \square$

$\left(\frac{5}{8} \times \frac{4}{1}\right) \div \left(\frac{1}{4} \times \frac{4}{1}\right) = \frac{20}{8} \div \frac{4}{4} = \frac{20}{8} \div 1$

$\frac{20}{8} \div \boxed{2\frac{1}{2}}$

E. $6 \div 2 = 3$

$(6 \times 2) \div (2 \times 2) = ?$

$(6 \times 3) \div (2 \times 3) = ?$

$(6 \times 4) \div (2 \times 4) = ?$

$(6 \times 5) \div (2 \times 5) = ?$

F. $\frac{1}{3} \times \frac{2}{2} =$

$\frac{1}{3} \times \frac{3}{3} =$

$\frac{1}{3} \times \frac{4}{4} =$

$\frac{1}{3} \times \frac{5}{5} =$

$\frac{1}{3} \times \frac{6}{6} =$

3. Common denominator method: (Example C)
Change to equivalent fractions of common denominator as in adding fractions. Then divide just the numerators, (since the denominator of the quotient will be one).

4. Multiplication by the reciprocal of the divisor: (Example D)
This is perhaps the hardest to understand and the easiest to use. It is based on the idea that you can multiply both "sides" (the divisor and the dividend) of a problem by the same number, and keep the same relationship between these numbers.

Precede this lesson with an exploration of division of whole numbers. What happens when the divisor and dividend in $6 \div 2 = 3$ are both multiplied by the number 2? Does the answer remain 3? (yes) Continue multiplying by other whole numbers. Ask students what conclusions they can draw about multiplying both "sides" by the same number - how is this like finding equivalent fractions? Does $\frac{3}{9}$ name the same number as $\frac{1}{3}$? (yes) Does $18 \div 6$ name the same number as $6 \div 2$? (yes)

By multiplying the divisor and dividend by the "reciprocal" of the divisor, the division problem becomes a multiplication problem divided by 1. This leads to the popular....

5. "Invert and multiply" method:
After pupils know the reasons behind this method, they can do computations in this fashion: copy the dividend; change the sign from (÷) to (x); invert the divisor; multiply straight across; reduce (if you wish).

$$\frac{5}{8} \div \frac{1}{4} = \frac{5}{8} \times \frac{4}{1} = \frac{20}{8} = 2\frac{1}{2}$$

Multiplication of decimal and mixed decimals through thousandths

Example	Estimated Product	Product with Decimal Point
7 x 9 = □		□ =
8.9 x 1.11 = N		N =
1.9 x .839 = □		□ =
8 x .97 = N		N =

Prerequisite: an understanding of place value of decimal fractions and whole numbers—an understanding of rounding decimals to whole numbers

Procedure:

Example: 2.8 x 6 = N

1. Estimate the product by rounding the decimals.

   Thus: $2.8 \longrightarrow 3$
   $6 \longrightarrow \times 6$
   $16.8 \qquad 18$ estimated product
   ↑

2. Multiply as whole numbers.

3. Place the decimal point according to the estimate (note the arrow).

4. Check by using the common fraction method.

   Example: $6 \times 2\frac{8}{10} = 6 \times \frac{28}{10} = \frac{6 \times 28}{10} = \frac{168}{10} = 16\frac{8}{10}$

5. After ample explanation complete table "A."

Division of decimals and mixed decimals

Prerequisite: an understanding of place value of decimal fractions and whole numbers--an understanding of rounding decimals to whole numbers

Procedure:

Example: 18 ÷ .9 = N

1. Estimate the quotient by rounding the dividend and divisor. Thus, 18 ÷ 1 = 18

2. Divide as whole numbers. Thus, 18 ÷ 9 = 2

3. Place the decimal point according to the estimate (note a zero will be annexed). Thus, 18 ÷.9 = N N = 20

4. Check by the inverse procedure (multiplication).

5. The common fraction method may also clarify this concept.

   Example: $\frac{180}{10} \div \frac{9}{10} = 18 \div 9 = 20$

6. After many examples pupils should discover that the decimal places in the quotient are equal to the number of decimal places in the dividend minus the decimal places in the divisor. Review the placing of decimal points in multiplication and the relationship between multiplication and division. (We are looking for a missing factor in division.)

7. Understanding the placement of the decimal point in the quotient can be developed through the use of the generalization we discovered in division of fractions, i.e., multiplying the divisor and dividend by the same number will not affect the value of the quotient. We simply multiply the divisor by a number which will change it from a decimal fraction to a whole number. Thus in the example below 3.7 is changed to 37 by multiplying by 10. The dividend 93.98 must also be multiplied by 10. If the students have not previously learned it, it should be shown that any number can be multiplied by 10 by simply moving its decimal point one place to the right. Carets (∧) are often used to show the "new" place of the decimal points.

.6 x 10 = 6

.8 x 10 = 8

.3 x 10 = 3

.53 x 100 = 53

.91 x 100 = 91

4.62 x 100 = 462

.437 x 1000 = 437

5.326 x 1000 = 5326

.333 x 1000 = 333

$$3.7\overset{\wedge}{)}93.9\overset{\wedge}{8}\quad\overset{25.4}{}$$

$$.12\overset{\wedge}{)}.01\overset{\wedge}{44}\quad\overset{.12}{}$$

$$.154\overset{\wedge}{)}2.700\overset{\wedge}{8}\quad\overset{17.5}{}\ \text{R } 58$$

$$37\ )939.8\quad\overset{25.4}{}$$

$$12\ )01.44\quad\overset{.12}{}$$

$$154\ )2700.8\quad\overset{17.5}{}\ \text{R } 58$$

Ratio is a comparison of two numbers. The expression $\frac{1}{5}$ shows one number that is five times greater than another. "What is another pair of numbers that are in the ratio of one-to-five?" (Have pupils give several pairs. Write them on the chalkboard.) "What property do all these have in common?" (The lower number is five times greater than the top.) Acquaint pupils with other ratios, such as one-to-eight, etc.

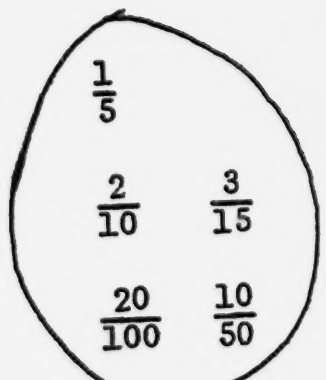
After pupils are familiar with ratios in the form of equivalent fractions, use some with larger numerals on "top," such as "five-to-four." If a pupil insists that this is an improper fraction, lead him to conclude that a "five-to-four" ratio is another way of saying "one-and-one-fourth to one."

Another way of noting ratio uses the colon to indicate a relationship between two numbers. "2 : 1" is read "two to one."

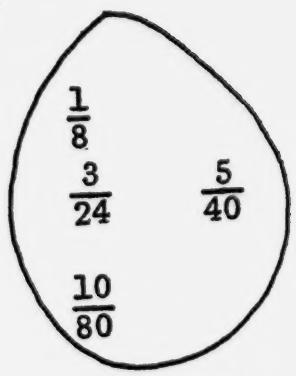
Ask pupils to find the missing numbers in ratio problems in this new form. Make certain that pupils understand the relationship of one number to another in terms of MULTIPLICATION. One is not "so many more," but "so many TIMES" the other.

When the larger of a pair is on the left, all other pairs of numbers in the same ratio will have the larger number on the left.

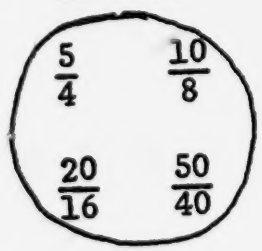
Ratio



Ratio of one-to-five



Ratio of one-to-eight



Ratio of 5 to 4

2 : 1

4 : 2

10 : 5

16 : 8

100 : 50

3 : 6

□ : 12

20 : □

25 : □

When the larger number is on the right, all equivalent pairs will likewise have the larger number on the right.

Ratio and proportion

a. $\frac{N}{3} = \frac{2}{3}$ N = 2

b. $\frac{2}{3} = \frac{\square}{12}$ □ = 8

c. $\frac{3}{5} = \frac{N}{15}$

Ratios may be used to solve problems in proportions. Find the missing number. (N represents a missing number.) This may be a good time to use (or review) the "multiplicative property of ONE." (One times any number equals that number.) This is still true when you are using a "different name for one," such as $\frac{3}{3}$ or $\frac{5}{5}$.

$\frac{3}{5} \times 1 = \frac{3}{5}$

$\frac{3}{5} \times \frac{3}{3} = \frac{N}{15}$

$\frac{9}{15} = \frac{N}{15}$ N = 9

Multiplicative property of one

Ratio used to compare scores

Many relationships can be shown best when given as a ratio. Have pupils work out the scores for several basketball players. (Select scores that can be handled by your particular class' ability.) Such a chart might look like this:

Player	Baskets	Throws	Ratio	Comparison
Tom	2	4	2:4	(18:36)
Bob	5	9	5:9	(20:36)
Carl	6	12	18:36	(18:36)
Ken	8	18	16:36	(16:36)

As shown, there is no simple way to decide who is doing the best. The scores must all be compared to the same number in order to show this. Choose a number that all scores can be changed into. (In the chart, all the values can be shown in relation to 36.) Fill in the equivalent ratio for each player. (Answers are shown in parentheses.) Who had the best score for the number of tries? (Bob)

Ratio

Ratio may be used to solve many types of problems, such as:

Equivalent ratio

1. Finding equivalent fractions

   Example: $\frac{1}{2} = \frac{2}{4} = \frac{50}{100}$, or any fraction where the denominator is twice as great as the numerator. The ratio is the comparison of one to the other.

Proportion and rate

2. Rate problems can be solved using ratio by forming a proportion (a comparison of two equivalent ratios). 7:305-308

   Example: 2 balloons sell for 10 cents; how many can you buy for 20 cents? Show the problem as a ratio of 2:10, or as a fraction, $\frac{2}{10}$.

The proportion would be: $\frac{2}{10} = \frac{4}{20}$, and would be read, "two is to ten as what is to 20?" Both 2 and 10 must be multiplied by the same number. What number? (2) What is the number of balloons for 20¢? (4) For 60¢? $\frac{2}{10} = \frac{12}{60}$ (= 12)

Percent shown as a ratio

$\frac{1}{4} = \frac{25}{100} = 25\%$

3. Percent may be shown as a ratio of any number to 100. In fact, "percentum" means "for a hundred." Thus, "1 percent" means "one per hundred," or "one out of every 100 or $\frac{1}{100}$.

Ratio can be shown by using a bicycle. Count the number of teeth in the front and rear sprockets. The ratios can be figured by dividing the number of teeth in the rear into the front. Check the answer by counting the number of turns of the rear wheel for one turn of the crank. Such a ratio may be written thus: 15:45, or 15 to 45; by dividing, an equivalent ratio is 1:3 or 1 to 3. A chalk mark on the tire makes it clear that the rear wheel turns the appropriate number of turns. (3 revolutions per 1 revolution of a pedal)

Multiplication of denominate numbers

Prerequisite: familiarity with "Tables of Measures," and an understanding of changing measures from small units to large units, and large units to small units.

Procedure:

Example:
$$\begin{array}{r} 2 \text{ lb. } 6 \text{ oz.} \\ \times \qquad 3 \\ \hline 6 \text{ lb. } 18 \text{ oz.} \end{array}$$

Method "A"

1. Multiply as whole numbers (vertical notation)

2. Regroup the product as whole numbers. Thus, 6 lb. 18 oz. = 7 lb. 2 oz.

Method "B"

1. The multiplicand may be expressed as a mixed number.
   Thus, 2 lb. 6 oz. = $2\frac{3}{8}$ lb.

2. Multiply using horizontal notation.

$$2\frac{3}{8} \times 3 = N$$

$$(2 \times 3) + (\frac{3}{8} \times 3) = N$$

$$6 + \frac{9}{8} = N$$

$$6 + 1\frac{1}{8} = N$$

$$N = 7\frac{1}{8} \text{ lb.}$$

Compare product of Method "A" with Method "B" as a check.

Concept: Division of denominate number

Prerequisite: familiarity with "Table of Measures," and an understanding of changing measures from small units to large units and large units to small units.

Procedure:

Example: 5 yd. 2 ft. ÷ 2 = N

Method "A"

1. Express the dividend (5 yd. 2 ft.) as a mixed number ($5\frac{2}{3}$ yd.) and divide.

Thus, $5\frac{2}{3} \div 2 = N$. Use common denominator method to solve. Thus, $5\frac{2}{3} = \frac{17}{3}$; $2 = \frac{6}{3}$

$$\therefore\ 5\frac{2}{3} \div 2 = N$$

$$\frac{17}{3} \div \frac{6}{3} = \frac{17}{6} = 2\frac{5}{6} \text{ yd.}$$

Method "B"

1. Express the dividend in the smallest unit. Thus, 5 yd. 2 ft. = 17 ft. Then divide as follows: $17 \div 2 = N$

$$N = 8\frac{1}{2} \text{ ft.}$$

As a check change the quotient in "A" to feet to correspond to the quotient in "B."

Thus, $2\frac{5}{6} \times 3 = N$ $\qquad$ $\frac{17 \times 3}{6} = N$

$\frac{17}{6} \times 3 = N$ $\qquad$ $\frac{51}{6} = N$

$$N = 8\frac{1}{2} \text{ ft.}$$

Exponential notation to express repetition of factors

1. $10 = 10^1$
2. $10 \times 10 = 10^2$
3. $2 \times 5 \times 2 \times 5 = 10^2$
4. $2^2 \times 5^2 = 10^2$
5. $10 \times 10 \times 10 = 10^3$

It is to be understood by all who do mathematics that a factor by itself is that number raised to the first power. This being understood and agreed upon by all, it is reasonable to assume that if you are going to take a factor times itself this would be the same as raising the factor (or base) to the second power. At this stage (step 2) the use of exponents as a shorthand is not a great saving. Suppose we factor these two factors and see how much longer the expression becomes (step 3). We can shorten these by collecting factors and using exponents. In step 7 we begin to see how valuable a shorthand exponential notation really is. To the physicist and chemist this mathematical

6. $10 \times 10 \times 10 \times 10 = 10^4$

7. $2 \times 5 \times 2 \times 5 \times 2 \times 5 \times 2 \times 5 = 10^4$

8. $2^4 \times 5^4 = 10^4$

9. $3 \times 3 = 3^2$

10. $3 \times 3 \times 3 = 3^3$

    $3 \times 3 \times 3 \times 3 = 3^4$

    $3 \times 3 \times 3 \times 3 =$ ___

10. $16 = 4 \times 4 =$ $\underline{4^2}$

    $25 = 5 \times 5 =$ ___

    $27 = 3 \times 3 \times 3 =$ ___

    ___ $= 2 \times 2 =$ ___

    $9 =$ ___ $\times$ ___ $= 3^2$

11. $27 = \underline{3} \times \underline{3} \times \underline{3}$

    $100 =$ ___ $\times$ ___

    $1000 =$ ___ $\times$ ___ $\times$ ___

12. $4^2 =$ ___

    $3^2 =$ ___

13. Give the meaning in words :

    3^4 = three to the fourth power

shorthand is invaluable. For example, 1.86×10^{23} as a large number and 1.36×10^{-25} as a small number. To help the student understand what an exponent is and how to read it, and how to write it give him plenty of patterns with sufficient opportunity to try his hand at completion.

10^4 = _____

5^3 = _____

7^2 = _____

14. Make a completion table as follows:

	Numeral	Repeated Factors	Exponential Form	Power
a		10 x 10 x 10		third
b	10,000			
c			10^5	
d		10 x 10		
e	1,000			
f			10^6	sixth
g		10 x 10 x 10 x 10		
h			10^4	
i	100			
j	1,000,000			
k		10		

Multiplication and Division - Games & Drills

Multiplying a number by itself
(squaring a number)

"When we multiply a number by itself, we say that we square the number."

24 x 24 may be written 24^2.

It is read "24 squared." It means multiply 24 by 24.

Examples:

24	24		
x24	x24		
96	400	⟵	(20 x 20)
48	160	⟵	(4 x 20 x 2)
576	16	⟵	(4 x 4)
	576		

Procedure for another way to think:

$$\begin{array}{r} 20 + 4 \\ \times 20 + 4 \\ \hline (20 + 4)\ (20 + 4) = N \end{array}$$

(This is squaring the number.)

$20(20 + 4) + 4(20 + 4) = N$ (distributive principle)

$(400 + 80) + (80 + 16) = N$

$480 + 96 = N = 576$

Check these answers by following the above procedure.

A = 17 x 17 = 100 + 140 + 49 = 289

B = 23 x 23 = 400 + 120 + 9 = 529

C = 46 x 46 = 1600 + 480 + 36 = 2116

Multiplying and dividing two and dividing two and three-place numerals (Cross number puzzles)

	A	B	C	
N→	24	0	20	4	
	168	100	60	8	7
	720	700	20	0	30
CHECK BOX →	888	800	80	8	37 ←N

N→	238	200	30	8	
	952	900	50	2	4
	7140	7100	40	0	30
CHECK BOX →	8092	800	90	2	34 ←N

The rules for this game are:

In column "A," only multiples of 100 are allowed.

In column "B," only multiples of 10 that are less than 100 are allowed.

In column "C," only numerals less than 10 are allowed.

A pattern for "construction" of a cross number puzzle becomes a problem of reconstruction. The number in the check box is given and enough information is given to see the regrouping of the initiating numerals (N) on top or right side.

To reconstruct an example of a given product and one factor—which is exactly the case at hand—is simply another name for "division."

MULTIPLICATION AND DIVISION - GAMES & DRILLS

6TH GRADE

Multiplication with one and two-place multipliers

Game 1 - Rules: Fill in the blanks in each example so the sentences are true--but do not use multiples of 5.

Examples:

A. $3(13 + \boxed{7}) = 60$

B. $7(4 + 16) = 140$

C. $9(6 + 16) = 198$

D. $15(8 + 2) = 150$

I. Method to solve type "A"

	Reason
$3(13 + \square) = 60$	Given
$(3 \times 13) + (3 \times \square) = 60$	distributive principle
$39 + 3\square = 60$	rename to include 39 thus 39 + 21
$\cancel{39} + 3\square = \cancel{39} + 21$	cancellation law of addition
$3\square = 21$	
$\square = \frac{21}{3}$	definition of division
$\square = 7$	

II. Method to solve type "B," "C" or "D"

$7(4 + 16) = 140$ ---- Given

$7 \square = 140$ ---- Frame is equivalent to the quantity inside parenthesis

$\square = 20$ ---- Rename the 20 so as to follow "Game 1 - Rules"

$\square = 4 + 16$ ---- Thus, 4 + 16 or 18 + 2 or 17 + 3 etc.

Game 2 - Rules: The first addend in each example should be a multiple of 5, and the second addend should not be a multiple of 5. All sentences must be true. To solve, choose Method I or II.

Examples:

$3(10 + 3) = 39$

$8(20 + 2) = 176$

$10(30 + 6) = 360$

$15(10 + 4) = 210$

Game 3 - Rules: All numerals used to make each sentence true must all be odd or must all be even. Choose Method I or II to solve.

Examples:

$4(12 + 14) = 104$

$6(18 + 18) = 216$

$9(20 + 14) = 306$

$12(9 + 9) = 216$

Game 4 - Rules: The first addend in each parenthesis must be a multiple of 10; the second addend must be less than 10. All sentences must be true.

Examples:

8(20 + 6) = 208

12(20 + 5) = 300

9(50 + 5) = 495

11(70 + 2) = 792

25(10 + 5) = 375

34(10 + 0) = 340

Game 5 - Rules: Same as fourth game except that the first member in the parenthesis is a multiple of 10 less than 100. The second member is reserved for multiples of 100. The third member is reserved for numerals less than 10.

Examples:

2(40 + 200 + 3) = 486

5(10 + 100 + 8) = 590

19(20 + 200 + 0) = 4180

25(20 + 300 + 8) = 8200

54(70 + 300 + 6) = 20,304

III. Method to solve "Game 5 problems"

$2(40 + \bigcirc + \triangle) = 486$

$2(40 + [\bigcirc + \triangle]) = 486$

$80 + 2\square = 486$ (rename)

$\cancel{80} + 2\square = \cancel{80} + 406$ (cancellation law of addition)

$2\square = 406$

$\square = \frac{406}{2}$

$\square = 203$ (rename with restrictions)

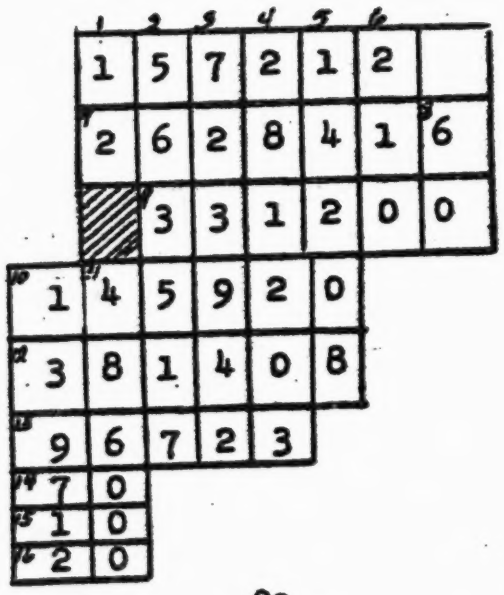
$\bigcirc + \triangle = 203$

$\bigcirc = 200$

$\triangle = 3$

Multiplication with one, two, three and four-digit multipliers

An Arithmetic Puzzle



Across

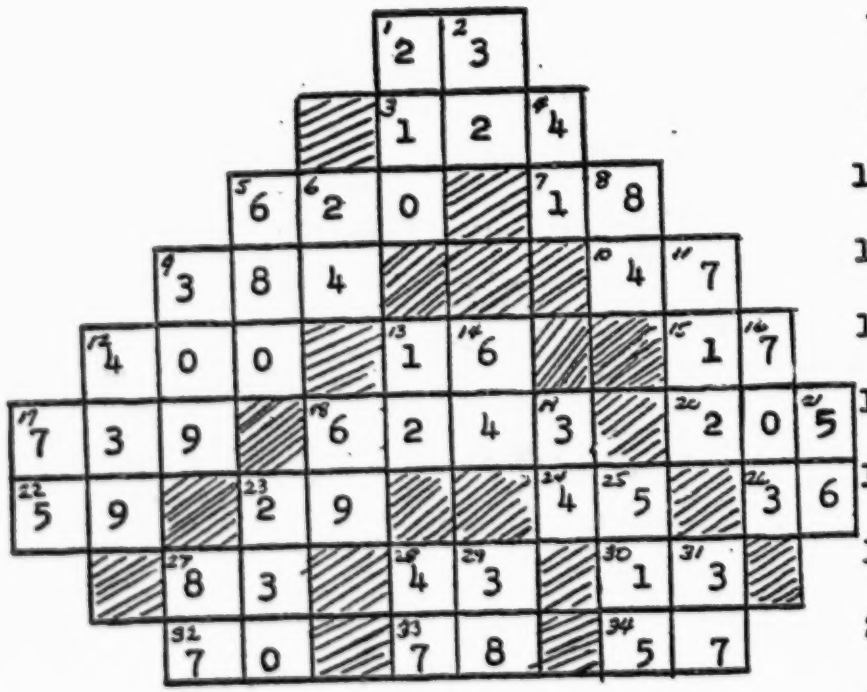
1) 397 x 396
7) 5867 x 448
9) 6900 x 48
10) 3648 x 40
12) 4384 x 87
13) 32,241 x 3
14) 10 x 7
15) 10 x 1
16) 10 x 2

Down

1) 6 x 2
2) 62613 x 9
3) 80,438 x 9
4) 2899 x 97
5) 1776 x 8
6) 30 x 7
8) 12 x 5
10) 17,464 x 8
11) 9000 x 54

Division with one and two-digit divisors

An Arithmetic Puzzle



Across

1) 69 ÷ 3

3) 248 ÷ 2

5) 19, 840 ÷ 32

7) 216 ÷ 12

9) 2304 ÷ 6

10) 94 ÷ 2

12) 6800 ÷ 17

13) 1104 ÷ 69

15) 1241 ÷ 73

17) 2217 ÷ 3

18) 143,589 ÷ 23

20) 8815 ÷ 43

22) 5133 ÷ 87

23) 1363 ÷ 47

24) 720 ÷ 16

26) 2484 ÷ 69

27) 2075 ÷ 25

28) 3741 ÷ 87

30) 806 ÷ 62

Down

1) 840 ÷ 4

2) 192 ÷ 6

4) 123 ÷ 3

5) 23,800 ÷ 35

6) 168 ÷ 7

8) 420 ÷ 5

9) 19,158 ÷ 62

11) 29,904 ÷ 42

12) 10,975 ÷ 25

13) 636 ÷ 53

14) 1088 ÷ 17

16) 2812 ÷ 4

17) 600 ÷ 8

18) 828 ÷ 12

19) 306 ÷ 9

21) 3472 ÷ 62

23) 17,480 ÷ 76

25) 19,055 ÷ 37

27) 2784 ÷ 32

12:44

(over)

All four operations of rational numbers

32) 3990 ÷ 57

33) 702 ÷ 9

34) 855 ÷ 15

28) 3901 ÷ 83

29) 2508 ÷ 66

31) 703 ÷ 19

The following pages lead to a "pattern hunt." Part of the activity in each hunt is the search for an appropriate name or title that seems to express the idea of the pattern or point it out. For these examples we will use the form ____, ____ → ____. The idea is: given a pair of numbers (____, ____) and a "secret rule," when the rule is applied to the pair it points (⟶) to a particular third number (____). For example: 3, 2⟶6

We can read this, "When a secret rule is applied to 3 and 2, it leads to 6," or in short, "three and two goes to six."

You suspect the rule 3 x 2 = 6, "multiply the two numbers to find the product—the third number." But you can't be sure. Some more examples. Show that your uncertainty was justified.

3, 2⟶ 6

3, 3⟶ 7

2, 7⟶ 10

Your first inclination was to see the pattern:

a, b⟶ a x b

But it turns out to be:

a, b⟶ a + b + 1

"What are some stories we could use with this headline?"

"After I spent 3¢ for a pencil and 2¢ for a pencil, I had 1¢ left. How much did I start with."

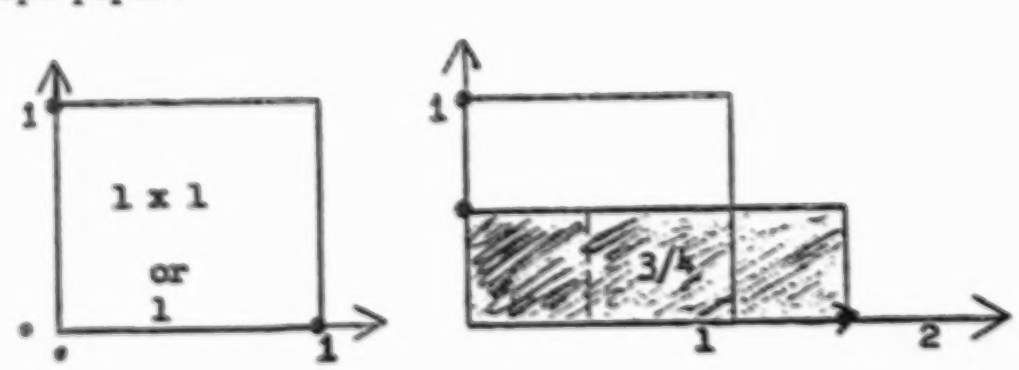
"I took some books back to the library and left some at home. The librarian said I was charged with one more than I can account for."

Fractions and operations

As we proceed, sketches show the idea of addition and subtraction, another sketch suggests the idea of multiplication and division. The idea is expanded beyond whole numbers to fractions--including division of a fraction by another.

Multiplication and division of fractions

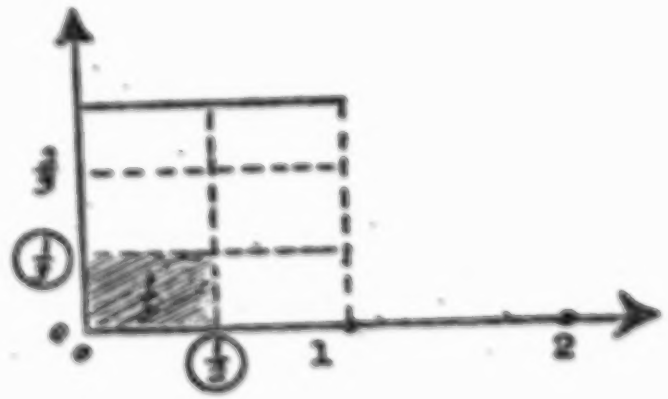
A picture that suggests an example of the idea of multiplication and division of fractions can be easily made with 1" graph paper.



If the sketch on the left above suggests that $1 \times 1 = 1$, then the other sketch suggests that $\frac{1}{2} \times 1\frac{1}{2} = \frac{3}{4}$. Changing the sentence form involving these factors and products. We say:

$\frac{1}{2} \times 1\frac{1}{2} = \frac{3}{4}$ $\qquad$ $\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$

$1\frac{1}{2} \times \frac{1}{2} = \frac{3}{4}$ $\qquad$ $\frac{3}{4} \div 1\frac{1}{2} = \frac{1}{2}$



Factors are $\frac{1}{3}$ and $\frac{1}{2}$. Product is $\frac{1}{6}$.

$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ $\quad\quad$ $\frac{1}{6} \div \frac{1}{2} = \frac{1}{3}$

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ $\quad\quad$ $\frac{1}{6} \div \frac{1}{3} = \frac{1}{2}$

Multiplication and subtraction of fractions and whole numbers

Pattern hunt

$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ (M)

$\frac{1}{2}$, $\frac{1}{3} \longrightarrow \frac{1}{6}$ (S) or (M)

0, $\frac{7}{8} \longrightarrow$ 0 (M) or (S)

7, 19 $\longrightarrow$ 133 (M)

6, 8 $\longrightarrow$ 2 (S)

$\frac{1}{3}$, 1 $\longrightarrow \frac{2}{3}$ (S)

$1\frac{1}{2}$, 8 $\longrightarrow$ 12 (M)

$\frac{1}{3}$, $1\frac{5}{6} \longrightarrow 1\frac{1}{2}$ (S)

Addition and division of fractions and whole numbers

$1\frac{1}{2}, \frac{1}{2} \longrightarrow \frac{3}{4}$ (M)

15, 15 $\longrightarrow$ 225 (M)

v, t $\longrightarrow$ v · t (M)

$\frac{1}{3}, \frac{1}{2} \longrightarrow \frac{1}{6}$ (S or M)

13, 13 $\longrightarrow$ 169 (M)

$2\frac{1}{2}, 1\frac{1}{2} \longrightarrow$ 4 (A)

3, 8 $\longrightarrow$ 11 (A)

19, 17 $\longrightarrow$ 35 (A)

$1\frac{1}{2}, \frac{1}{3} \longrightarrow 1\frac{5}{6}$ (A)

58, 94 $\longrightarrow$ 152 (A)

a, n $\longrightarrow$ a + n (A)

$\frac{1}{3}, 1\frac{1}{2} \longrightarrow 1\frac{5}{6}$ (A)

$\frac{1}{3}, \frac{1}{6} \longrightarrow \frac{1}{2}$ (A)

$\frac{1}{3}, \frac{1}{2} \longrightarrow \frac{1}{6}$ (M or S)

c, d $\longrightarrow$ d - c (S)

255, 500 $\longrightarrow$ 245 (S)

.9, 1 $\longrightarrow$ .1 (S)

3, 2 $\longrightarrow 1\frac{1}{2}$ (D)

75, 15 $\longrightarrow$ 5 (D)

144, 12 $\longrightarrow$ 12 (D)

$\frac{3}{4}, \frac{1}{2} \longrightarrow 1\frac{1}{2}$ (D)

$\frac{1}{6}, \frac{1}{3} \longrightarrow \frac{1}{2}$ (D)

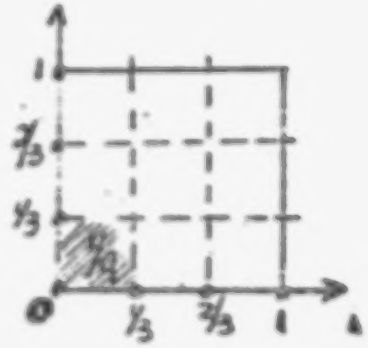
x, y $\longrightarrow$ x ÷ y (D)

$\frac{3}{4}, 1\frac{1}{2} \longrightarrow \frac{1}{2}$ (D)

132, 11 $\longrightarrow$ 12 (D)

Multiplication and division of fractions

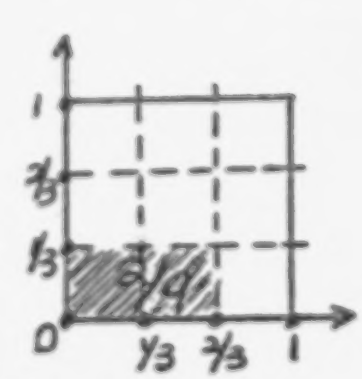
This picture suggests:



$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

$\frac{1}{9} \div \frac{1}{3} = \frac{1}{3}$

This picture suggests:



$\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

$\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

$\frac{2}{9} \div \frac{2}{3} = \frac{1}{3}$

$\frac{2}{9} \div \frac{1}{3} = \frac{2}{3}$

Pattern hunt

$\frac{1}{4}, \frac{1}{4} \longrightarrow \frac{1}{16}$ (M) $\frac{1}{3}, \frac{2}{3} \longrightarrow \frac{2}{9}$ (M) $\frac{1}{16}, \frac{1}{4} \longrightarrow \frac{1}{4}$ (D) $\frac{3}{4}, \frac{1}{4} \longrightarrow \frac{3}{16}$ (M)

$\frac{1}{2}, \frac{1}{2} \longrightarrow \frac{1}{4}$ (M) $\frac{1}{5}, \frac{4}{5} \longrightarrow \frac{4}{25}$ (M) $\frac{1}{25}, \frac{1}{5} \longrightarrow \frac{1}{5}$ (D) $\frac{1}{5}, \frac{4}{5} \longrightarrow \frac{4}{25}$ (M)

$\frac{1}{6}, \frac{1}{6} \longrightarrow \frac{1}{36}$ (M) $\frac{1}{8}, \frac{5}{8} \longrightarrow \frac{5}{64}$ (M) $\frac{1}{9}, \frac{1}{3} \longrightarrow \frac{1}{3}$ (D) $\frac{2}{3}, \frac{1}{3} \longrightarrow \frac{2}{9}$ (M)

$\frac{1}{n}, \frac{1}{n} \longrightarrow \frac{1}{n \cdot n}$ (M)

$\frac{1}{7}, \frac{1}{7} \longrightarrow \frac{1}{49}$ (M)

Missing product

$\frac{a}{b}, \frac{c}{b} \longrightarrow \frac{a \cdot c}{b \cdot b}$ (M)

$\frac{2}{9}, \frac{4}{9} \longrightarrow \frac{8}{81}$ (M)

Missing product

$\frac{1}{x \cdot x}, \frac{1}{x} \longrightarrow \frac{1}{x}$ (D)

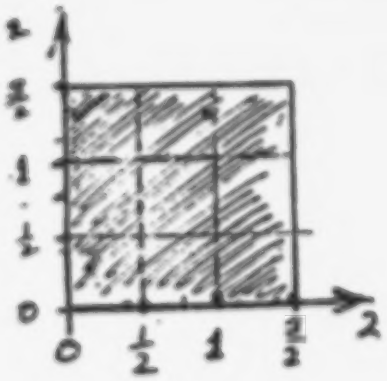
$\frac{1}{49}, \frac{1}{7} \longrightarrow \frac{1}{7}$ (D)

Missing factor

$\frac{r}{s}, \frac{t}{s} \longrightarrow \frac{r \cdot t}{s \cdot s}$ (M)

$\frac{3}{4}, \frac{3}{4} = \frac{9}{16}$ (M)

Missing product



This picture suggests multiplication and division in which at least one of the factors is greater than one.

$\frac{3}{2} \times \frac{3}{2} = \frac{9}{4} = 2\frac{1}{4}$

$\frac{9}{4} \div \frac{3}{2} = \frac{3}{2} = 1\frac{1}{2}$

"Draw as many sketches as you need to find the product of the factors given in the multiplication table on the next page.

Some shorthand to use when multiplying:

a x c = a · c = ac or c x a = c · a = ca

1 x x = 1 · x = x or x x 1 = x · 1 = x

3 x r = 3 · r = 3r or r x 3 = 4 · 3 = 3r

X	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{2}{5}$	$\frac{1}{8}$	$\frac{4}{7}$	$\frac{8}{11}$	$\frac{7}{10}$	$\frac{c}{d}$	$\frac{4}{5}$	$\frac{11}{8}$	$\frac{x}{Y}$	$\frac{7}{8}$	$\frac{m}{a}$
$\frac{1}{3}$													
$\frac{9}{11}$									$\frac{36}{55}$				
$\frac{3}{5}$				$\frac{3}{40}$									
$\frac{a}{b}$								$\frac{ac}{bd}$					
$\frac{r}{t}$													

Multiplying common and decimal fractions

Build on student knowledge of common fractions to bring understanding to operations with decimals.

$4 \times 2\frac{9}{10} =$

$4 \times \frac{29}{10} = \frac{116}{10}$

$$\begin{array}{r} 2.9 \\ \times \quad 4 \\ \hline 11.6 \end{array}$$

Think: 2.9 is almost 3

$4 \times 3 = 12$

the product is almost 12

$5 \times 1\frac{2}{10} =$

$\overset{1}{\cancel{5}} \times \frac{12}{\cancel{10}_{2}} = \frac{12}{2}$

$= 6$

$$\begin{array}{r} 1.2 \\ \times\ 5 \\ \hline 6.0 \end{array}$$

Use what you know about rounding numbers to help you estimate the product before multiplying.

$3 \times 3\frac{152}{1000} =$

$3 \times \frac{3152}{1000} = \frac{9456}{1000}$

$= 9\frac{456}{1000}$

$$\begin{array}{r} 3.152 \\ \times\ \ 3 \\ \hline 9.456 \end{array}$$

Multiplying decimals

.3 x 5 = 1.5 tenths x ones = tenths

.3 x .5 = .15 tenths x tenths = hundredths

.3 x .05 = .015 tenths x hundredths = thousandths

.03 x .05 = .0015 hundredths x hundredths = ten-thousandths

.3 x .005 = .0015 tenths x thousandths = ten-thousandths

Don't multiply, tell whether the product will be tenths, hundredths, thousandths or ten thousandths.

8	.5	.4	3.3	.31
x.3	x.7	x 7	x .4	x .5
(tenths)	(hundredths)	(tenths)	(hundredths)	(thousandths)

Decimal places:

1 - place decimals	{.4,	.8,	1,076.1}
2 - place decimals	{.32,	.06,	4.61}
3 - place decimals	{.342,	.791,	5.007}

Place a decimal point in each product.

.7 x .6 = 42	.3 x .42 = 126
.3 x .7 = 21	.9 x .25 = 225
.6 x .5 = 30	.7 x .78 = 546
1.2 x .3 = 36	4.8 x .03 = 144
4.8 x .7 = 3 36	.59 x .8 = 472
9.3 x .9 = 8 37	.86 x .7 = 602

Placing the decimal point in the product: "Study these examples. Compare the number of decimal places in the factors with the number of decimal places in their product. Do you see a pattern?"

a. 2.3	b. .18	c. .76	d. 45.6
x .6	x 2.3	x .51	x .12
1.38	.3876	.3876	5.472
e. 538	f. .019	g. 7.256	h. 9.986
x .46	x 7	x .4	x .08
247.48	.133	2.9024	.79888

Complete this chart.

	Decimal Places in Multiplicand	+	Decimal places in Multiplier	= Sum	Decimal places in Product
A.	ONE	+	ONE	= TWO	TWO
B.					
C.					
D.	ONE	+	TWO	= THREE	THREE
E.					
F.					
G.					
H.	THREE	+	TWO	= FIVE	FIVE

Multiply - Use the patterns a through h above to find the decimal products.

Example:

$$\begin{array}{r} 436 \\ \times\ 1.3 \\ \hline 1308 \\ 436 \\ \hline 5668 \end{array} \qquad \begin{array}{r} 21.8 \\ \times\ .9 \\ \hline \end{array} \qquad \begin{array}{r} .562 \\ \times\ \ 5 \\ \hline \end{array}$$

$$\begin{array}{r} 1.25 \\ \times .16 \\ \hline \end{array} \qquad \begin{array}{r} 2.77 \\ \times\ .3 \\ \hline \end{array} \qquad \begin{array}{r} 4.369 \\ \times\ .007 \\ \hline \end{array}$$

Dividing decimals by whole numbers

$$\begin{array}{r} 3\ 4 \\ 4\overline{)13.6} \\ 12\ 0 \\ \hline 1\ 6 \\ 1\ 6 \\ \hline 0 \end{array}$$

1) Divide just as you divide whole numbers.
2) Is the quotient 34, 3.4, or .34?
3) Estimate by rounding 4 x $\boxed{3}$ < 14
4) Whole number x 1-place decimal = 1-place decimal.
5) Do you see the quotient must be 3.4?

Follow the steps on the previous page closely to help you find the quotient.

$3 \times \square < 76$ $\quad 7 \times \square < 28$ $\quad 18 \times \square < 1$ $\quad 74 \times \square < 44$

$$3\overline{)75.6} = 25.2 \qquad 7\overline{)27.86} = 3.98 \qquad 18\overline{)1.332} = .974 \qquad 74\overline{)44.40} = .6$$

$$\begin{array}{r} 600 \\ \hline 156 \\ 150 \\ \hline 6 \\ 6 \\ \hline 0 \end{array}$$

Dividing decimals by decimals

$8 \div 4 = \frac{8}{4}$

$= 2$

"It is easier to divide by whole numbers than fractions. How was the divisor .4 changed to a whole number? Do $\frac{8}{4}$ and $\frac{.8}{.4}$ name the same number?" (yes)

Complete the examples:

$.8 \div .4 = \frac{.8}{.4}$

$= \frac{.8}{.4} \times \frac{10}{10}$

$= \frac{8}{4}$

$= 2$

$.9 \div .3 = \frac{.9}{.3}$

$= \frac{.9}{.3} \times \frac{10}{10}$

$= \frac{9}{3}$

$= 3$

$$.6 \div .2 = \frac{.6}{.2}$$

$$= \frac{.6}{.2} \times \frac{10}{10}$$

$$= \frac{6}{2}$$

$$= 3$$

Rewrite the example so that the divisor becomes a whole number.

$.3\overline{).6}$ $3\overline{)6}$ $.8\overline{)2.4}$ $8\overline{)2.4}$

$.7\overline{)3.5}$ $7\overline{)3.5}$ $.5\overline{)4.5}$ $5\overline{)45}$

Multiplying decimals by powers of 10

10 x 2.31 = 23.1

10 x .231 = 2.31

10 x 23.1 = 231

100 x 2.31 = 231

100 x .231 = 23.1

100 x 23.1 = 2310

1000 x .5231 = 523.1

1000 x 5.231 = 5,231

1000 x 52.31 = 52,310

Multiplying decimals by 10, 100, and 1,000. "When we multiply a decimal by 10 the decimal point moves (one) place(s) to the right."

"When we multiply a decimal by 100 the decimal point moves (two) place(s) to the right."

"When we multiply a decimal by 1000, the decimal point moves (three) place(s) to the right."

Dividing mixed decimals

$.4\overline{)\ 13.64}$

"To change .4 to a whole number multiply by (ten). To multiply 13.64 by 10 move the decimal point (one) place(s) to the right."

Use a caret (^) to show where the decimal points should be placed when changing the divisor to a whole number. Then, divide.

$3.7\overline{)\ 93.9\ 8}$ $\quad 25.4$

$.12\overline{)\ .01\ 44}$ $\quad .12$

$.3\ \overline{)\ 17.\ 7\ 6}$ $\quad 59.2$

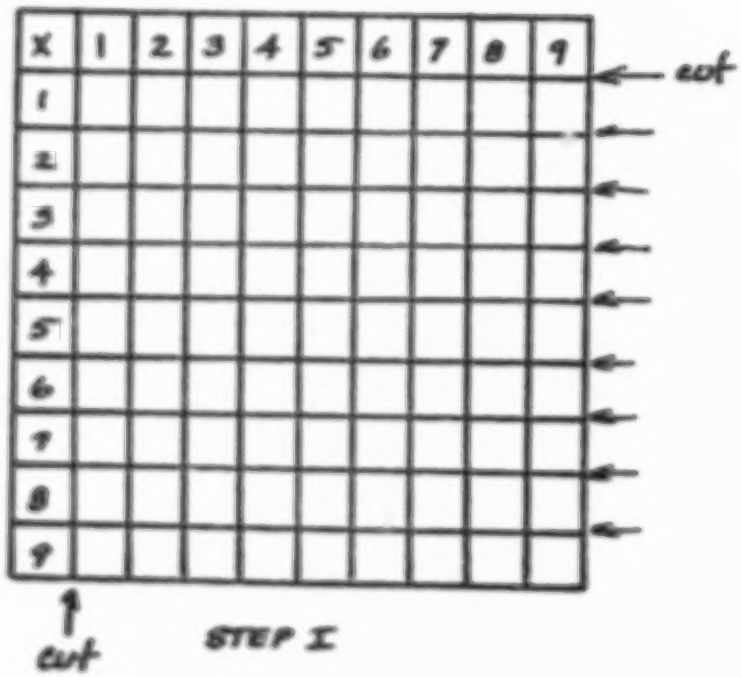
$1.5\ \overline{)\ 22.5}$ $\quad 12$

$.029\overline{)\ .49329}$ $\quad 1701$

$.84\ \overline{)\ 3.5868}$ $\quad 427$

Equivalent fractions

Here is a "slide rule" approach to finding equivalent fractions. Have students make a large multiplication table out of graph paper. Cut off and discard the squares shown in the example. Cut the remaining rows into horizontal strips. If the first two strips are placed alongside each other a series of equivalent fractions are seen. (See Step II) The top strip represents the numerators and the bottom strip, the denominators. Now let students match up different strips and share their discoveries with the class. Let's apply this "slide rule" tool to finding missing numerators and denominators.



$\frac{1}{2} = \frac{\square}{10}$ $\quad \frac{\square}{2} = \frac{5}{10}$ $\quad \frac{1}{\square} = \frac{5}{10}$ $\quad \frac{1}{2} = \frac{\square}{10}$

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18

STEP II

$\frac{1}{2} = \frac{\square}{6}$

$\frac{2}{4} = \frac{1}{\square}$

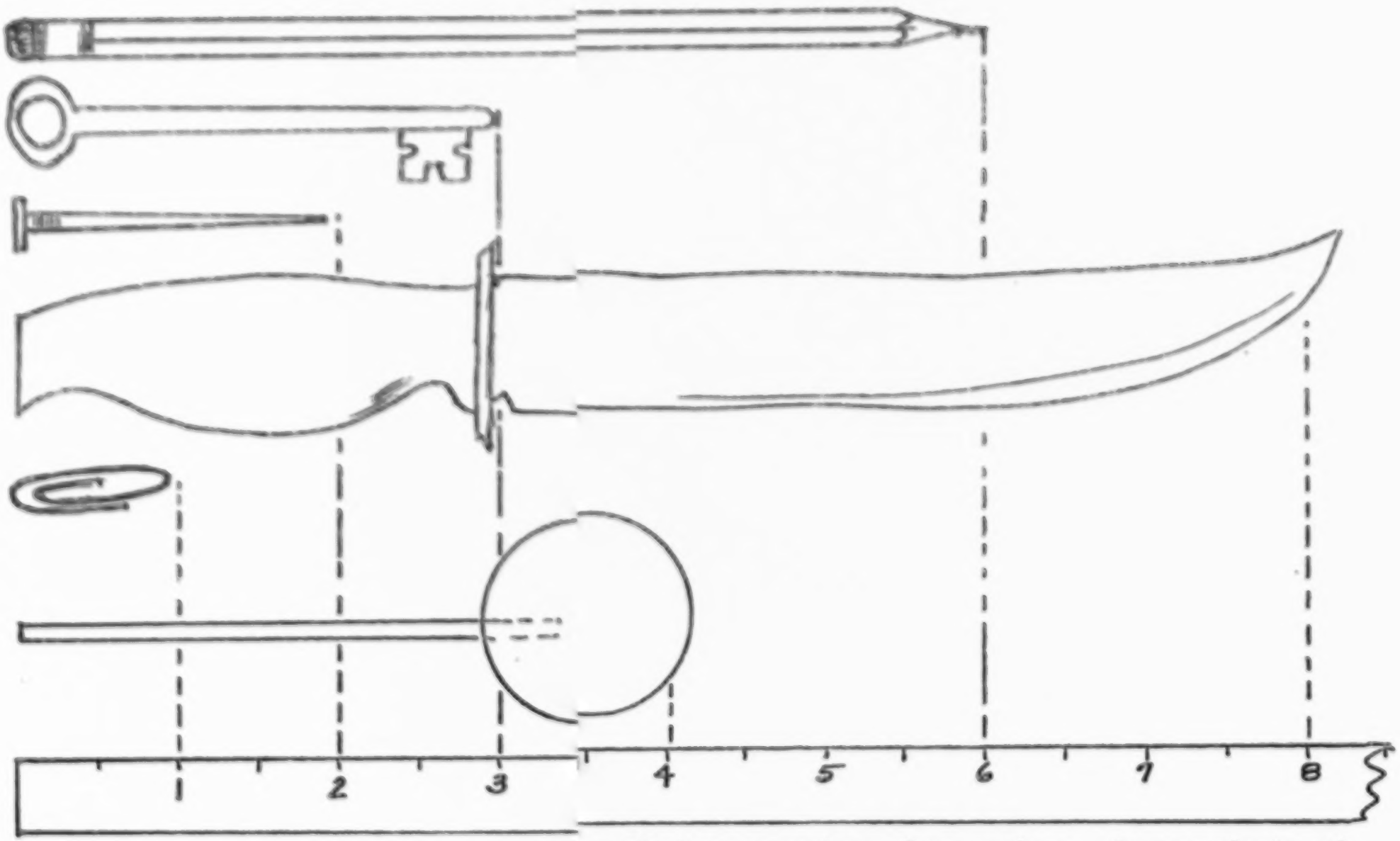
$\frac{4}{\square} = \frac{3}{6}$

$\frac{\square}{6} = \frac{1}{2}$

By selecting the right pair of strips and lining them up properly, the numerator and denominator of the known fraction are aligned and the known numerator or denominator of the equivalent fraction locates the unknown number.

Ratio

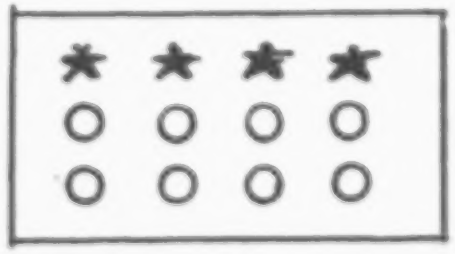
Comparing numbers with ratios.



"A ratio is a comparison of two numbers. Compare the length of these objects by completing the table. 6:3 is read '6 to 3'."

Pencil to Key	6:3	Paper clip to pencil	
Key to the pencil	3:6	Pencil to paper clip	
Knife to nail	8:2	Nail to sucker	
Nail to knife		Sucker to nail	
Paper clip to sucker		Nail to key	2:3
Sucker to paper clip		Key to nail	
Key to sucker	3:4	Paper clip to sucker	
Sucker to key		Sucker to paper clip	
Nail to pencil		Pencil to sucker	
Pencil to nail		Sucker to pencil	

Equivalent ratios



"The ratio of the number of stars to the number of circles is 4:8. The ratio 4:8 has the simpler name 1:2. Can you explain why this is true?" (The fraction $\frac{4}{8}$ can be changed to simplest form.)

"Each column of this table contains a set of equivalent ratios. Find the missing numbers."

a	b	c	d	e	f
1:2	3:2	4:5	3:4	2:5	5:2
2:4	6:4	8:10	6:8	4:10	10:4
3:6	9:6	12:15	9:12	2:15	20:8
4:8	12:8	16:20	12:16	10:25	60:24
5:10	15:10	20:25	15:20	14:35	15:6

"Do you see that working with equivalent equations is much like working with equivalent fractions?"

Exponential notation

It may be helpful for students to use a "longhand" notation chart before reducing their answers to the base-and-exponent. Such a chart is shown, and emphasizes the "what is done?" idea, rather than just calling for an answer. After pupils grasp the meaning and mechanics of the computation, they are ready to move directly into the "shorthand" scientific notation.

2^0 is read "two to the zeroth power." "Does it fit the pattern in the columns?"

		Short-hand	is equal to:
2 taken as a factor 0 times		2^0	1
2 taken as a factor 1 time	2	2^1	2
2 taken as a factor 2 times	2x2	2^2	4
2 taken as a factor 3 times	2x2x2	2^3	
2 taken as a factor 4 times	2x2x2x2		

3 as a factor 0 times		3^0	1
3 as a factor 1 times	3	3^1	3
3 as a factor 2 times	3·3	3^2	
3 as a factor 3 times	3·3·3		
3 as a factor 4 times			
3 as a factor 5 times			

Pupil's Answer Card:

4	8	1	64
12	9	6	25
81	16	42	54
144	7	2	72

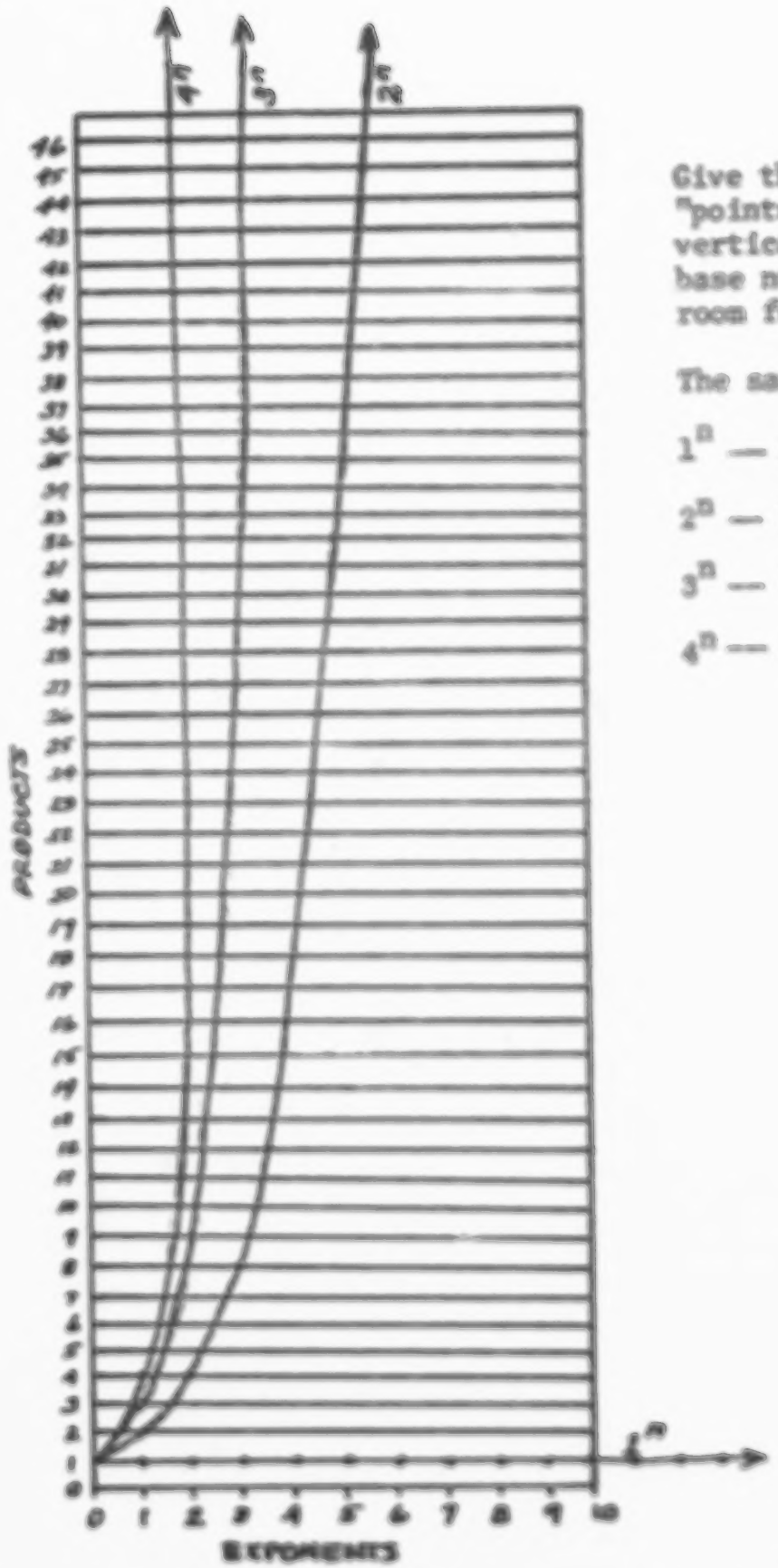
A Bingo-type game may be played to reinforce any arithmetic area, and mixed examples may be used. Decide whether you want pupils to do mental or pencil-and-paper computation. Have pupils make answer cards by folding a sheet of paper into 16ths (4 by 4). Each space is filled with a numeral from the chalkboard, where you have written the answers to 16 problems you will "call." Pupils are to "scramble" the 16 answers into any order on their answer card. When all cards are made, the game begins. Call off a problem, say 8^2. All pupils have this answer, and must find and mark it with a crayon. (Decide on a color or two for each round.) The players who have four in a row call out "multi!" or "I got it!", or whatever you choose. The game continues "as long as class interest is high," or until you decide to move to something else. When four or five have "won," pupils change cards with a neighbor and mark the new cards with a different color. It may save time to name two colors, since someone won't have a blue, or a red. "For this round, use either blue or green," etc.

Sample Questions:

1. 2^3
2. 8^2
3. 8×2
4. $27 \div 3$
5. 12^2
6. 2^2
7. $\sqrt{36}$
8. 6×7
9. 9×8
10. $16 - 4$
11. $\sqrt{49}$
12. 8^0
13. 2^1
14. 6×9
15. 9^2
16. 5^2

"Tell whether the following sentences are true or not true. Mark them T or F." 12:165

1. $2^5 < 6^2$ T
2. $3^2 < 2^3$ F
3. $8^2 = 4^3$ T
4. $10^2 > 5^3$ F
5. $10^3 > 5^5$ T
6. $9^2 = 3^4$ T



Give the pupils graph paper and show them how to plot "points." The left-right axis is the exponent, and the vertical axis is the product. Plot a curve for each base number, raised to as many powers as the chart has room for.

The sample shows the following curves:

1^n — 1^0, 1^1, 1^2, 1^3, 1^4, 1^5, 1^6, 1^7, 1^8, 1^9, 1^{10}

2^n — 2^0, 2^1, 2^3, 2^4, 2^5

3^n — 3^0, 3^1, 3^2

4^n — 4^0, 4^1, 4^2

Finding percent of a number

Principal	Rate	Int.	Rate	Int.
$ 1	4%	$.04	3%	$.03
2	4%	.08	3%	
3	4%		3%	
4	4%		3%	
5	4%		3%	
10	4%		3%	
20	4%		3%	
50	4%		3%	

Use charts to give practice in figuring interest. Pupils can "gain" answers by doubling or tripling earlier answers of other principals. Finding such a shortcut is just as valuable as the practice of multiplying by a new number.

Percent interpreted as a ratio or fraction

Give pupils experience in changing around the different types of fractions namely,

1. "the long way," as 1 out of 100
2. as a common fraction, as $\frac{1}{100}$
3. as a decinal fraction, as .01
4. as a percent, or 1%
5. as a ratio, or 1:100

A "Hundred board" can be made, using doubleheaded nails driven into a plywood board in 10 rows of 10. Using this, loop a rubber band around, say five of the nails. Ask several pupils to express this value in different "names," writing them on the chalkboard.

Sample Answers:

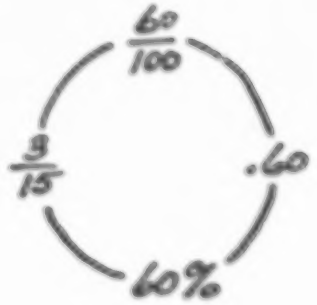
5%

5 out of 100

$\frac{5}{100}$

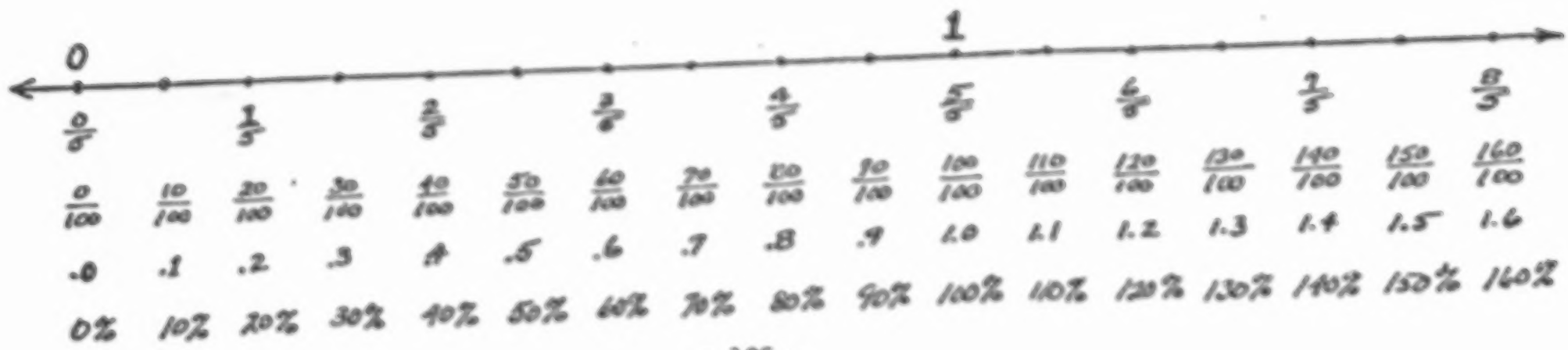
$\frac{1}{20}$

5 : 100



Common fractions, ratios, decimals, and percents are all members of the fraction family. Let students fill in the missing numerals in a "number family circle" like the one to the left.

As shown on the number line, $\frac{3}{5}$, $\frac{60}{100}$, .60, and 6% are all names for the same point.



Use the number line to find answers to the problems below.

a. $90\% = \frac{}{100} = \frac{}{10}$

b. $\frac{1}{5} = \frac{}{100} = \_\_\_\%$

c. $80\% = \frac{}{100} = \_\_\_$

d. $30\% = \frac{}{100} = \frac{}{10}$

e. $\frac{3}{5} = \frac{}{100} = \_\_\_\%$

f. $60\% = \frac{}{100} = \_\_\_$

g. $70\% = \frac{}{100} = \frac{}{10}$

h. $\frac{6}{5} = \frac{}{100} = \_\_\_\%$

i. $\frac{12}{10} = \frac{}{100} = \_\_\_\%$

j. $\frac{10}{10} = \frac{}{100} = \_\_\_$

k. $20\% = \frac{}{100} = \_\_\_$

l. $\frac{5}{10} = \frac{}{100} = \_\_\_\%$

Using the same type of number line, relationship problems may be used.

"Use the symbols $<$, $>$, or $=$ to make the sentences true. Use the numberline."

$10\% \; < \; 20\%$

$50\% \; > \; 20\%$

$.5 \; > \; 30\%$

$10\% \; = \; \frac{1}{10}$

$100\% \; = \; 1.0$

$.9 \; = \; \frac{90}{100}$

SCOREBOARD		
Players	**Tries**	**Hits**
Bill	50	35
Tim	25	6
Joe	10	8

"Three boys were playing darts. The scoreboard tells how many darts each boy threw, as well as how many times each boy hit the target. Which boy was the best dart thrower?"

Set up a ratio that will compare each boy's hits with his number of tries. Then, to help us compare these ratios, we write each one with the same denominator. We will use 100. We can then easily change our <u>ratios</u> into <u>percents</u>.

Bill = $\frac{\text{Hits}}{\text{Tries}} = \frac{35}{50} = \frac{70}{100} = \underline{\quad 70 \quad}$ %

Tim = $\frac{\text{Hits}}{\text{Tries}} = \frac{6}{25} = \frac{24}{100} = \underline{24}$ %

Joe = $\frac{\text{Hits}}{\text{Tries}} = \frac{8}{10} = \frac{80}{100} = \underline{80}$ %

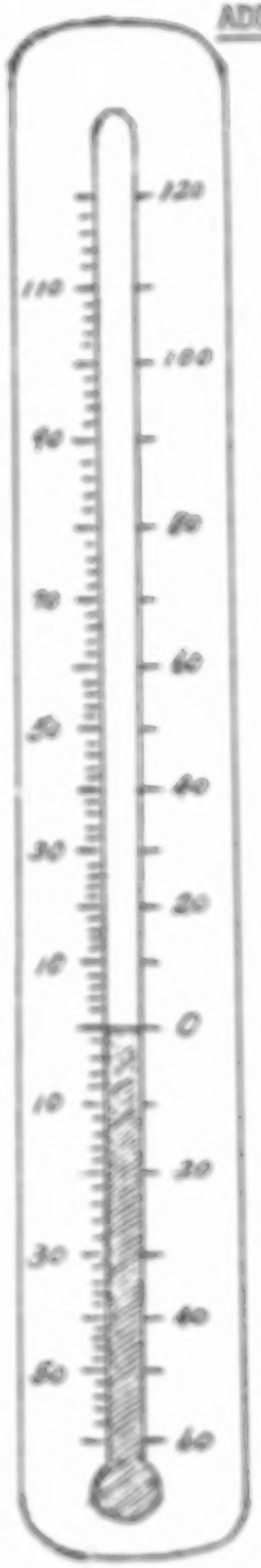
ADDITION OF NEGATIVE INTEGERS

6th Grade

Grade 6

ADDITION OF NEGATIVE INTEGERS

Adding negative numbers.

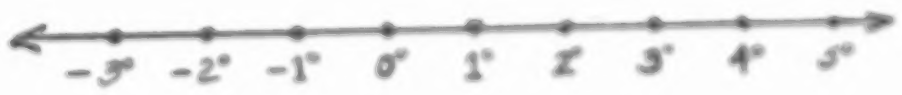


The pupils are familiar with the Fahrenheit thermometer. The pupils may not know that temperatures below zero are represented by negative numbers. 10:264-265

"On the thermometer at the left, the temperatures from 60° below zero to 120° above zero are shown on the scale.

How many spaces are there between 0° and 10° on the scale? (5) How many degrees are represented by each space? (2°)

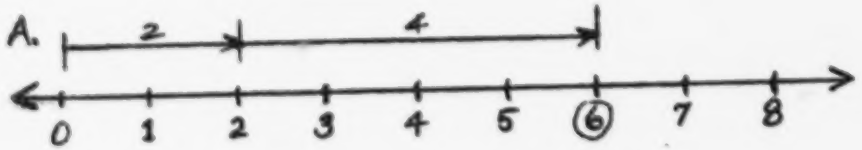
A temperature of 10° below zero is usually represented as '-10°'."

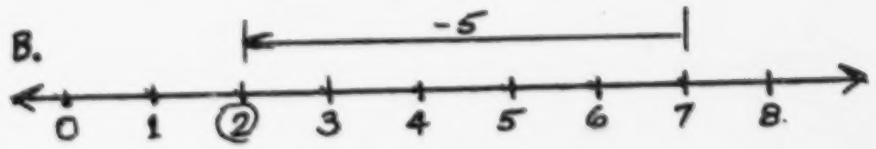


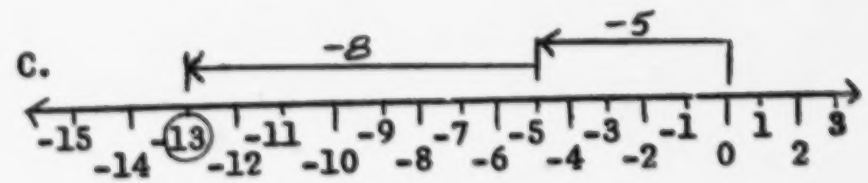
"Think of this drawing as a thermometer scale in a horizontal position. The marks for temperatures above 0° are to the right of the point marked 0°. A mark has been made for each degree, and each of these points has been labeled."

"How many unit spaces to the right of 0° is 4°? (4) How many unit spaces are there from 0° to -4°? (4) We can call a pair such as -4° and 4° opposites. What is the opposite of 6°? (-6°) the opposite of -30°?" (30°) How many things can you think of that are opposites? What would be the opposite of a profit of $5? (a loss of $5) What would be the opposite of a gain of three pounds? (A loss of 3 pounds)

What would be a good name for these numbers to the left of 0°? ("In the hole," "below," "in the red," "left," etc.) Could we call them minus numbers? Perhaps we could, but we will call these "negative numbers" to avoid confusing them with subtraction. What would the numbers to the right of 0° be called? (positive)

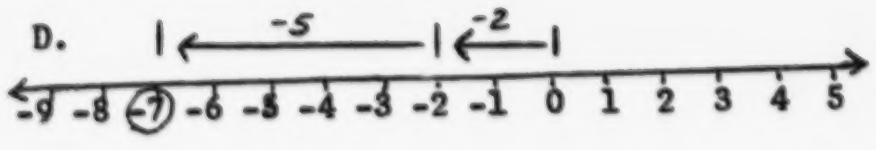






The number line may be used to show addition. On the number line A, to show the addition of 2 and 4, you begin at 0 and draw an arrow that shows a move of two spaces to the right. From this point a second arrow shows a move of four spaces. Number line B shows subtraction.

Imagine that a news dealer has a loss of $5 one day, and the next day he suffers an $8 loss. Would his total loss be the sum of the two losses? (Yes) How would you represent this problem in a mathematical sentence? Suppose you represent a loss of $5 as -5. How would you represent a loss of $8? (-8) Would this be the mathematical sentence: $-5 + (-8) = -13$? (Yes) Why the parenthesis around the -8? (So that the sentence can be read without the ambiguity of trying to read a mathematical sentence where two numerals are separated by two opposite signs.)

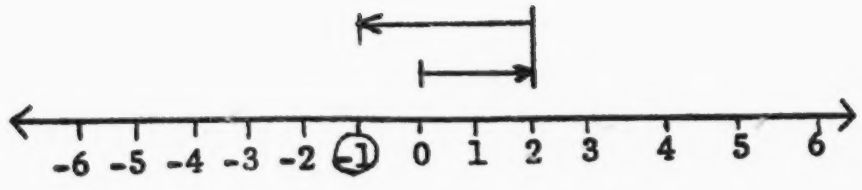


Number line "D" represents the sentence $(-2) + (-5) = \triangle$. We begin at 0, draw an arrow to show a move of two spaces to the left. Then, from -2, we draw an arrow to show a move of five spaces to the left. At what numeral does the second move end? (-7)

12

"Make number lines to show the addition of 3 and 6. Did you begin at zero and draw the arrows to the right? 3 + 6 = 9.

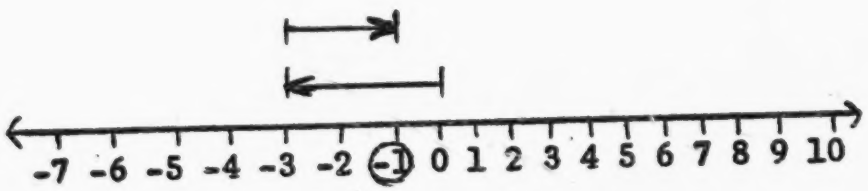
Now use a number line to represent an addition of -3 and -6. At what numeral does the second move end? (-9) What is the replacement for the frame in the following sentence? (-3) + (-6) = -9



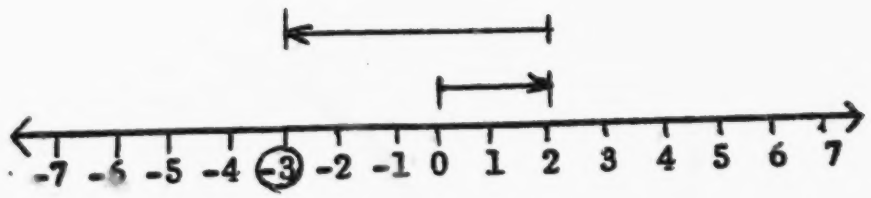
Find the correct replacement for the frame:

$$2 + (-3) = \boxed{-1}$$

The move is made to the right 2 spaces; then from 2, you represent a move of 3 spaces to the left. "At what numeral does the second move end?" (-1)

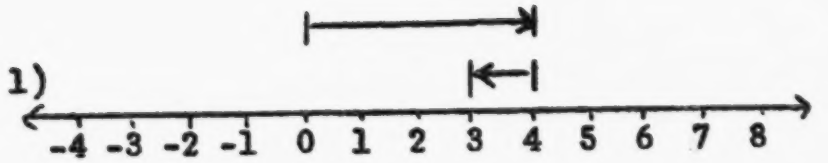


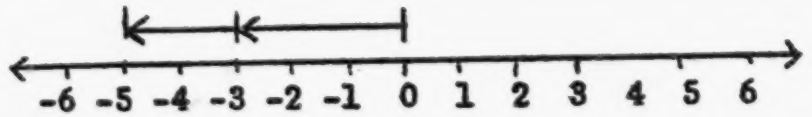
To show (-3) + 2 = $\square$, start at 0 and draw an arrow to represent a move of three spaces to the left. Then, from the point for -3, you represent a move of two spaces. "In which direction?" (the right) "What is the replacement for the frame?" (-1)

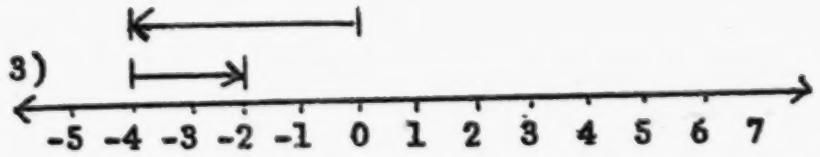


"What addition sentence is represented by the drawing to the left? Name the sum."

$$2 + (-5) = \square \quad (-3)$$







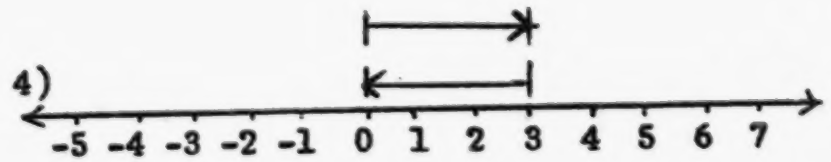
"Using the drawings of the number line, represent each of these pairs of sentences and name the sum in each case."

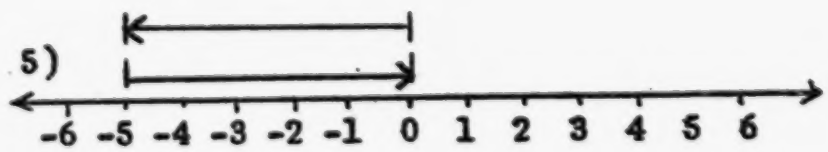
1) $4 + (-1) = \square$ (3)

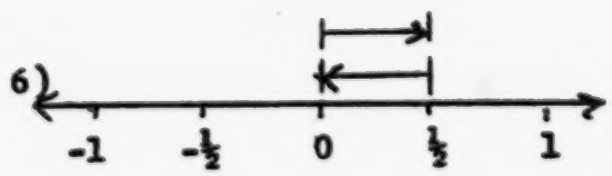
2) $-3 + (-2) = \square$ (-5)

3) $(-4) + 2 = \square$ (-2)

"Study the results of the examples. Do you agree that the order in which we add does not change the sum?" (yes)







Using the drawing of a number line, find the sums."

4) $3 + (-3) = \square$ (0)

5) $-5 + 5 = \triangle$ (0)

6) $\frac{1}{2} + -\frac{1}{2} = N$ (0)

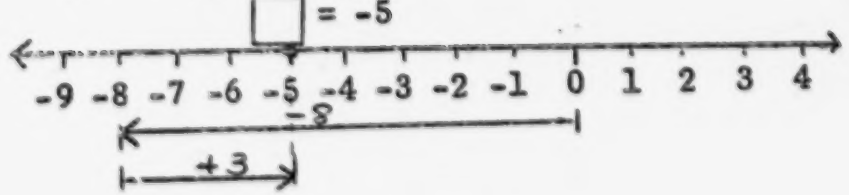
"Are we finding the sum of a number and its opposite? (yes) Is the sum 0 in each case?" (yes)

"Add each pair of numbers shown and then prove your sums by using number line pictures:"

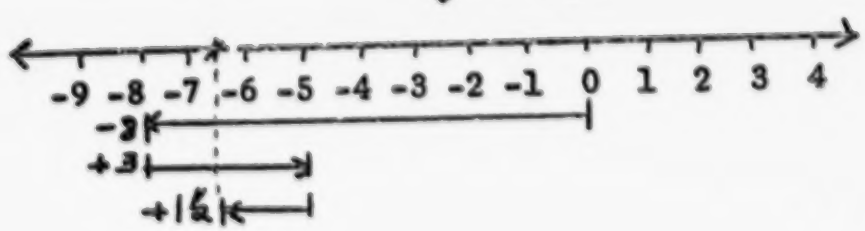
12

a. $-3 + (-4) =$ _____
b. $-4 + (-3) =$ _____
c. $-8 + (-5) =$ _____
d. $4 + (-7) =$ _____
e. $9 + (-9) =$ _____
f. $-5 + 2 =$ _____
g. $-3 + (-8) =$ _____
h. $-7 + 9 =$ _____

A. $-8 + 3 = \square$
$\square = -5$



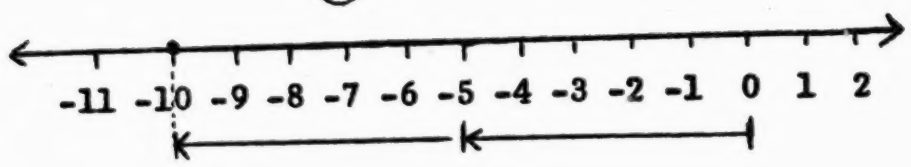
B. $(-8 + 3) + (1\frac{1}{2}) = \diamondsuit$
$\diamondsuit = -6\frac{1}{2}$



A football game is an example of a use of negative integers. "on one play the Bayville High's football team lost eight yards. On the next play it gained three yards. If the loss is represented by the numeral '-8,' what numeral would represent the gain of three yards?" (3) A loss of $1\frac{1}{2}$ yards? $(-1\frac{1}{2})$ "Does sentence A to the left tell about the over-all results of the first two plays? (yes) What does sentence B tell about? (over-all results of 3 plays)

C. $(-5) + (-5) = \bigcirc$

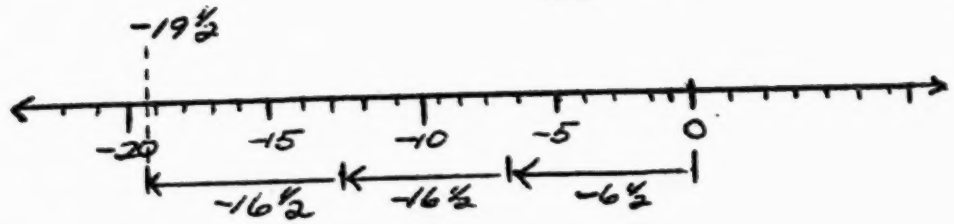
$\bigcirc =$



"Find the correct replacement for the $\bigcirc$ and tell what the numeral means." (A loss of 5 yards in two plays.) See Example C.

D. $(-6\frac{1}{2}) + (-6\frac{1}{2}) + (-6\frac{1}{2}) = \boxed{N}$

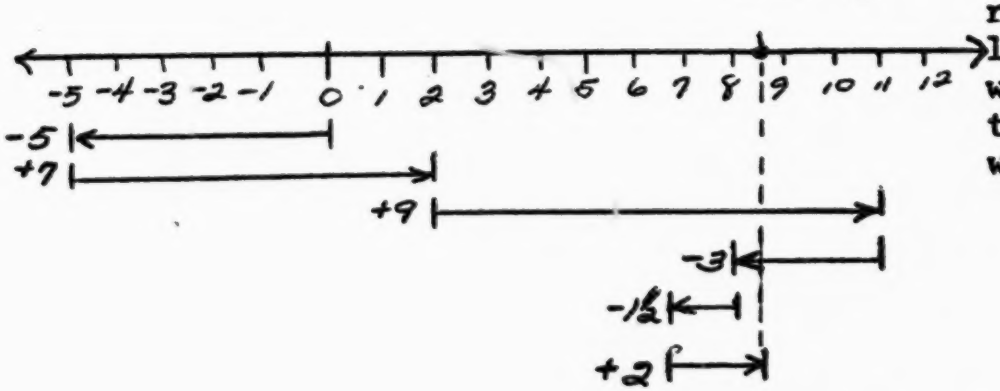
$\boxed{N} = -19\frac{1}{2}$



Find the correct replacement for the $\square$ and tell what the numeral means. (A loss of $6\frac{1}{2}$ yards in three plays.) See Example D.

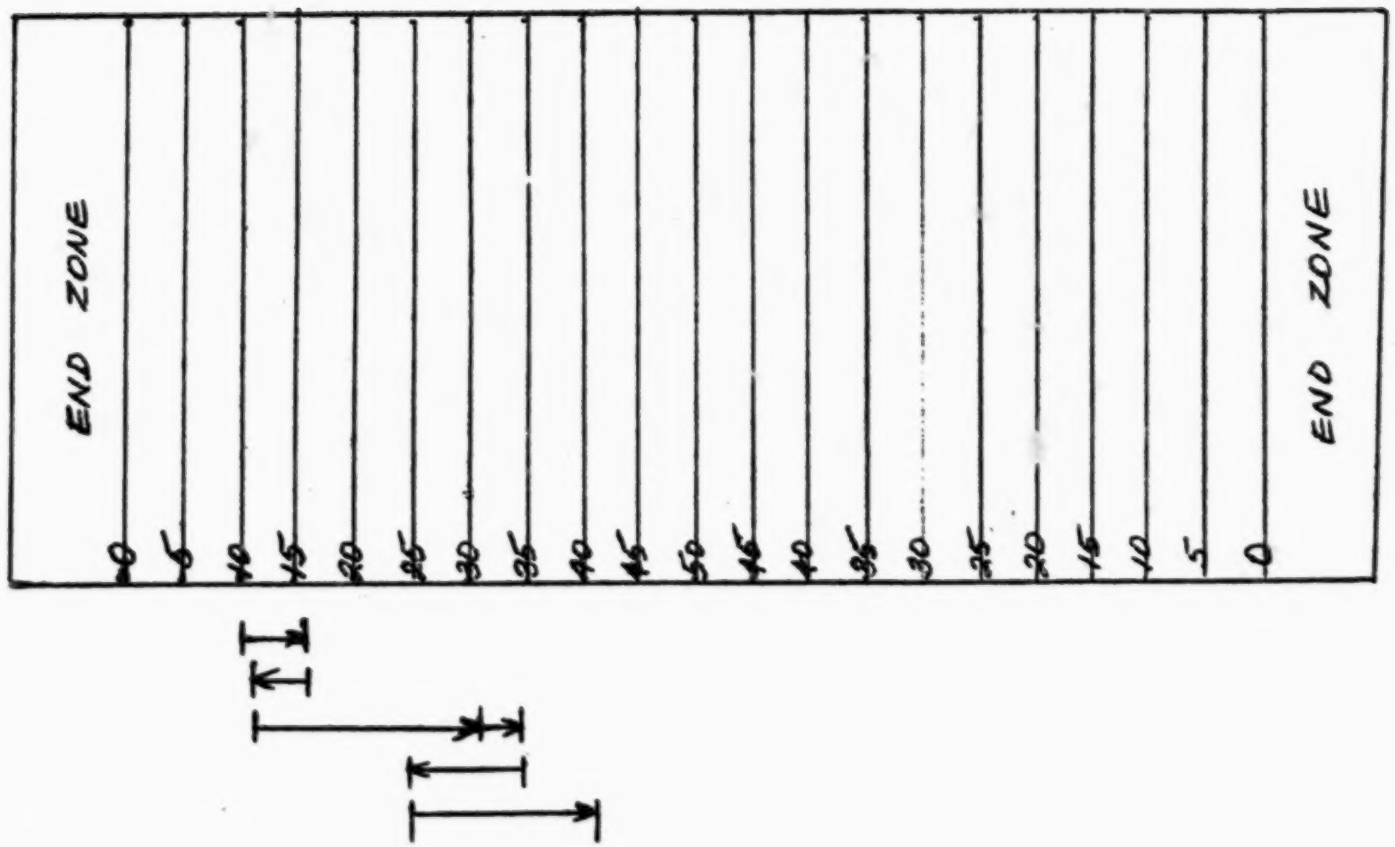
E. $(-5) + (7) + (9) + (-3) + (-1\frac{1}{2}) + (2) = N$

$N = 8\frac{1}{2}$



"Betty kept a record of the yards that her brother gained or lost when he carried the ball. The record is shown at the left. Using any method you wish, find a single numeral that represents the sum of these gains and losses. After you find the numeral, tell what it means in the situation." ($8\frac{1}{2}$ yards; the number of yards Betty's brother gained when he carried the ball). See Example E.

The football team began a drive on the 10 yard line. The team made two first downs and the yards gained or lost on these downs were as follows: 6 yards gain; 5 yards lost; 20 yards gained; 3 yards gained; 10 yard penalty; 17 yards gained. On what yard line would the team be? (41st) What was their gain in yards? (31 yards). See diagram F.



The diagrams shows the yard line on which the team would be (41st).

Since the team started on the 10 yard line, you would think 41 - 10 = 31 yards gained.

"Blast Off is at T minus 15 and counting . . . 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, ignition, Lift-off, 1, 2, 3, . . . T plus 10. . .T plus 20. . ." Everyone has heard these familiar words at the launching of a space vehicle. How much time passes between T-10 and T-6? (4 seconds), between T-7 and T-4? (3 seconds), between T-2 and T+2? (4 seconds), between T+4 and T+4 and T+6? (2 seconds)

Other activities that will show negative and positive integers are:

1. Altitudes above and below sea level
2. Games that cost points to miss
3. Thermometers
4. Profit and loss
5. Installment payments

"Complete the headlines and circle the one that best fits the story." 16:73

(1) "Bill thought: We are at 3 minutes before blast-off time. Where will we be in 5 minutes?

(-3) + (-5)→[ ] -8 (-3) + 5→[ ] 2

(-3) + ([ ] 8)→5 [ ] -2 + (-3)→(-5)

(2) "I lost a nickel and found a dime."

(-5) + [ ] 15 → 10 5 - 10→[ ] -5

(-5) + (-10)→[ ] -15 -5 + 10→[ ] 5

(3) "After I opened the store, Mrs. Jones returned a $4.00 book and bought a $3.00 book instead."

$(-3) + \underset{2}{\boxed{}} \longrightarrow (-1)$ $\qquad$ $(-4) + \underset{3}{\boxed{}} \longrightarrow (-1)$

$(-4) + (-3) \longrightarrow (\underset{-7}{\boxed{}})$ $\qquad$ $\underset{10}{\boxed{}} + (-3) \longrightarrow 7$

GRADE 6

PROBLEM SOLVING

PROBLEM SOLVING

Computational skills are basic to problem solving. Addition, subtraction, multiplication, and division of rational integers must be understood even though the task of calculation is being taken over by machine. Today and even more so in the future, the need is for people who can, through disciplined imagination, create new problems to solve and new ways to solve them.

Man is still the key. Machines are capable of fantastic computations, but constructing computer systems, programming them, and interpreting the vast amounts of data they produce are tasks for well trained, analytical minds. The computers have afforded man time for creative thinking -- for developing and expressing the complex problems to be solved by machines. This necessity for abstract thought demands critical and analytical ability -- and creative imagination -- to discover new questions.

Keeping these goals in sight, our approach at the elementary school level must be inductive. (Children work a great number of problems, using different methods to find solutions.) From this they find their own "rules" or generalizations, which they can apply to future work.

Since by the sixth grade the children have been given at least one set of steps by which they should be able to solve any story problem, it seems that one approach would be to have the children write their own story problems about social situations they have experienced. Another approach would be to have several typical story problems prepared without numerals and have the pupil supply his own numerals. The first approach will naturally be

readable by the pupils but the second approach should be watched for words that the pupil's can't read. Story problems can also take the form of a chart of facts or a graph or a set of pictures with several questions following. With this type it is suggested that the teacher put the chart on the chalkboard (or on the overhead projector) and have a class discussion about it, i.e., the facts it gives, how many questions can be made with the facts, if it is necessary to use all the facts in each question, if there are some facts that do not need to be used at all, comparison of facts without computation, estimating answers to suggested problems. All of these require a good deal of thought and these thoughts should be the pupils' and not the teacher's or textbook author's thoughts. Story problems are not the only type of problem solving and neither should they be considered the only test of a child's arithmetic reasoning ability.

Another type of problem solving does not require large numerals nor does it require a large reading vocabulary. This is called finding the solution set. (A solution set is a set of values that can be substituted for a place holder in a given problem to yield a true statement.) Look at an open sentence such as △ + □ = 8.

The values to be substituted for △ and □ can be different values or the same values. The following is the set of ordered pairs that make up the solution sets for this sentence:

(0,8); (1,7); (2,6); (3,5); (4,4); (5,3); (6,2); (7,1); (8,0)

Another example would be to change the addition sign to that of subtraction and then multiplication and then division. The pupils will find a finite set of ordered pairs in the solution set with addition and multiplication but with subtraction and division the set of ordered pairs is infinite.

In the area of geometry, problem solving can take forms other than just finding the area or the perimeter of a given geometric figure. For example, given a rectangle and a triangle discuss the relationship. Another, given a square and a circle discuss their relative sizes, i.e., how large must the diameter of the circle be to give a circle with the same area as the square. Consider the problem solving in trying to arrive at a way to determine if a given figure (of straight lines) can be drawn without retracing any line or part there of -- (Konigsberg Bridge).

Denominate numbers provide much work in problem solving as it lends itself to every computational skill as well as conversion to fractions and also other number bases. To illustrate the latter; what number base do we use in changing from inches to feet, from feet to yards, from ounces to pounds, from pints to quarts, quarts to gallons, seconds to minutes, minutes to hours, hours to days, days to weeks, weeks to months, months to years, pennies to nickels, nickels to dimes, nickels to quarters, quarters to dollars, and many more.

Determining relationships requires a knowledge of these symbols:

1. (a) $=$ 1. (b) $\neq$

2. (a) $>$ 2. (b) $\not>$

3. (a) $<$ 3. (b) $\not<$

4. (a) $\geq$ 4. (b) $\not\geq$

5. (a) $\leq$ 5. (b) $\not\leq$

1. (a) is read, "the same as" or "equal to"
1. (b) is read, "not the same as" or "not equal to"
2. (a) is read, "greater than"
2. (b) is read, "not greater than"
3. (a) is read, "less than"
3. (b) is read, "not less than"
4. (a) is read, "greater than or equal to"
4. (b) is read, "not greater than or equal to"
5. (a) is read, "less than or equal to"
5. (b) is read, "not less than or equal to"

Many mathematical principles can be investigated by giving the pupils just a few equations and asking that they supply an equation for each of the other relationships.

The development of logical and critical thinking ability in students.

The following geometrical figures are to be drawn on the chalkboard. The students are to make their own predicitions concerning the retraceability of the geometric figures and to try retracing the figures individually on their own paper at their desk. Instruct the students <u>not</u> to worry about making a wrong prediction.

Fig. 1

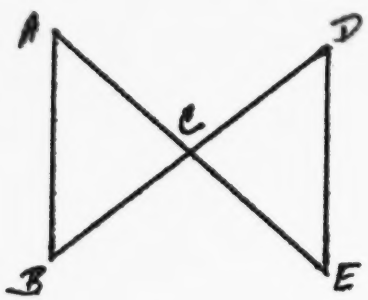


Fig. 2

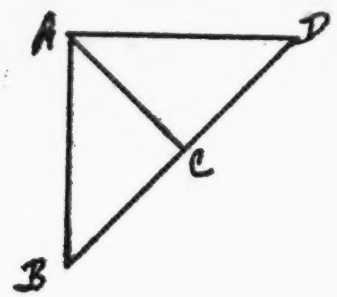


Fig. 3

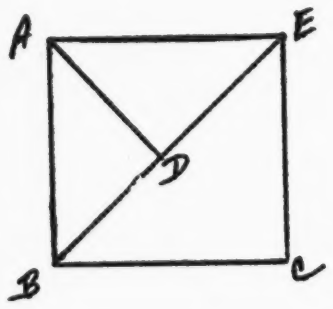
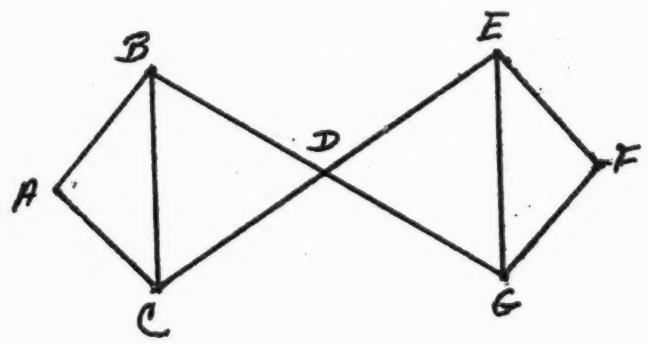


Fig. 4



In the four geometric figures predict whether the figures are retraceable by numbering your paper from 1 to 4, and write YES or NO next to each numeral. After you have made your predicitions, try to retrace the figures. You may cross a line, but be sure you do not go over a line more than once.

Were your predicitions correct?

After much effort and discussion the students should have discovered that 1 and 2 were retraceable and that figures 3 and 4 were impossible to retrace.

If you found these four predicitions and the retracing to be a challenge, try figures 5 through 13. Be sure to make your predicitions before you try the retracing.

Fig. 5

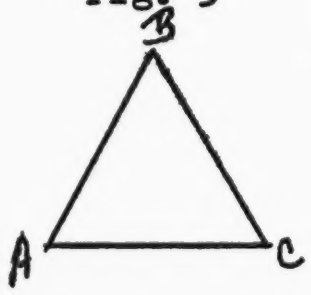
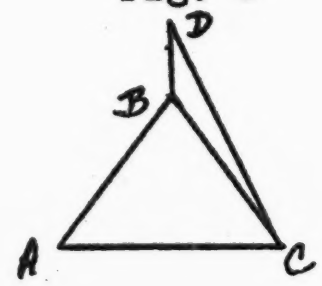


Fig. 6



Guide the students in developing a general consensus about the traceability of figures 5 through 13. Discuss with the students the traceability of each figure. Write YES under all the figures on the chalkboard that are traceable and NO under all those that are not traceable. Some of the students will insist that a nontraceable figure can be retraced. Be willing to have the students come to the chalkboard one at a time and try to retrace them. They will soon discover for themselves that they were making some little error and the figure itself really cannot be retraced under the conditions established in this lesson. Ask the students if they found any factor or mental tool they could use in helping them to make their predictions more accurate. Test the students' ideas concerning aids to determine retraceability of geometric figures on the problem figures 1-13. Discuss with the students the concept that a factor has to fail only once in order to have its validity nullified. Allow the students to come up to the chalkboard one at a

Fig. 7

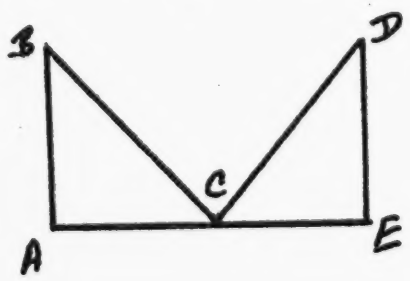


Fig. 8

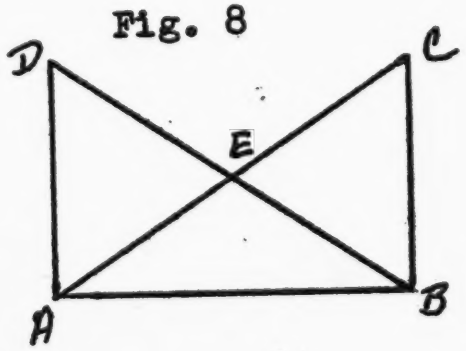


Fig. 9

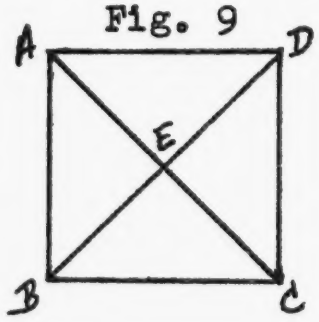


Fig. 10

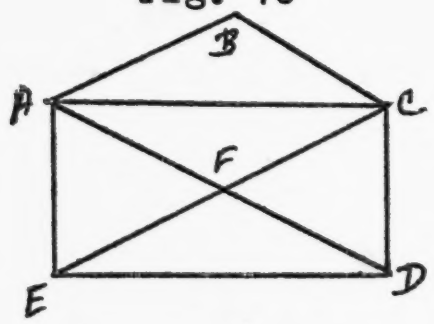


Fig. 11

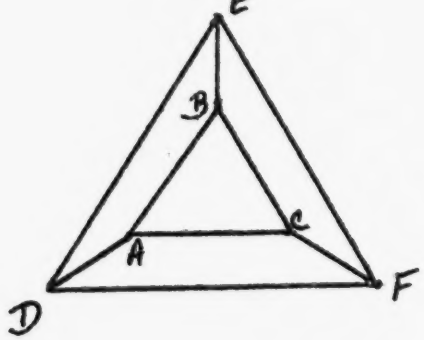


Fig. 12

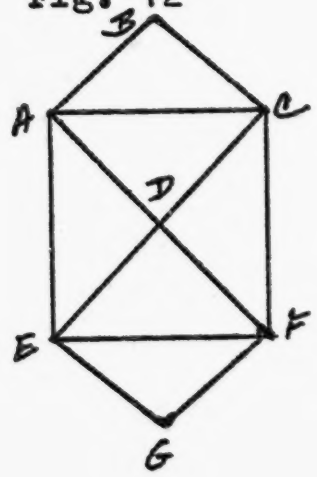
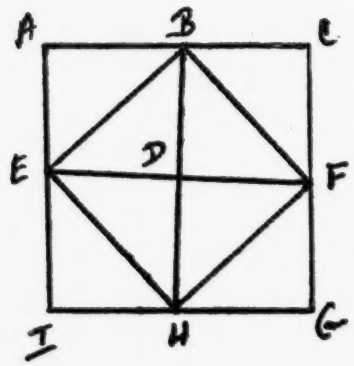


Fig. 13



time and test their ideas of a factor which will determine retraceability of a figure without first trying to draw it. Let the other students be judges and critics of the factors as they are proposed. The students will soon discover that they are unable to design a tool for determining the retraceability of a geometric figure just by examining it and not drawing it. Tell the students that there is a tool that can be used, and for their homework assignment they are to compare the characteristics of all the figures that were traceable and the characteristics of all the figures that were nontraceable. Then compare the characteristics of the traceable to the non-traceable figures to find a factor that can be used in guiding you to make a more accurate predicition concerning the retraceability of a geometric figure.

The next day's activity will involve a group discussion for testing the students' theories for a predicition factor. Remind the students that in order for a theory or factor to be operational, it has to work every time without exception. Allow the students to present their predicition factors one at a time using the thirteen examples and the chalk-board. Also allow the other students to finds dis-crepancies in each others predicition factors.

When the students seem to have exhausted their ideas (this may be stretched over several class periods), put the following chart on the chalk-board. Start filling in the chart, but allow the students to complete it. For each of the thirteen geometric figures count the number of lines originating from each lettered point and record this number on the chart. Then in the last column on the far right write YES or NO as to whether the figure is or is not retraceable.

Ask the students as a homework assignment to again make comparisons between the traceable and non-traceable figures, but this time use the completed chart. The next day again allow a group discussion and testing of theories of traceability. Again when the students seem exhausted for ideas suggest they count the number of points in each of the thirteen figures that have an odd number of lines originating from it and the number of points that have an even number of lines originating from it. This task may be accomplished by adding <u>two</u> more columns to the chart. Once these columns have been completed the students again can make comparisons of traceable and nontraceable figures. Again allow them to test theories while the other students attempt to find discrepancies in them. At this point the students should be able to discover the factor for determining the retraceability of a geometric figure.

	A	B	C	D	E	F	G	H	I	Traceability	Number of Points Even	Odd
1	2	2	4	2	2					Yes	5	0
2	3	2	3	2						Yes	2	2
3	3	3	2	3	3					No	1	4
4	2	3	3	4	3	2	3			No	3	4
5	2	2	2							Yes	3	0
6	2	3	3	2						Yes	2	2
7	2	2	4	2	2					Yes	5	0
8	3	3	2	2	4					Yes	3	2
9	3	3	3	3	4					No	1	4
10	4	2	4	3	3	4				Yes	4	2
11	3	3	3	3	3	3				No	0	6
12	4	2	4	4	4	4	2	5		Yes	7	0
13	2	5	2	4	5	5	2		2	No	5	4

Solution

The solution to this thought problem lies in the idea of counting the lettered points of the geometrical figure, that have an odd number of lines originating from it. If the number of these points is three or more then the figure is not re-traceable. However, this concept should be left to the students to discover for themselves, and the role of the teacher is only to guide the students to this discovery.

Problems without numbers

Why would any woman rather wash a mirror than wash a window of the same size? You assume that both are equally easy to work on. (The window has two sides to wash).

Two sons and two fathers went hunting, the teacher tells the class. They killed three rabbits. When the hunt was over, they divided the rabbits equally without cutting the rabbits into pieces. How was this possible? (Only three people on the hunt - a grandfather, his son, and his grandson).

A farmer seeing robins eating the cherry tree, fired his gun to scare them. Half the number flew away, but one returned. He fired a second time and again half the number flew away, but one returned. The farmer saw that there were exactly as many birds eating the cherries as there were when he fired the first shot. How many were there? (Two)

None of the following problems can be solved until some additional information is supplied. Study each problem and supply the information that will permit you to solve the problem. Be sure your information is reasonable. Then solve the problem. 10:181

We need to know the number of pounds in a bag.

1. A bag of sugar costs 65¢. What is the cost of one pound?

We need to know the width of the plot.

2. Bill was told by his father to find the perimeter of a rectangular plot he wanted to fence. The length was 22 ½ feet.

The length of the pane of glass is unknown.

3. Find the number of square inches in a pane of glass that is 2 feet 6 inches wide.

The amount of money Harold gave to the clerk is unknown.

The amount of Jim's savings is unknown.

4. The sign on the door of the candy store read, "Candy bars - 6 for 31¢." Harold bought 6 candy bars. How much change did he receive?

5. Jim earned $1.75 shoveling snow. How much did he have then to buy a birthday gift for his mother.

Tell what operation is used to answer each statement. Sometimes more than one operation can be used. Give a problem illustrating each statement.

1. The total number there are in several different groups. (Addition)

2. How far a plane can fly in five hours at a given speed. (Multiplication)

3. How many there are in several groups of the same size. (Multiplication)

4. How many equal smaller groups of a certain size can be made from a large group. (Division)

5. The total cost of several things sold at the same price. (Multiplication)

6. The number of ounces in several pounds. (Multiplication)

7. The total of several amounts of money. (Addition)

8. The average weight of four boys when you know their total weight. (Division)

9. How much greater one number is than another. (Subtraction)

10. The number there are in half a group. (Division)

11. The difference between two numbers. (Subtraction)

12. The other number if you know one of the members and the sum of both. (Subtraction)

13. The number remaining after several of a group leave. (Subtraction)

14. The price of one when you know the cost of several. (Division)

15. The number of quarts in a certain number of pints. (Division)

16. How many more cents you need before you can buy something costing more than the number you have. (Subtraction)

17. The number of pieces of a certain length that can be cut from a long piece of ribbon. (Division)

18. How many times as much one number is as another number. (Division)

19. The change from $1.00 when you buy a loaf of bread. (Subtraction)

20. A man counted his change and found that he had 100 coins equal in value to $5.00. He had no nickels. What coins did he have?

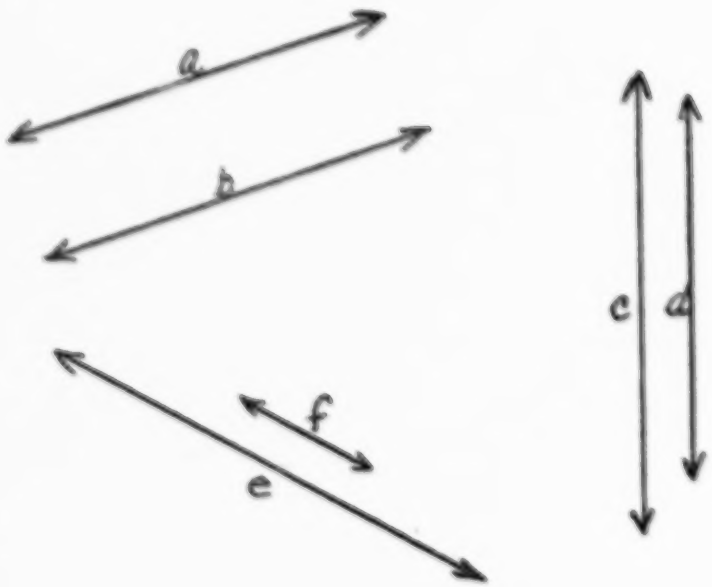
GEOMETRY

Grade 6

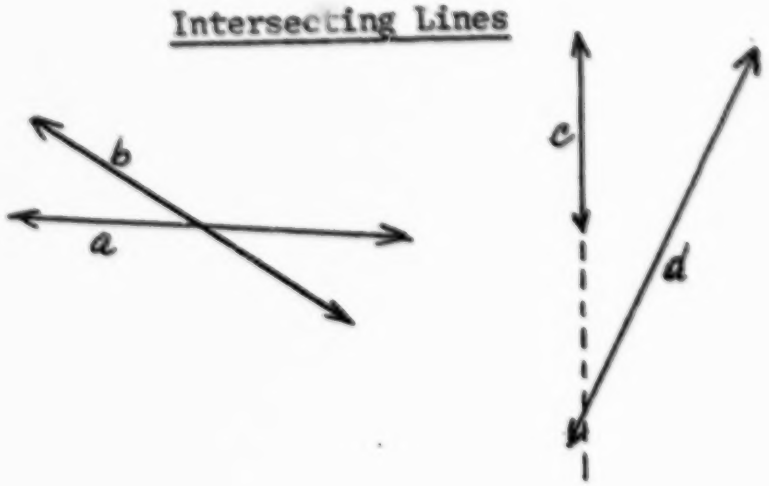
Grade 6

Lines: parallel, intersecting, and perpendicular

Parallel Lines



Intersecting Lines



Place pairs of lines on the chalkboard that represents all the possible relationships two lines have in a plane, i.e., the lines are parallel or intersecting. Show several pairs of intersecting lines that are perpendicular to each other. Bring students attention to the labeling of the lines, using a small case letter rather than naming two points of a line.

9:247

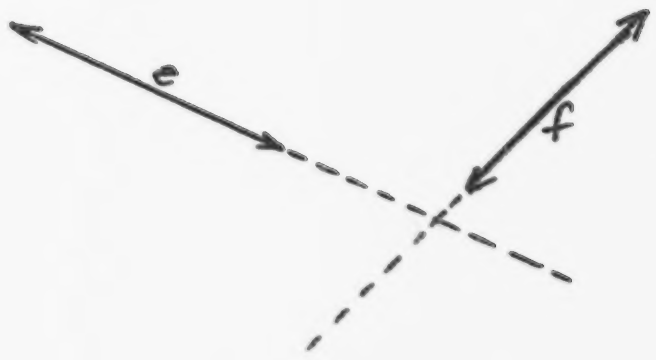
2:234-235

11:234-236

Ask students to decide which pairs of lines will intersect (meet, touch, or cross) in the plane represented by the chalkboard. Remind students that they are looking at pictures of lines and that the chalkboard is a model of a plane. Some intersecting lines are not easily seen. It may be necessary to show their intersection by extending the models of lines using chalk or string.

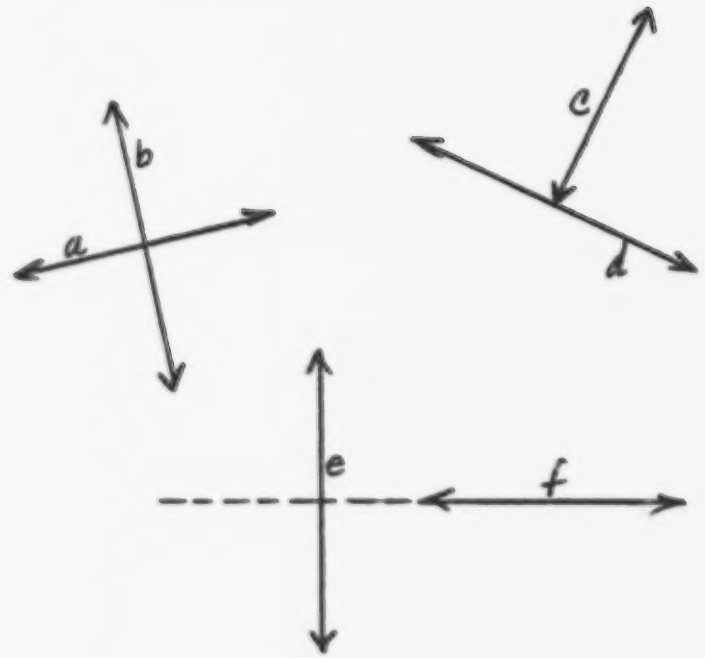
Tell students that the pairs of lines which never intersect are <u>parallel lines</u>.

Ask students to find examples of intersecting and parallel lines about them. Find lines whose intersections would not be obvious but would perhaps lie outside the model of the lines, even outside the classroom itself. Be careful to see that pairs of lines chosen by the students do lie in the same plane.



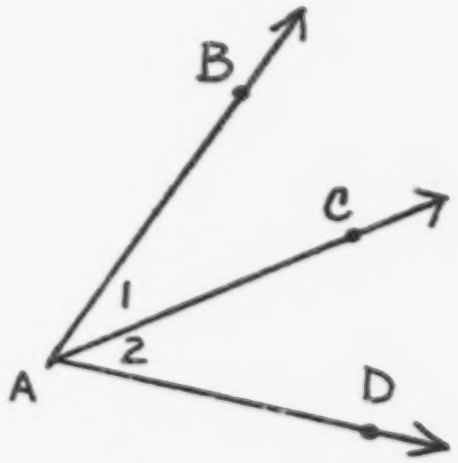
Perpendicular Lines

Show many pairs of intersecting lines on the chalkboard, some of which contain lines that are perpendicular to each other. "What angles are formed by these intersecting lines?" (right 90° angles) Explain that lines whose intersections form right angles are called perpendicular lines. Have students name pairs of perpendicular lines in the classroom. Make students prove their finding by measuring the angles with protractor or right angle model. Again be careful that models chosen to represent these intersections lie in one plane.



Angle notation

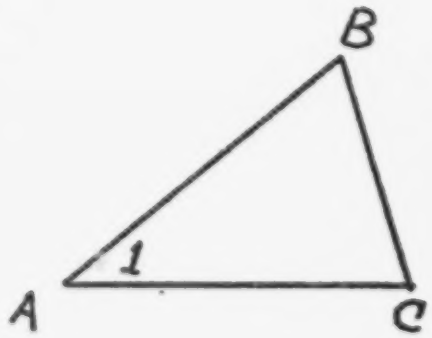
Draw angles on chalkboard like the example on the next page. Ask students to suggest ways of naming one of the angles. Encourage various responses, accepting those that truly discriminate one angle from the other.



Suggestions might include:

1) The "top" angle.
2) Angle 1
3) Angle BAC
4) Angle CAB

Lead students to see that naming an angle by some numeral or combination of three letters avoids confusion. Be sure to ask why "angle A" is not a good way to name angle 1. "Can we name an angle clearly by naming only one of its rays? (no) Why not?" (One of the rays may belong to more than one angle.)



Draw △ ABC on chalkboard. Ask students to give the best name for ∠ 1 using the names of the vertices shown. "Why doesn't ∠ ABC identify ∠ 1?" Explain that by agreement the letter of the vertex of the angle is always placed between the letters of the points named on the rays of the angles. Thus, 1 is named ∠BAC or ∠CAB.

Figure 1

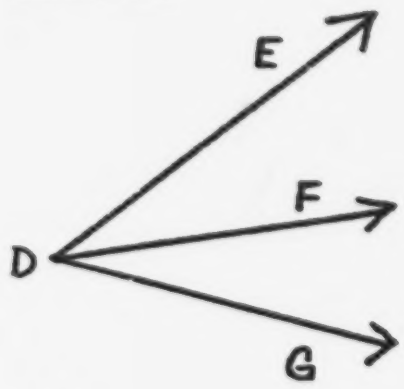


Figure 2

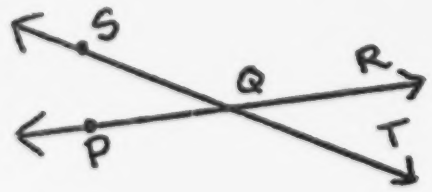
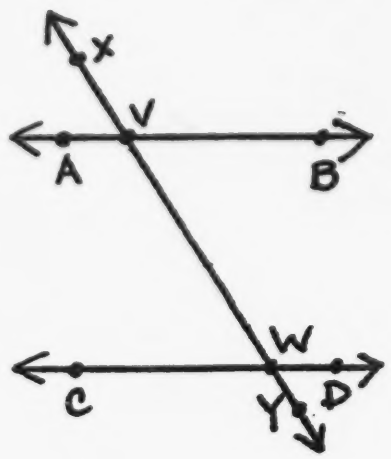


Figure 3



Give students dittoed sheets of paper showing angles formed by the union of rays and the intersections of lines and line segments:

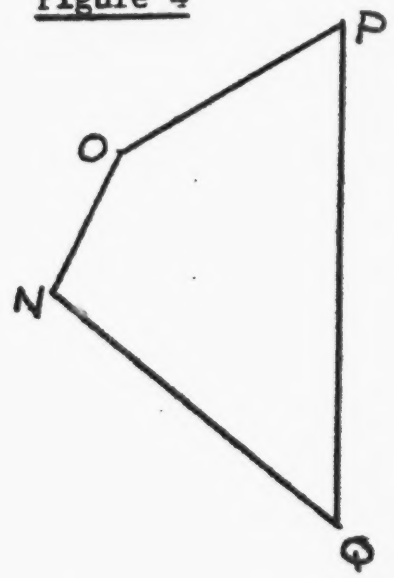
1. Name three angles in Figure 1. (∠EDF, ∠FDG, and ∠EDG)

2. Name four different angles in Figure 2. (∠SQR, ∠RQT, ∠TQP, ∠SQP) Can you find more angles? (Yes, several. ∠SQT is one)

3. Name eight angles in Figure 3. (∠AVX, ∠XVB, ∠BVW, ∠AVW, ∠CWV, ∠VWD, ∠DWY, ∠CWY)

If trouble occurs, erase letters and write numerals in the angles until students see the eight possibilities. There are, of course, more angles which we could name (see 2 above)

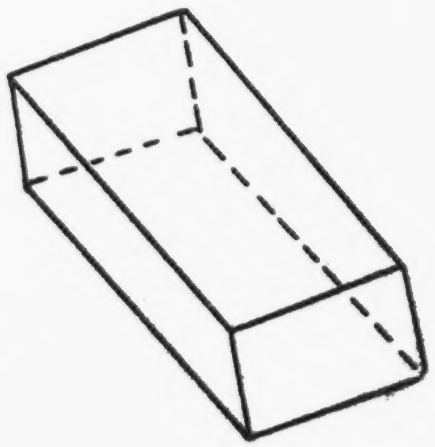
Figure 4



4. Name four angles in Figure 4. (∠NOP, ∠OPQ, ∠PQN, and ∠QNO)

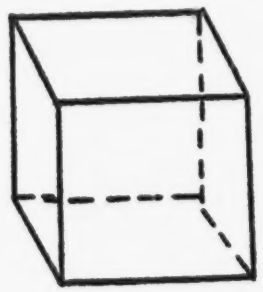
Properties of prism and cube

Give students models of rectangular prisms (including cubes) to handle and explore. (Chalk boxes, paper clip boxes, block sugar cubes, and dice) 2:210



Ask: (Emphasize vocabulary words) 11:250-251

1. How many faces does the prism have? Count them now. (6 faces)
2. How many edges does the prism have? (12 edges) 13:305-311
3. How many vertices (corners) does the prism have? (8)
4. What is the shape of a face of a prism? (square or rectangle).
5. What part of your model represents part of a line? (edge)



6. What part of your model represents part of a plane? (face)

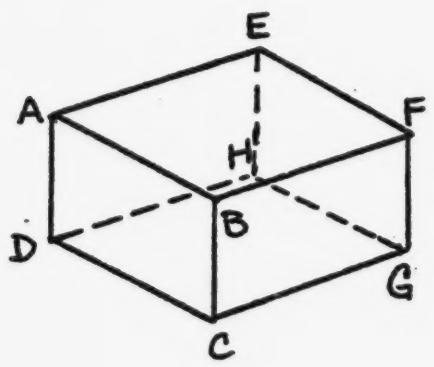
7. What part of your model represents a point? (Any part - but the most obvious would usually be a vertex)

8. How is the cube different from other rectangular prisms? (square face)

Properties of prism and cube

Draw a model of a rectangular prism on the chalkboard and label the vertices. Ask students to:

1. Name the faces of the prism by naming their vertices. (Example: ABCD names the <u>left</u> face)



2. Name the vertices of the prism. (A,B,C,D,E,F,G,H)

3. Name the edges of the prism. (AB, AD, BC, CD, etc.)

4. Name a pair of parallel line segments. (AB and EF, etc.)

5. Name two line segments perpendicular to each other (AB and BF, etc.)

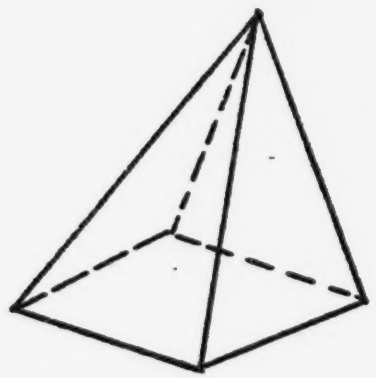
6. Name the intersection of $\overline{AB}$ and $\overline{BC}$. (point B)

7. Name the intersection of $\overline{AD}$ and $\overline{BF}$. (no intersection)

8. Name the intersection of $\overline{AD}$ and $\overline{BC}$. (no intersection)

9. Name the right angles you can see in the prism. (24 of them: ABF, ABC, FBC; ADC, ADH, CDH; etc.)

Properties of pyramid



Show the class a model of a square pyramid. Have each student construct a pyramid so he can explore the model with his hands and eyes. Ask students to describe the pyramid in their own words. "What do you see when you look at a pyramid?" Leading questions might include:

13:312-315

11:298

1. How many faces does the square pyramid have? (five)

2. Which face is called the base? (the square)

3. What is the shape of the faces other than the base? (triangle)

4. How many vertices does this pyramid have? (five)

5. How many edges can you count? (eight)

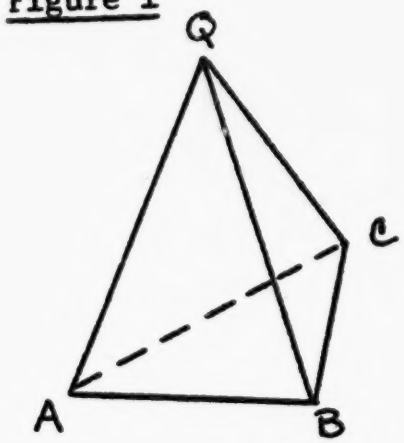
6. Can you find a vertex where only three edges intersect? (yes, four such vertices on the base)

7. Can you find a vertex where four edges intersect? (yes, the one vertex which lies outside of the base)

Properties of the pyramid

On the chalkboard, place drawings of pyramids whose bases are not rectangular or square like the examples at left. Point out to the students that all faces of a pyramid must be triangular except the base. The base of a pyramid can have three or more sides.

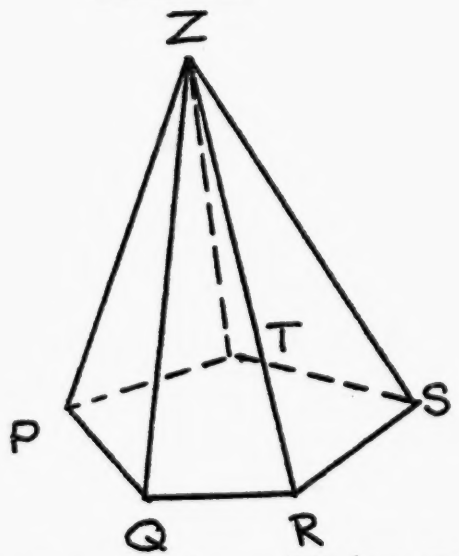
Figure 1



Pyramid - Figure 1

1. How many faces? (four)
2. Vertices? (four)
3. Edges? (six)
4. Shape of base? (triangle)
5. Name the edges of the base. ($\overline{AB}$, $\overline{BC}$, and $\overline{AC}$)
6. Name the intersection of $\overline{BQ}$, $\overline{AQ}$, and $\overline{QC}$. (Point Q)

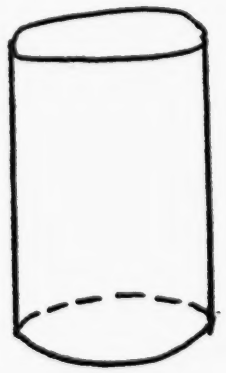
Figure 2



Pyramid - Figure 2

Help students explore Figure 2 by asking questions similar to those mentioned above. Students may make up their own questions. This is an excellent opportunity to review naming of points, lines, and planes; and intersection of sets of points.

Properties of the cylinder and cone



Collect objects which represent cylinders and cones to give to the students for inspection. Include various cans (some with top and bottom intact) mailing tubes, ice cream cones, party hats and cone-shaped drinking cups.

13:316-317

Ask:
1. How many edges does a cylinder have? (two). Trace them with your fingers.

2. How many edges does a cone have? (one)

3. How many faces does a cylinder have? (three, two ends and the lateral surface)

4. How many faces does a cone have? (two)

5. What is the shape of the base of a cylinder? (circle)

6. What is the shape of the base of a cone? (circle)

7. How is the cone like the cylinder? (Circular base)

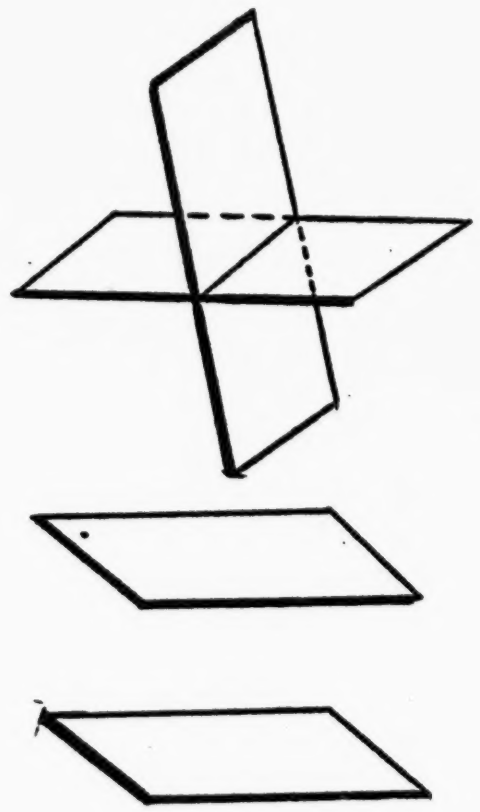
8. How is the cone different from the cylinder? (two faces only, one edge only, etc.)

9. How is the cone like the pyramid? (Related to the triangle in appearance) Both have one vertex opposite their bases.

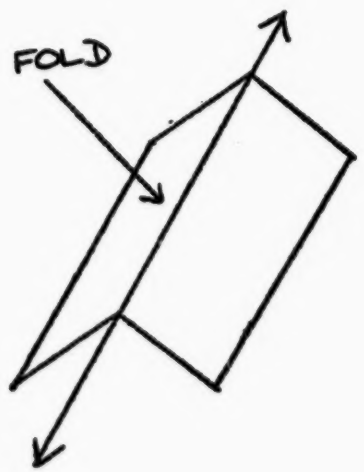
10. How is the cone different from the pyramid? (circular rather than polygonal base)

Parallel and intersecting planes

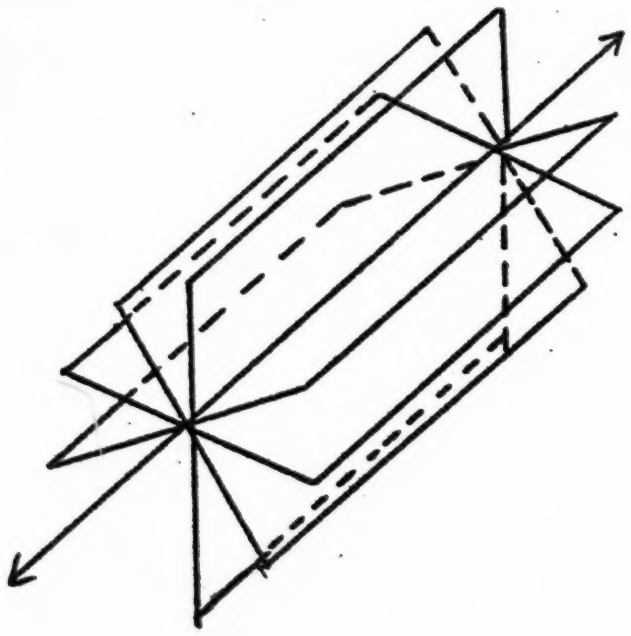
Give each student two index cards to use as models of planes. Tell students to remember that the cards represent only parts of planes. 10:216-217



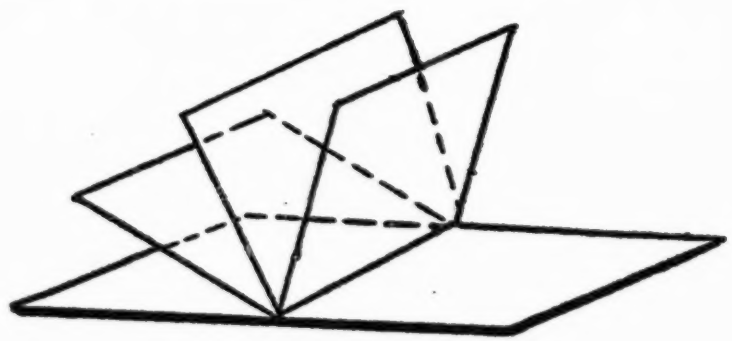
Begin lesson by reviewing the positions of two lines to each other. That is, two lines are parallel to each other or intersect each other at a point. Ask students "How can two planes be related to each other? Can planes intersect? (yes) Show the intersection of two planes with our cards." Let some students cut each index card half-way through so that the cards may easily be joined. "What set of points do the two intersecting planes share? Is the intersection a line segment or a line? (line) Why?" (Planes go on forever and so do their intersections)



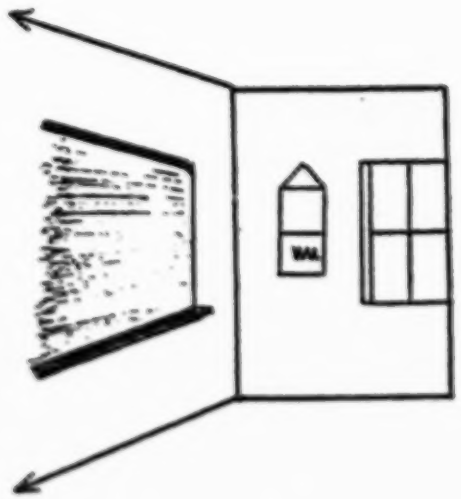
Another model of intersecting planes can be made by simply folding an index card in half lengthwise. The line of folding represents the line of intersection.



Ask students: "How many planes can intersect a line?" Have two students hold a long piece of yarn representing a line between them. Let students see how many index cards they can hold to the yarn line. "Are there more planes intersecting this line than we are showing? (yes) How many planes can intersect a line? (more than we can count)



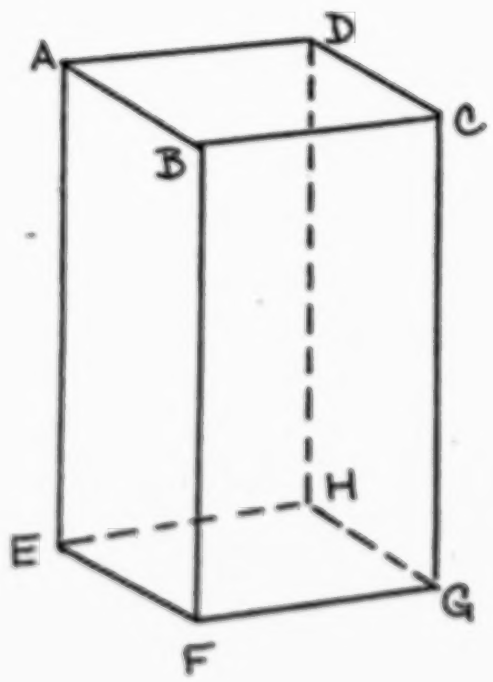
Hold up a thin book before the class. Ask "Can anyone use this book as a model to show what we have learned about planes and lines?" (a variety of suggestions can be made)



Lead students to find examples of intersecting planes in the classroom. The ceiling and one wall suggests intersecting planes whose line of intersection is obvious to students. Many examples should be found and their intersections named.

Ask class to think of the ceiling and floor as planes. "Do the planes suggested ever intersect?" (no) Explain that these planes are <u>parallel</u> planes.

Invite students to use their index cards again; this time to show parallel planes. Look for examples of parallel planes in the classroom, i.e., opposite walls, book shelves, the top and bottom sheets of a stack of paper, the desk top and floor, etc.



Using a large rectangular box for demonstration, ask students to point out parts of the box that illustrate parallel planes, intersecting planes, and lines of intersection. Draw a picture of a rectangular prism on the chalkboard.

Ask:

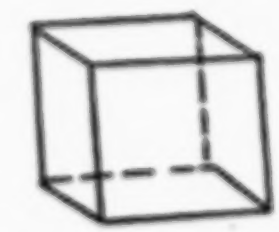
1. What two planes intersect at $\overline{BC}$? (ABCD and BCGF)
2. What plane is parallel to ADHE? (BCFG)
3. Name the intersection of ABEF and ADHE. ($\overline{AE}$)
4. What two planes have point G in common? (Several pairs: BCGF and DCHG, BCGF and EFGH, DCHG and EFGH)

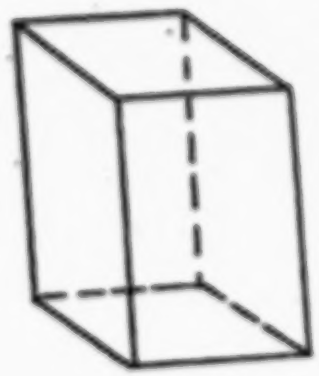
5. Name three pairs of parallel planes. (ABCD and EFGH, BCGF and ADHE, ABFE and DCGH)

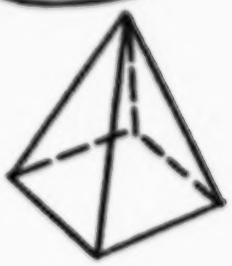
Constructing three dimensional models

Shown on the following pages are patterns to help students make their own three-dimensional models. The models should be made of heavy paper so that they will resist sagging, tearing, and frequent handling.

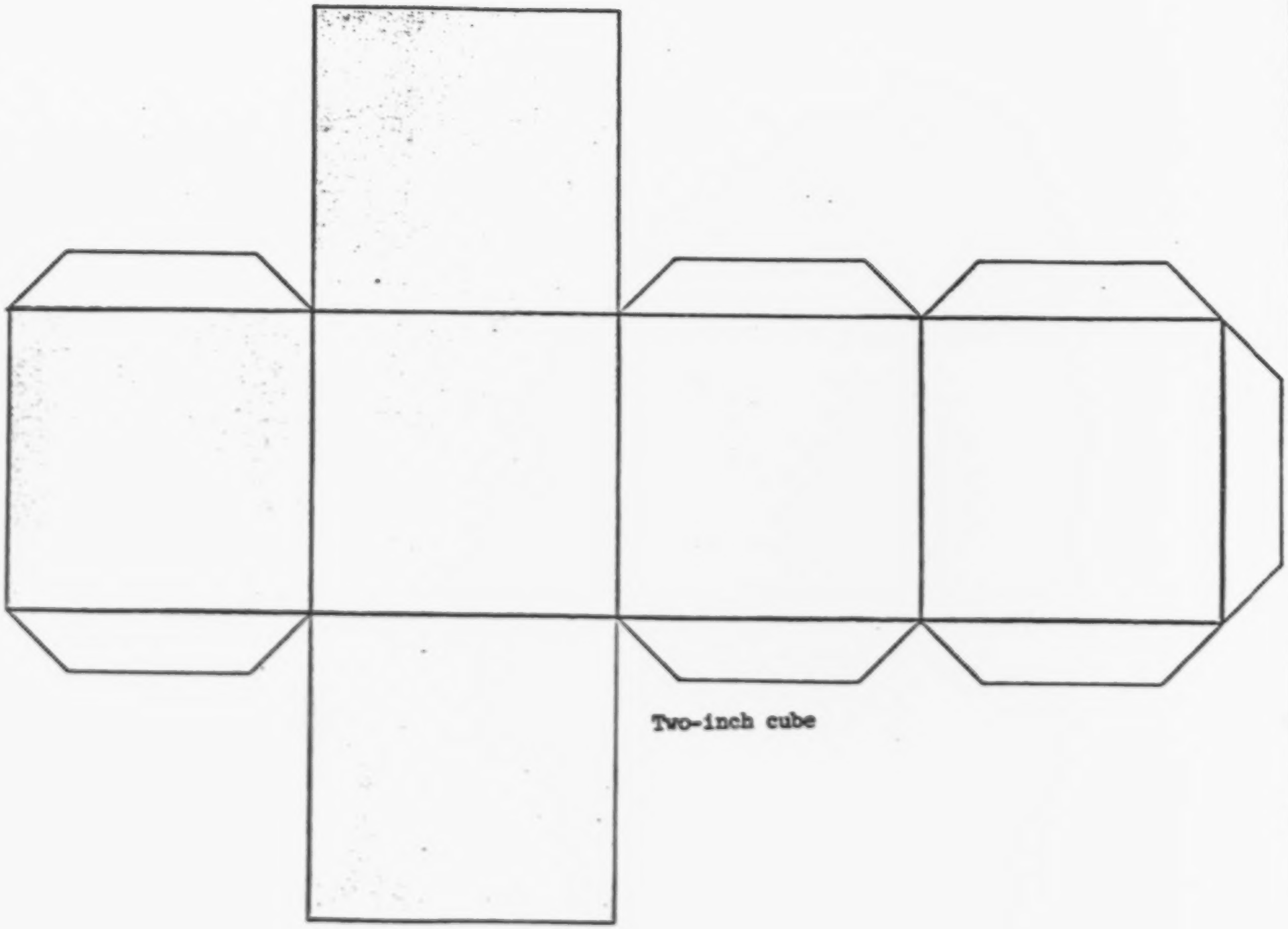
2:212
4:74-79
8:26-32





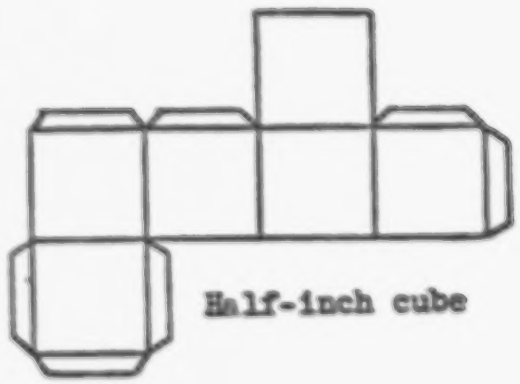


Cube

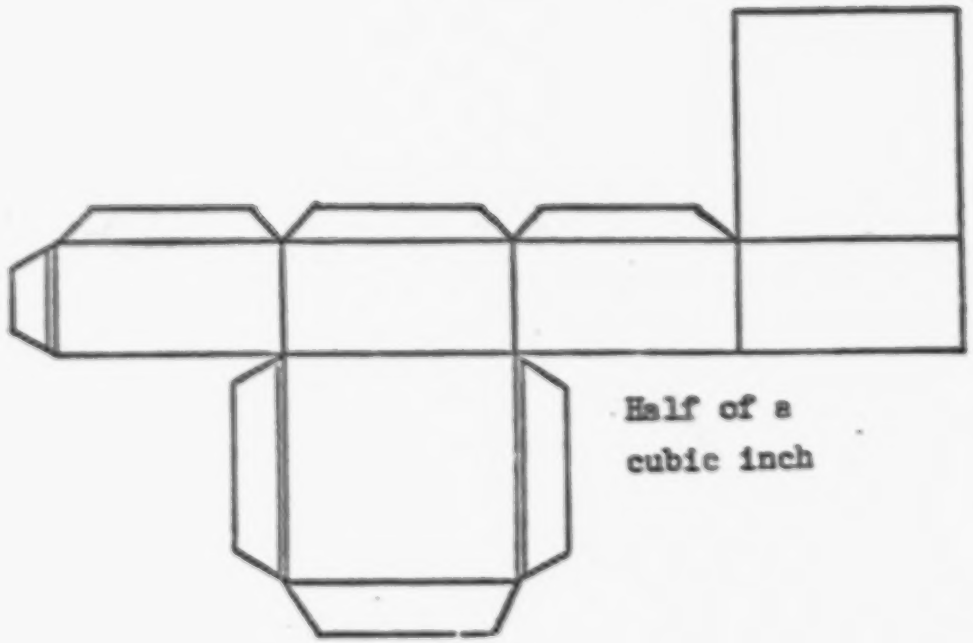


Two-inch cube

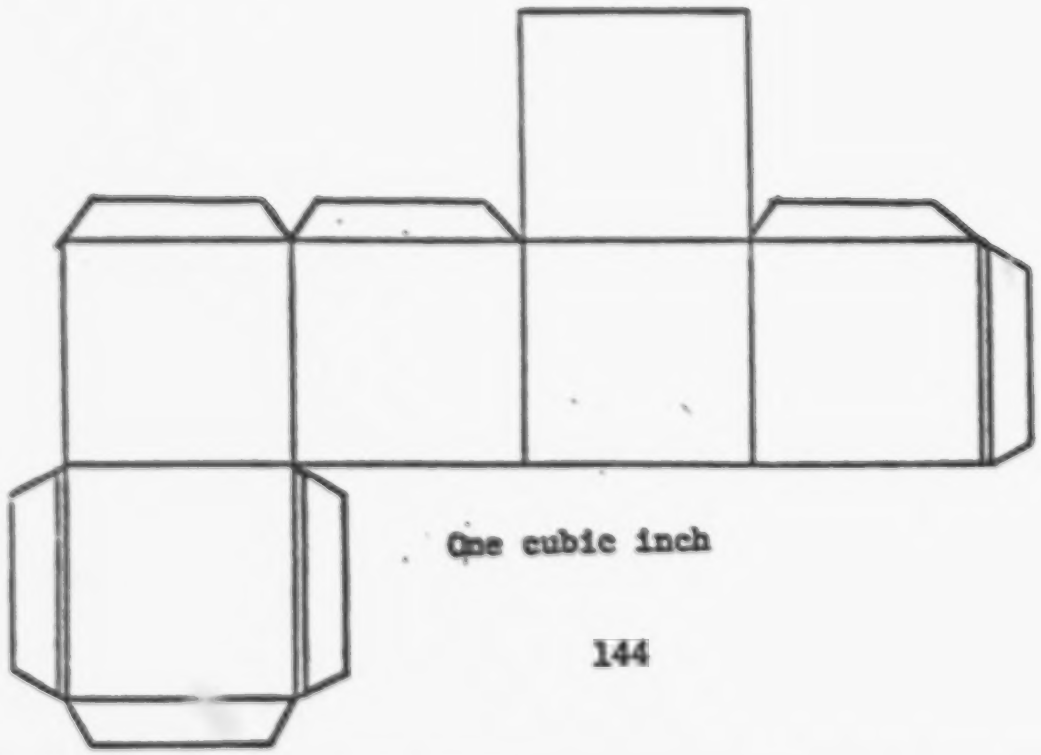
The Cube



Half-inch cube



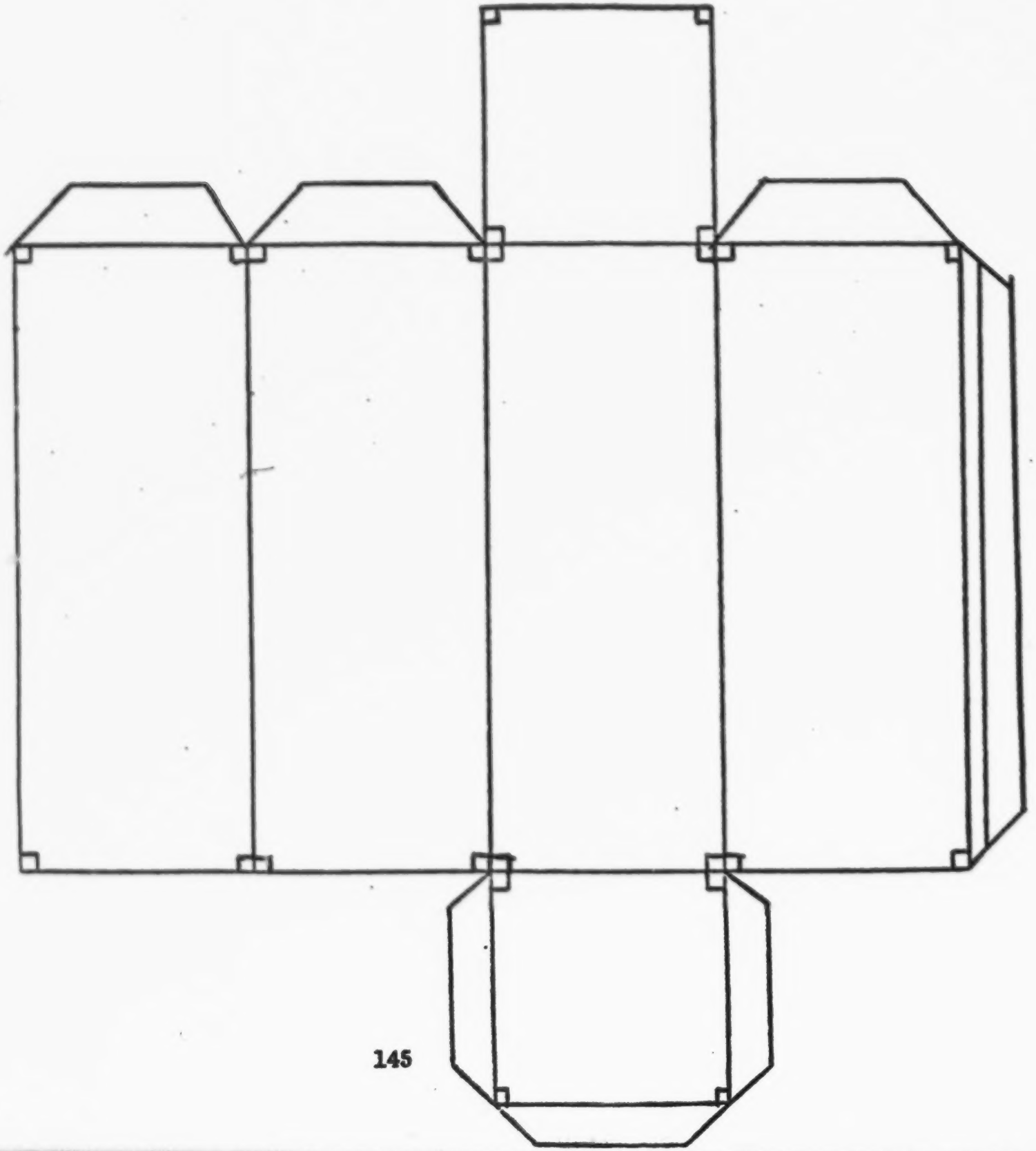
Half of a
cubic inch



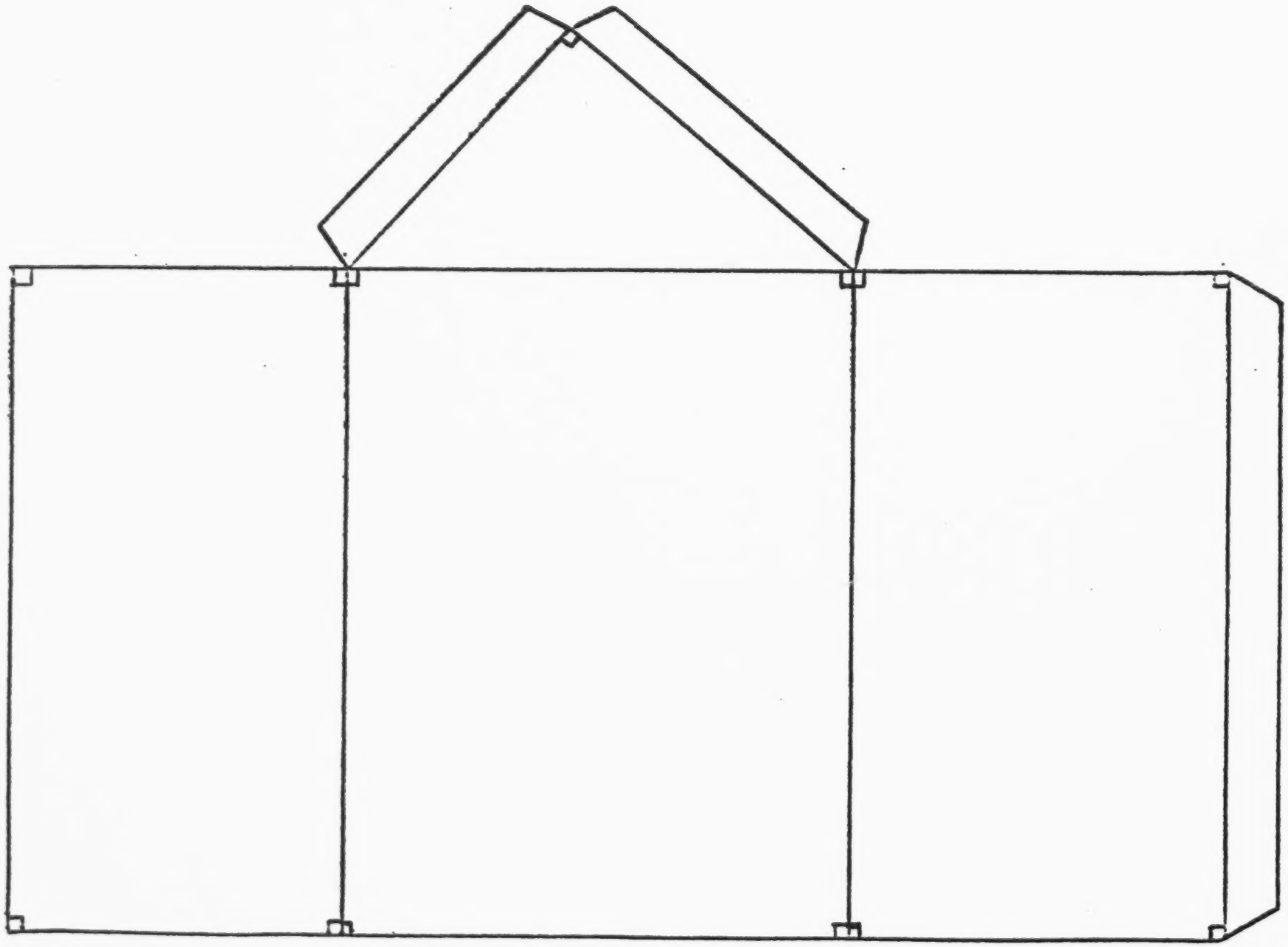
One cubic inch

Rectangular

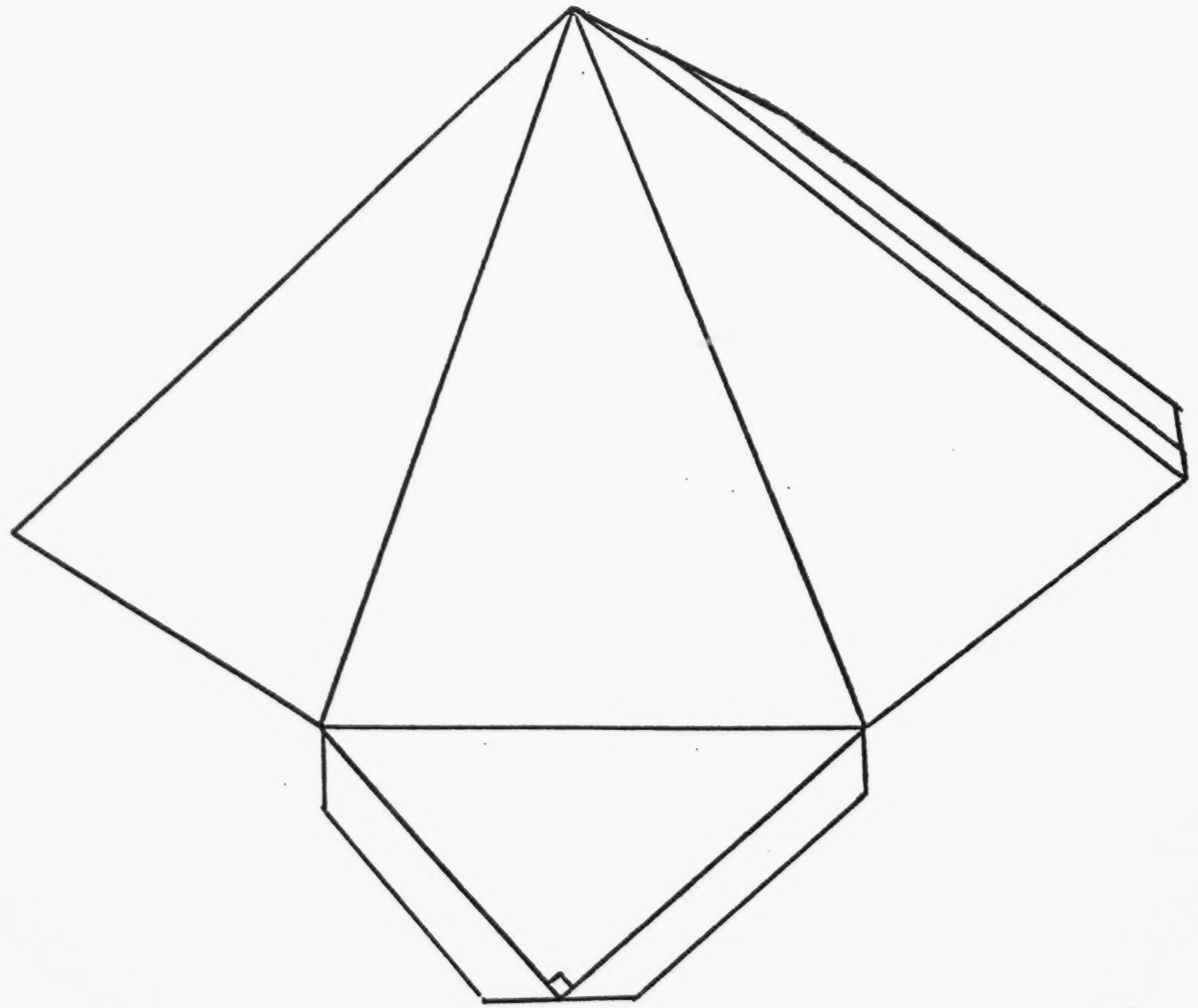
Right Prism



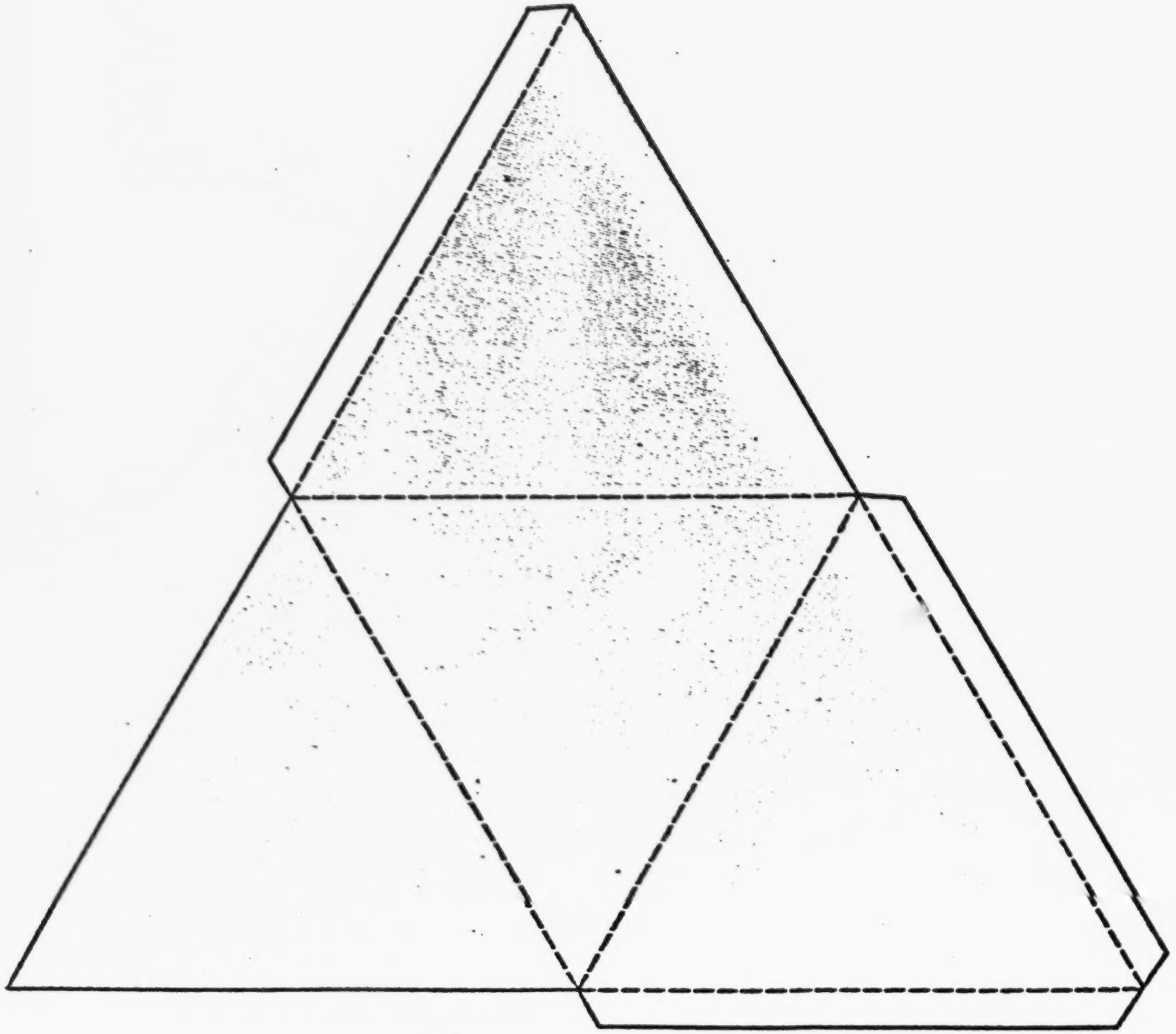
Right Triangular Prism



Triangular Pyramid
or
tetrahedron

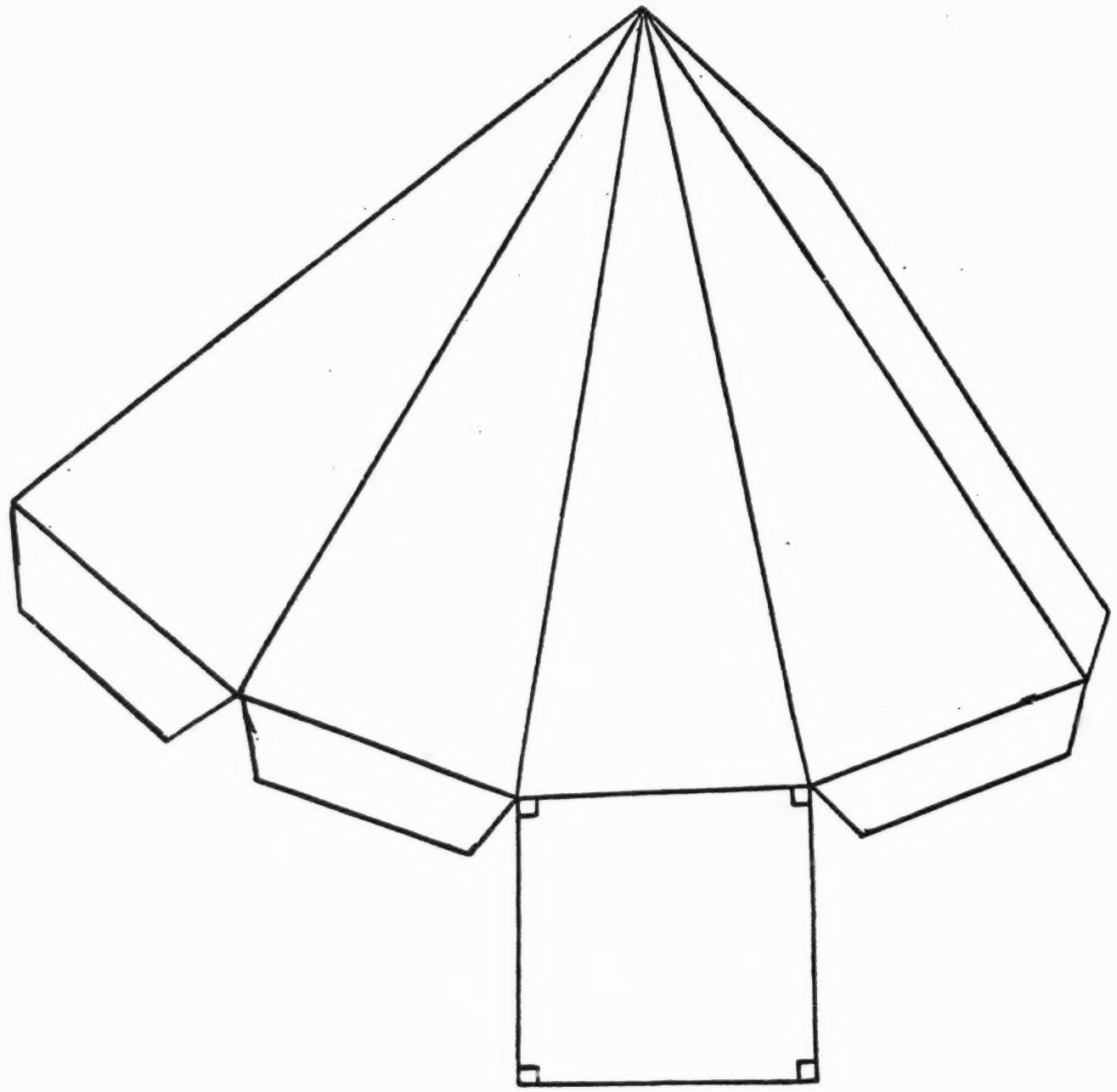


Right Triangular Prism or regular tetrahedron



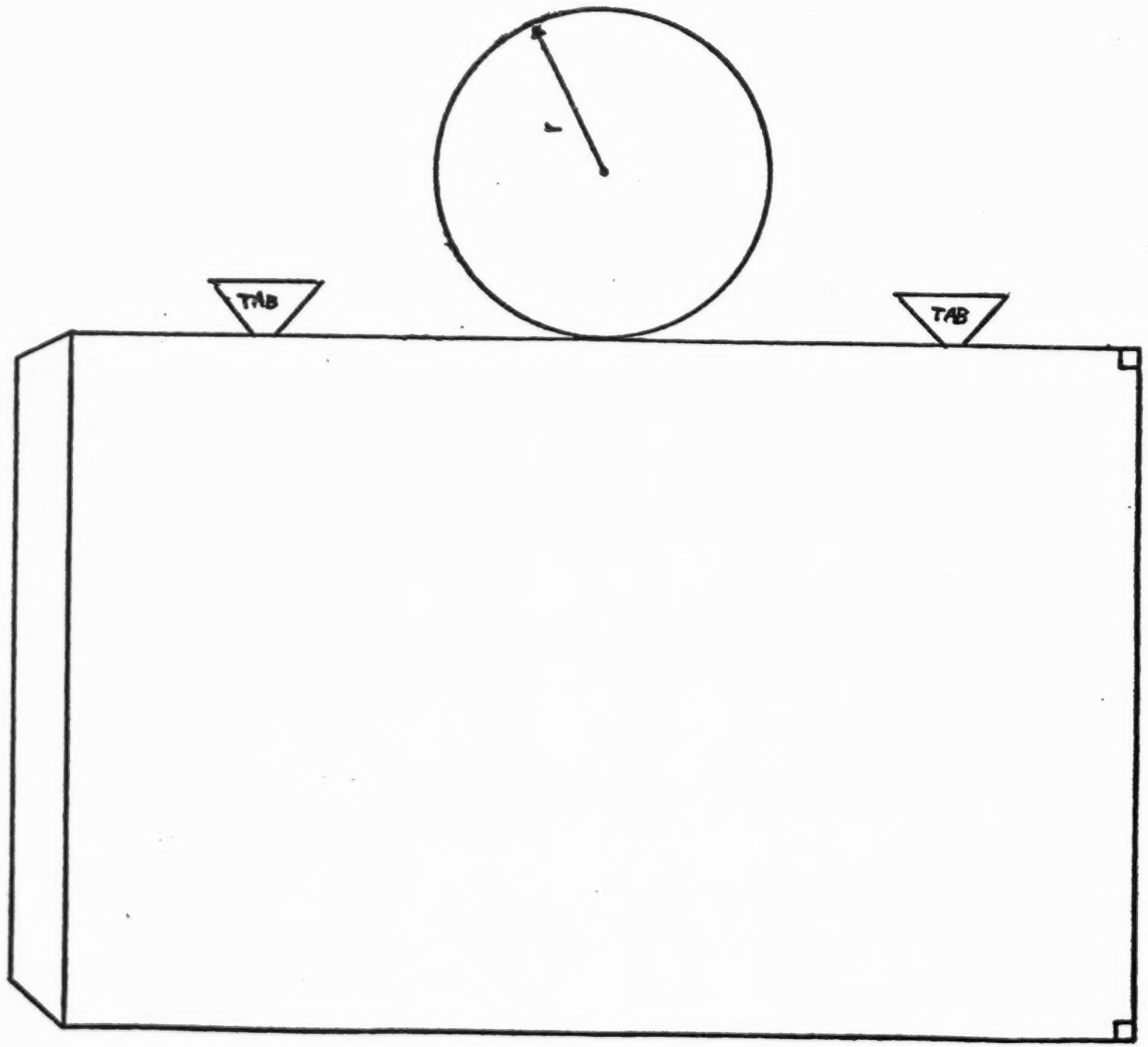
Regular Square

Pyramid



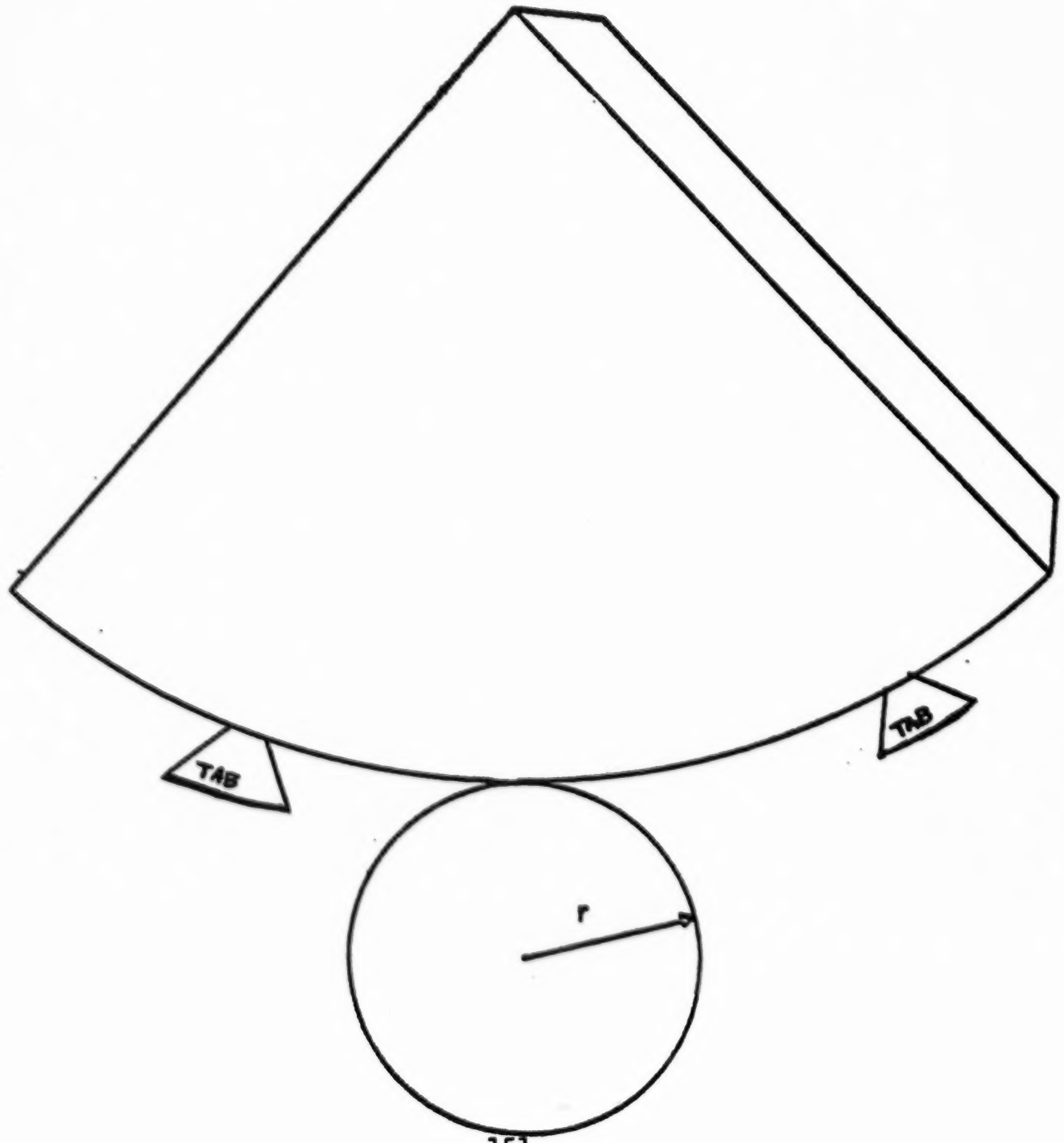
Right Circular

Cylinder



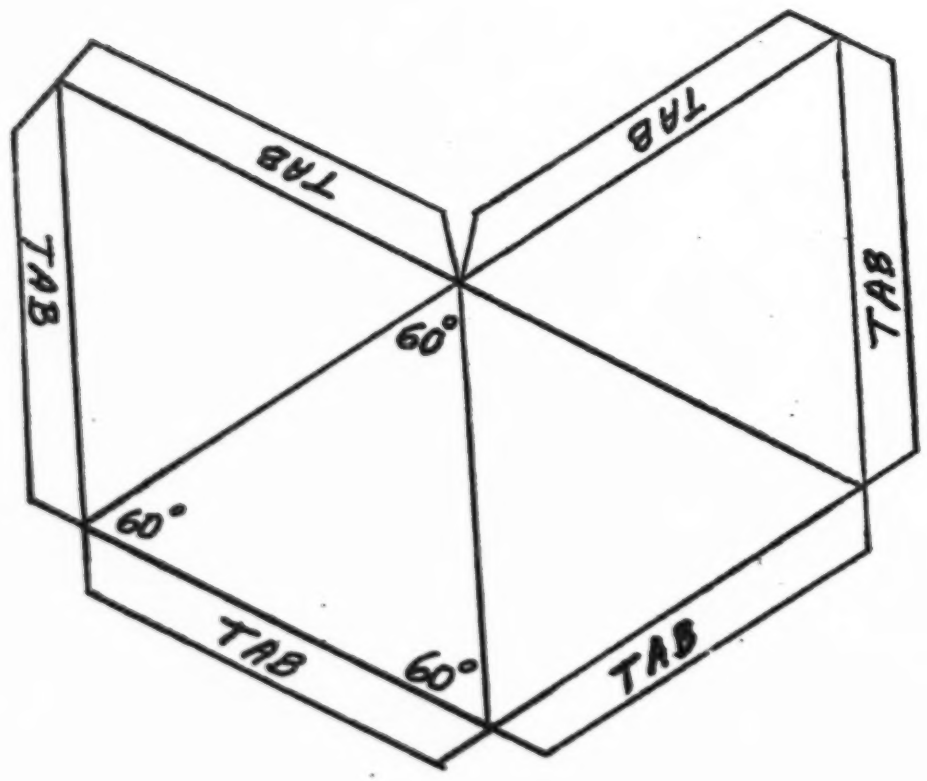
Right Circular

Cone



Advanced constructions:

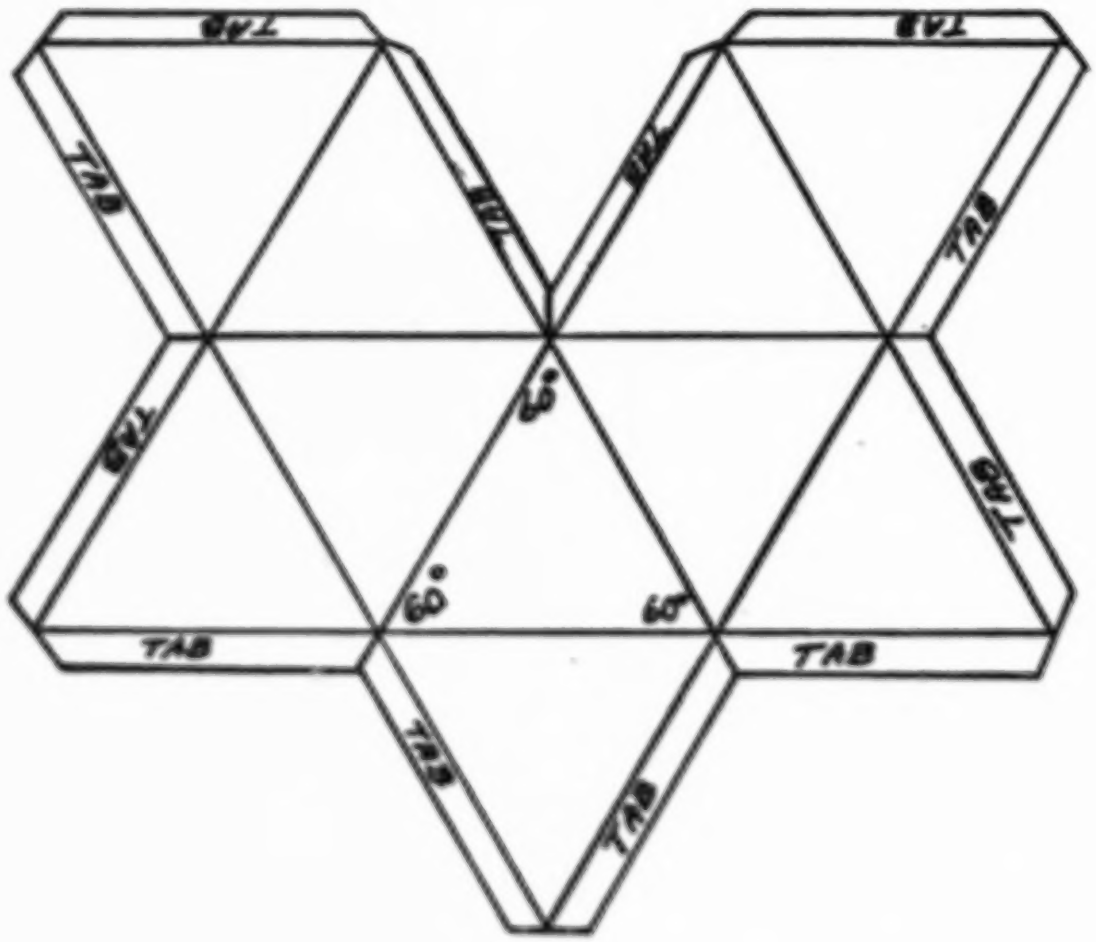
A. Octahedron



The octahedron to the left consists of two pyramids base to base. The length of the sides of the triangle can be of any value, however this pre-determined length must remain constant throughout the construction. The triangles making up the figure are equilateral and have interior angles of 60°.

The pattern shown is for half of the figure. Make up two identical pyramids, then join them together.

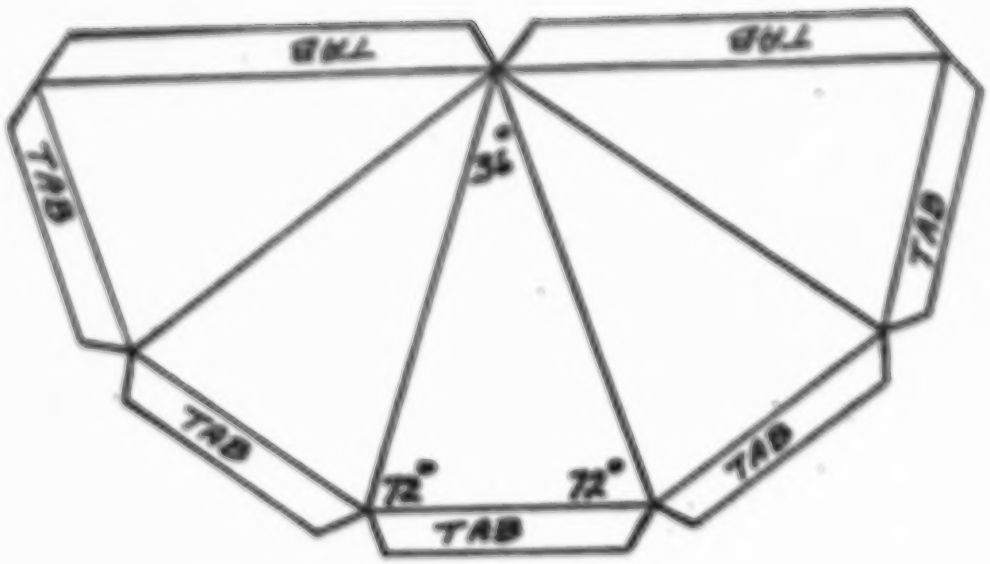
B. Icosahedron



The icosahedron consists entirely of equilateral triangles all the same size. The length of the sides of the triangle can be any value, however this pre-determined length must remain constant throughout the construction. There are 20 equilateral triangles making up the construction, each having interior angles of 60°.

Start by making up a group of 10 triangles as shown in the diagram. Next construct a second group of 10 and join the two pieces together.

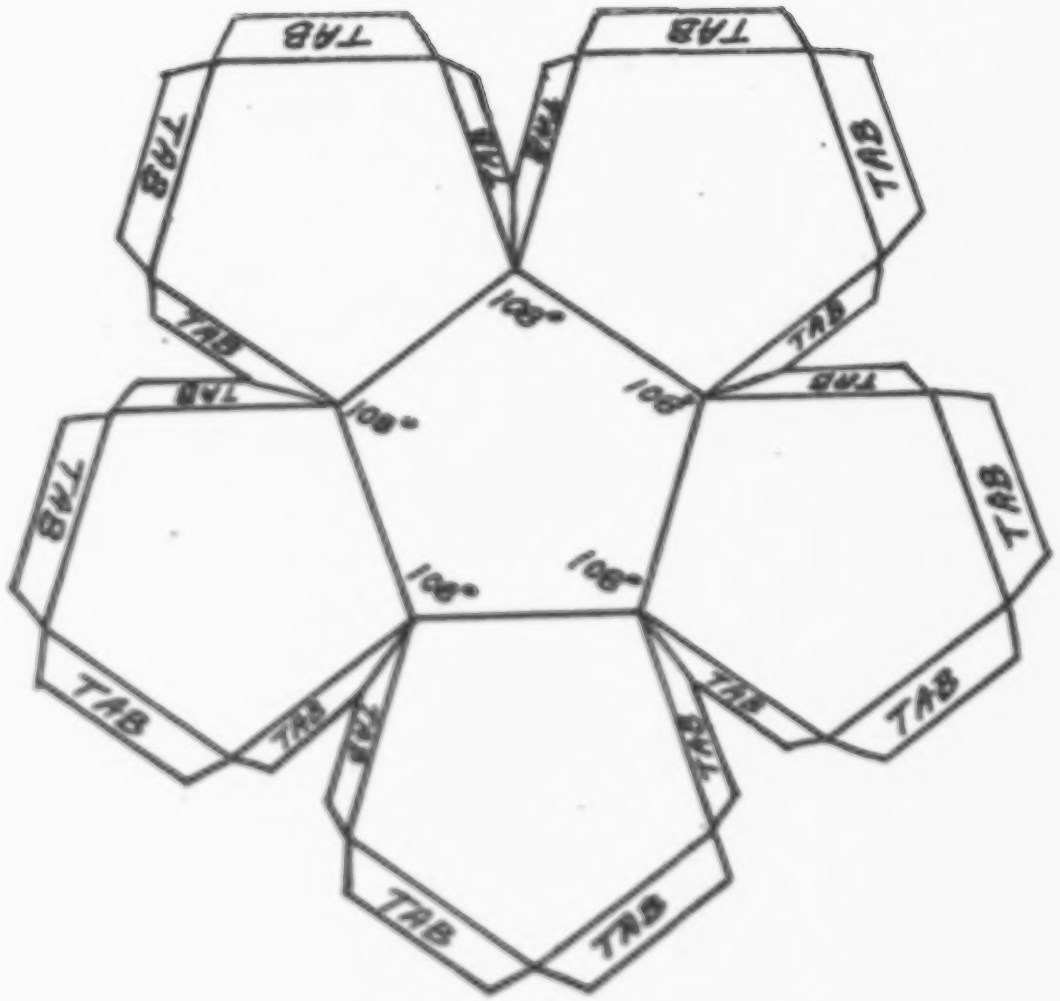
C. Stellated Dodecahedron



The small stellated dodecahedron is easier to make than it looks. Each of its points is a five-sided pyramid. The pattern shown is for one pyramid. You will need a total of 12. Start by making up one pyramid and attaching five more around its base. A second similar group of six completes the figure.

Remember to measure the indicated angles of each triangle carefully. The size of the pyramid is determined by controlling either the length of the base of each triangle or its vertical sides. However, whatever length is chosen, it must remain constant throughout the construction.

D. Dodecahedron



The dodecahedron consists of 12 identical five-sided pentagons. The length of the sides of the pentagon can be any value, however this predetermined length must remain constant throughout the construction. The pattern at the left is for half of the figure. Make up two identical groups of six each, then join them together.

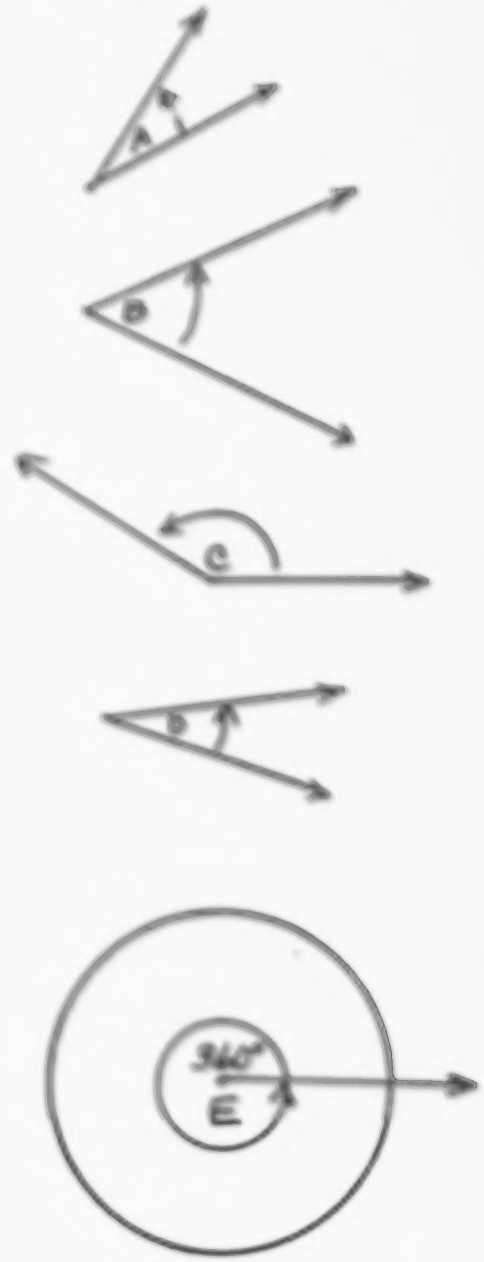
Remember that each interior angle of the pentagon consists of 108°.

MEASUREMENT

Grade 6

Grade 6

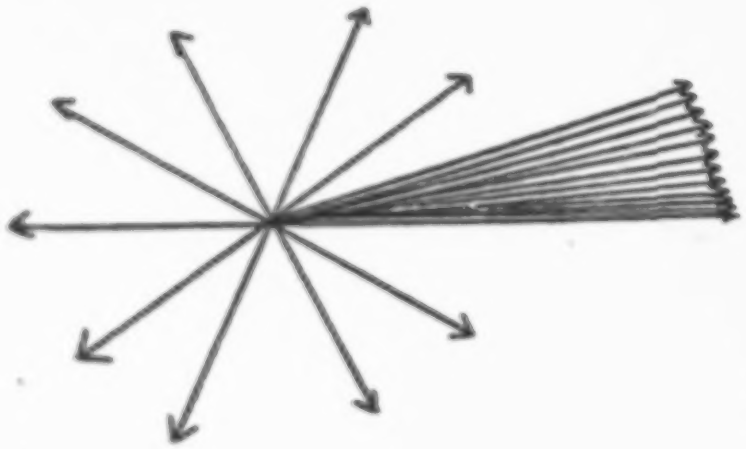
Measuring angles with a protractor



MEASUREMENT

Remember - an angle is the union of two rays; an open geometric figure formed by two rays which have the same endpoint. This common endpoint (the only common element of the two sets of points) is called the vertex of the angle.

Up to this point, we have measured angles by our eye alone. How reliable is this kind of measuring? Can you tell by looking that angle A is smaller than angle B, angle C, and angle D? The decision is easy enough for angle C, but angle D is difficult. What we need is a unit of measure. The need was satisfied by the Sumerians over 4000 years ago. It was such a good choice that we have used it ever since. We call this unit of measure a degree. There are 360 degrees in a full circle.



If we lay off 360 of these unit angles using a single point as a common vertex, then these angles together with their interiors cover the entire plane.

Provide each student with white drawing paper, a centimeter ruler, a compass, a pair of scissors, and the model of a 15° angle.

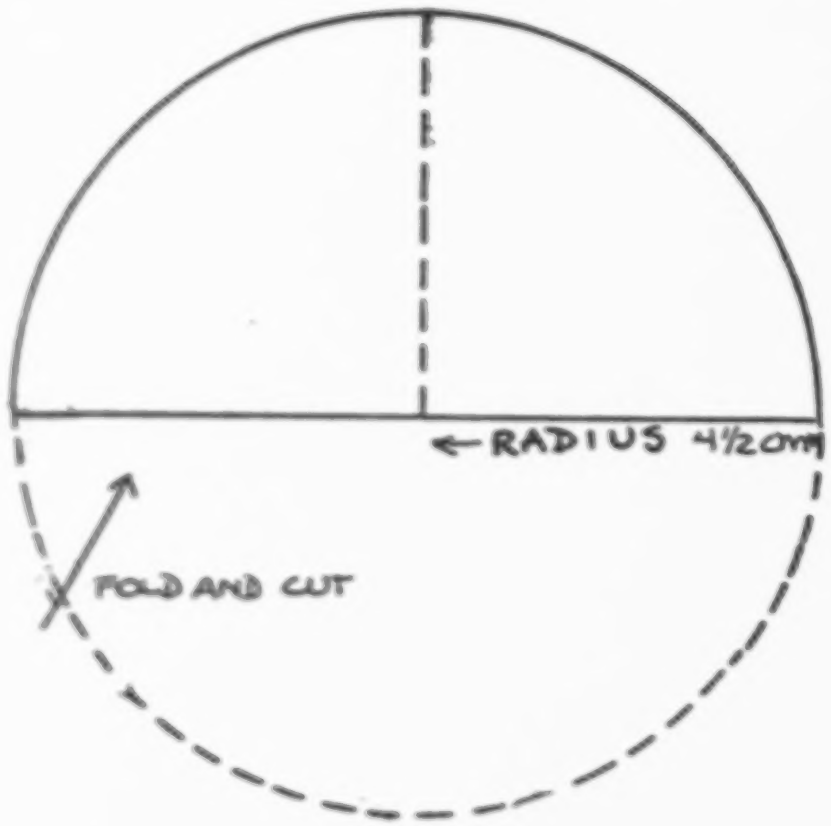
1) Draw a circle with a $4\frac{1}{2}$ cm. radius.

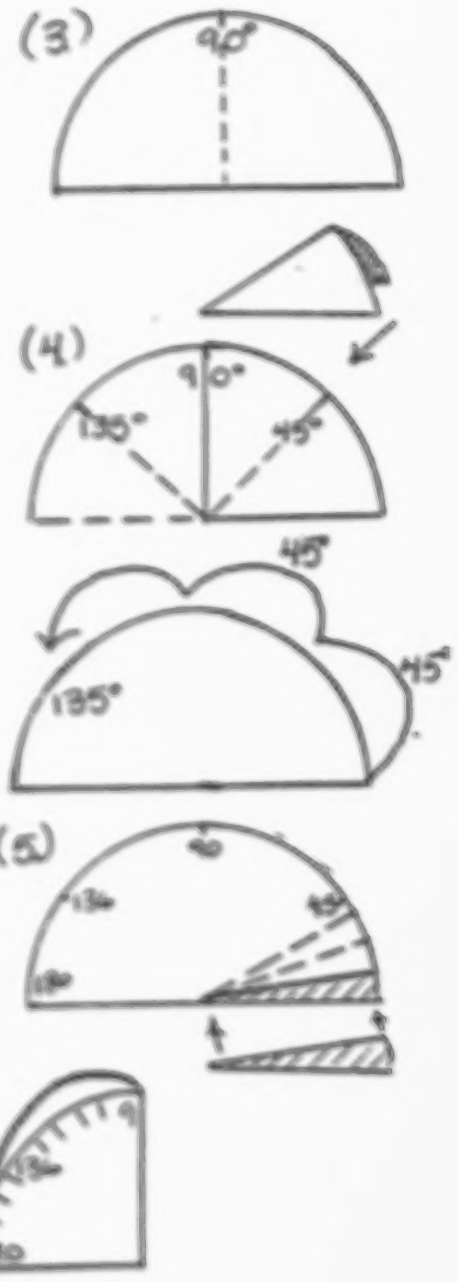
2) Fold the circle carefully and cut it in half.

3) Fold it in half again. What kind of angle have we?

A right angle (or 90°) with which we are already familiar.

Open the paper and mark the fold "90°."

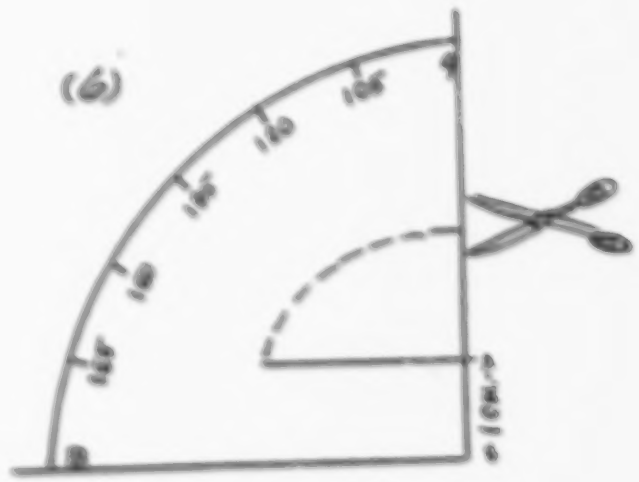




4) Fold in half again. This is half of your 90° angle, therefore it is a 45° angle. Mark your 45° angle and your $\underline{135^{o}\text{ angle}}$, which is three 45° angles.

5) Unfold your protractor and carefully place your model 15° angle unit upon it. Trace 15° angles side by side until you get to 180°. You now have a single-scale protractor.

6) As a last step, have the class fold their protractor in half again. Draw a circle on the interior with a $2\frac{1}{2}$ cm. radius. Draw a bar 1 centimeter thick at the bottom. Cut out this shape on the fold. (Draw directions and measurements for this cutting step on the blackboard).



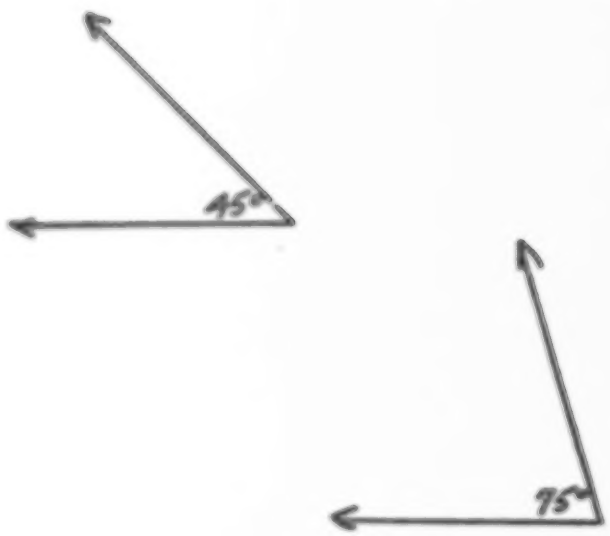
Measuring angles which open to the right

On the blackboard, using a large model of a single-scale protractor, the teacher should measure one or two angles, whose base rays point toward the right of the vector. (Only angles which open to the right should be measured at first, since the students' protractors have a single scale). Remind them to align two points: the index mark (in the center of the protractor's diameter) lines up with the vertex; and the zero on the scale lines up with one of the rays. Provide dittoed work sheets and have the children measure several angles (to the nearest 15°).

Adding another scale to the protractor

Discuss angles which open to the left. Can we measure things from the left as well as from the right? (yes). In order to measure these, we must have a new scale on our protractors: one whose zero point is toward the left of the scale.

Have the children put the second scale on their protractor (this time from left to right) and measure angles using the new scale.



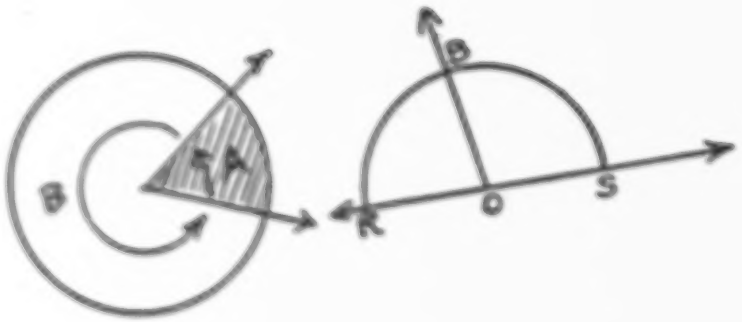
Have the students draw pictures, using both scales, of angles having these measures:

(a) 135° (b) 15° (c) 75° (d) 90° (e) 20°
(f) 165° (g) 45° (h) 120°

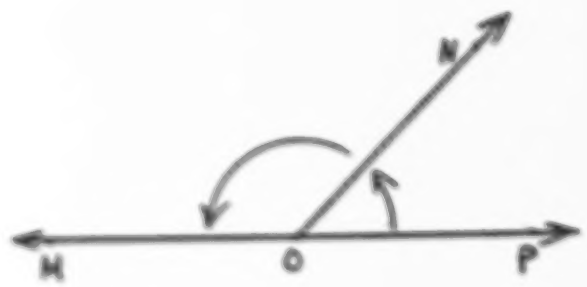
Using a commercial protractor

Provide the children with the protractors from the backs of their 6th grade books, or use ones you may have available in the school. Compare the one they made with the commercial one. Using the commercial one, have them measure angles and construct angles to fit specified measures.

Draw pictures of angles having these measures:
(a) 42° (b) 91° (c) 193° (d) 12° (e) 33°



1. What is the measure of the shaded angle? (60°)
2. What is the measure of the unshaded angle? (300°).
3. What is the measure of $\angle$ ROB? (80°).
4. What is the measure $\angle$ BOS? (100°). $\begin{array}{r} 180 \\ -\ 80 \\ \hline 100^{\circ} \end{array}$



5. Angle NOP is 45°. By <u>subtraction</u>, what is $\angle$ MON? ($180^{\circ} - 45^{\circ} = 135^{\circ}$)
6. What is the sum of both numbers indicated at any point on the scale of a protractor? (180°) (Example: 45° and $135^{\circ} = 180^{\circ}$).

Sums of angle measures

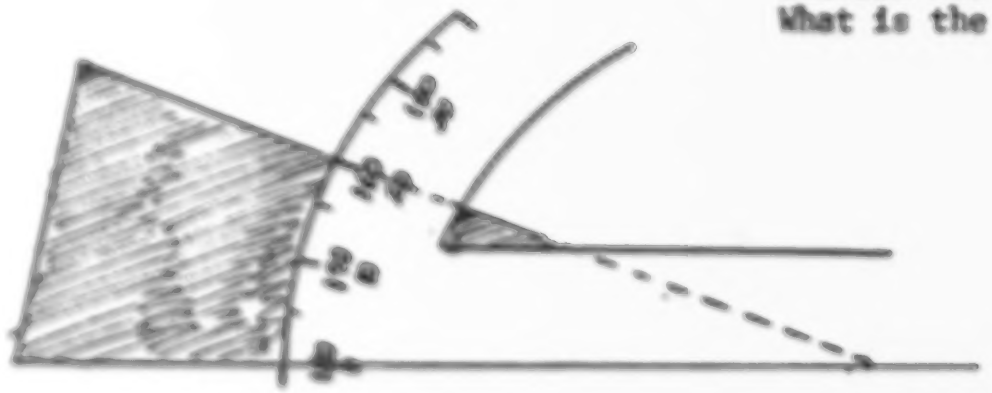
The measure of ∠ ABC is 35°.

The measure of ∠ EDF is 15°.

The measure of the two angles is 50°.

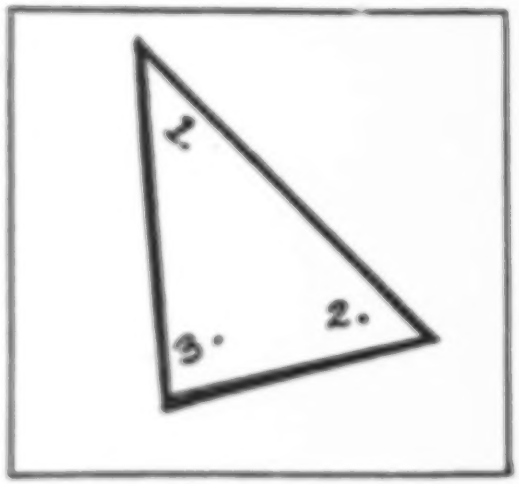
Use a protractor to measure several angles and find their sums. When we say the sum of two angles, we mean the sum of the degree measurements. 2:142-143

Sum of the angles of a triangle

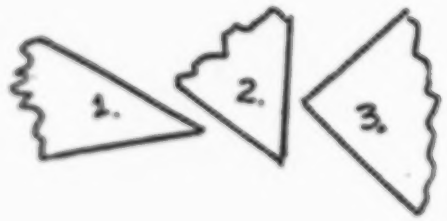


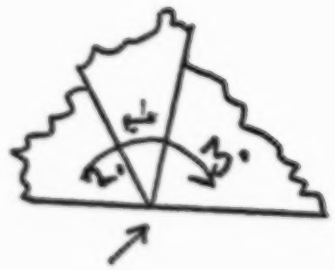
Put several large triangles on the blackboard and ask for volunteers to measure their angles. What is the sum of the angles of this triangle? (180°)

Put up a different triangle and have someone measure its angles. What is the sum of the angles of the second triangle? (180°). Have the children construct, with their ruler, a triangle of any size at their desks. Have them cut out their triangle, tear off three corner, and fit the three pieces together at a given point. They will see the 180° angle

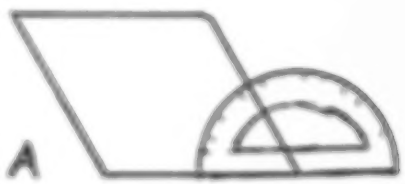


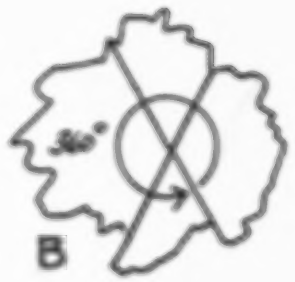
which is formed and can then assume that the sum of the angles of a triangle is 180°.



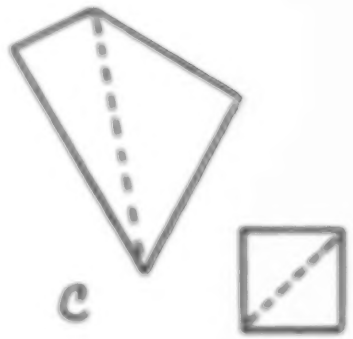


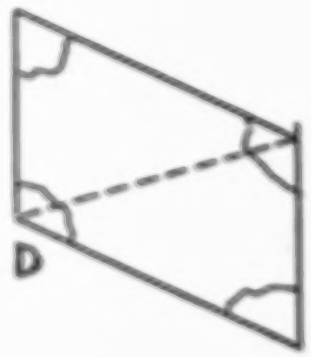
Sum of the angles of a quadrilateral (4-sided polygon)





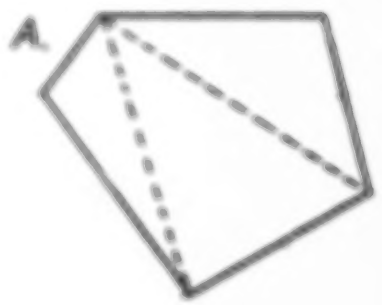
Have the children construct at their desks any four-sided polygon. Then have them use their protractors to measure the four angles (See Figure A). What is the sum? (360°) Have them draw another quadrilateral, cut it out, tear off the corners, and see if all the pieces fit on a point to make a complete 360° circle (See Figure B). They can now see that the sum of the angles for any quadrilateral is 360°.





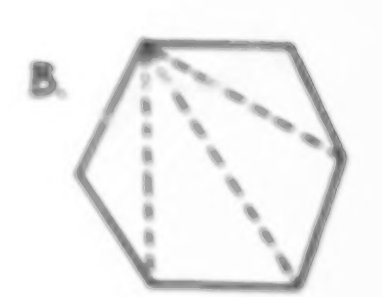
A quadrilateral can be divided into two triangles if you construct a diagonal line from one corner to the opposite corner (See Figures C). If the sum of the angles of a triangle is always 180°, then wouldn't a quadrilateral be twice 180°, or 360°? (yes, $2 \times 180^{o} = 360^{o}$)

Sum of the angles of a 5-sided polygon



How many triangles in a pentagon if one constructs two diagonals that don't cross (See Figure A)? (3) How many degrees in the angles of one triangle? (180°). Three triangles? ($3 \times 180^{\circ}$ or 540°) Measure the angles of a 5-sided polygon on the chalkboard and see if it comes to 540°.

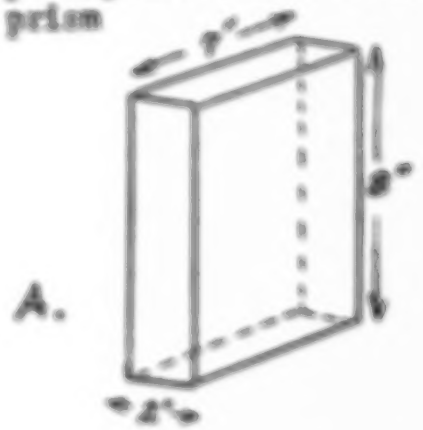
Sum of angles of other polygons



Now ask the children to tell you what the sum of the angles of a 6-sided polygon (hexagon) is, by dividing it into triangles where two lines do not cross (See Figure B). ($4 \times 180^{\circ} = 720^{\circ}$).

Use the same procedure to find the sum of the angles of a 7-sided polygon (septagon). (900°) A 10-sided polygon (decagon) (1440°), a 12-sided polygon (duo-decagon) (1800°).

Computing the lateral area of a prism

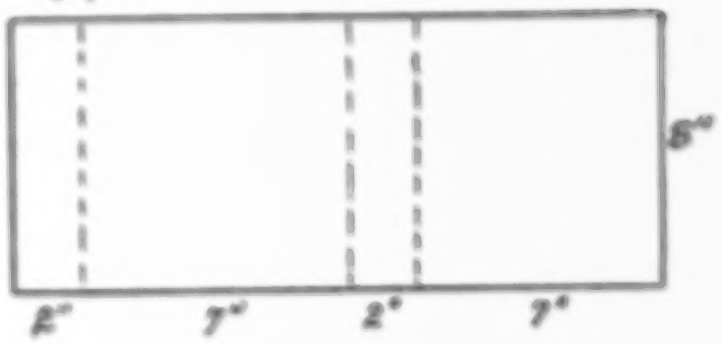


A prism is a space figure. It's <u>lateral faces</u> are the faces that are not bases. It's faces and bases, (i.e., the cube has 6 faces, 2 of which are bases), depend entirely on its resting position. The base is the face on which the figure is resting.

At the left (Figure A) we have a box which we intend to cover with wallpaper. It is open at the top, and we do not wish to cover the bottom. Therefore, we are covering <u>only</u>

its lateral faces. What is the lateral area of its faces? (A = 2(2 x 8) + 2(8 x 7).

Computing the lateral area of a prism



A = 8(2 + 7 + 2 + 7)

A = 8(18)

A = 144 sq. in.

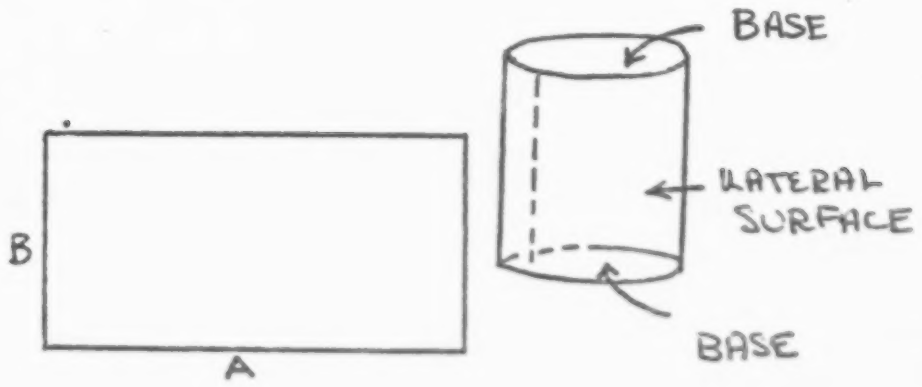
Bring in various boxes to be covered and have teams decide how they will accomplish this. They must first figure the area of each face, then find the sum of the areas of the faces.

What is a shorter way to find the lateral area, other than figuring the area of <u>each</u> face and adding?

Suggestion: Cut the box open along one edge and lay it out flat (See Figure A). It becomes one large rectangle. So the lateral area of a prism equals the perimeter of the base times the height of the prism. Have the teams cover their boxes using this formula (P. x h.). Then, provide match boxes, graph paper, and wallpaper. Have each child compute the lateral area and cover it with square-inch graph paper. (Thus, lateral area in square standard units of measurement.)

Lateral area of a cylinder

Give each math "team" some wallpaper and a 3 gallon ice cream container to make into a waste paper basket. This is a cylinder with bases that are circular in shape.

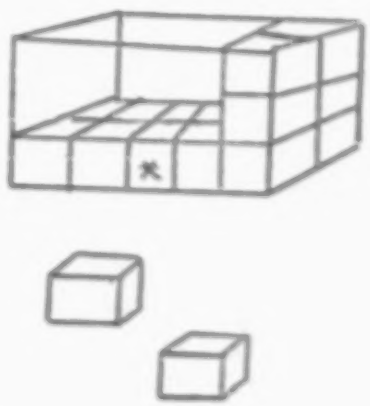


Ask the children what shape will cover their cylinder when laid flat. (a rectangle) You can prove this by cutting a rectangle out of paper and rolling it into the form of a cylinder. How can you find the area of a cylinder? (Perimeter times height of cylinder. [A x B]).

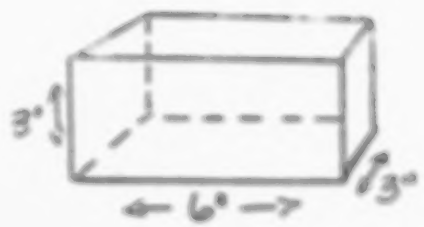
Volume of a rectangular prism

In the 5th grade the children learned that to find area they multiplied length times width, (A = lw) and to find volume they needed to multiply three dimensions. V = lwh.

Volume in cubic centimeters, cubic inches, cubic feet, and cubic yards



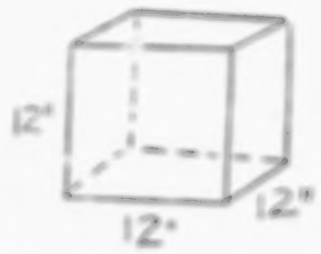
Sugar cubes are approximately 1½ centimeters square. Estimate the volume of various small boxes in centimeters, (match box, paperclip box, etc.), and check by filling with sugar cubes and then measuring with a centimeter ruler, using the formula, V = lwh.



How many cubic inch blocks could be put in one layer in the box at the left? (18 blocks). How many layers of cubes will fit in the box? (3 layers).

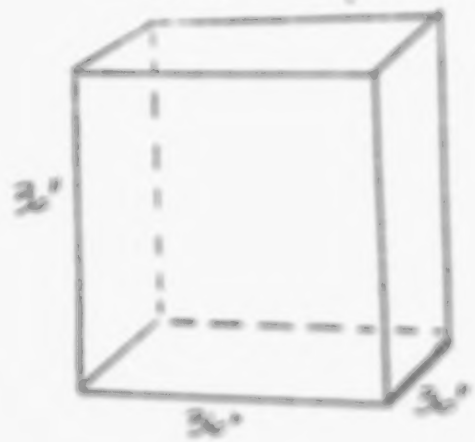
How many 1-inch cubes will fill the box? (54) What is the measure of the box in cubic inches? (54 cubic inches).

Cubic Foot



Have the children draw a picture of a cubic foot and a cubic yard. They should label the dimensions. Ask them what cubic measure they would use to measure the space inside a box with dimensions of 4", 7", and 1". (cubic in.) Ask why the lengths of the dimensions are in inches. What measure would they choose to use to measure the sand in a pick-up truck? (cu. yd.) Why ? (It is a large measure)

Cubic Yard



Nautical and Statute Miles

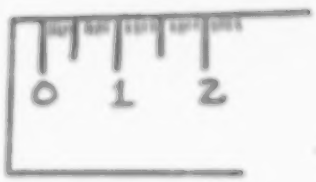
To measure distance by ship and airplane we use a unit of measure called the INTERNATIONAL NAUTICAL MILE. A nautical mile is 6,076.1155 feet, while a statute mile is 5,280 feet. Therefore the nautical mile is 796.1155 feet longer than a statute mile. 6076.155 - 5,280 = 796.1155 feet.

Bring in airplane and ship maps and charts. Have teams of children devise questions for a one-page quiz. Have the teams do some research in the area. The study of the nautical mile would be suitable were it made part of the 6th grade social studies unit.

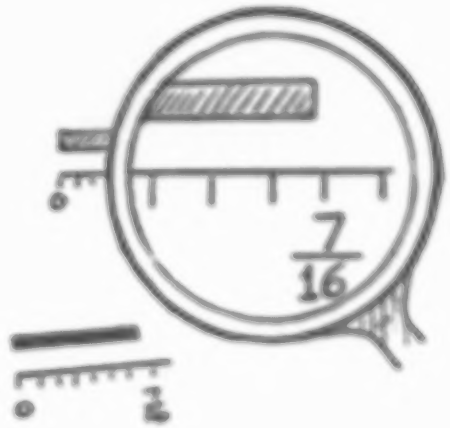
Greatest error of measurement

All measurement is approximate. No matter how carefully you measure, there will always be a difference between the measurement you get and the exact measurement.

The smaller the divisions on our measuring instrument, the more precise will be the measurement. The greatest possible error is one half of the smallest division on the scale used.

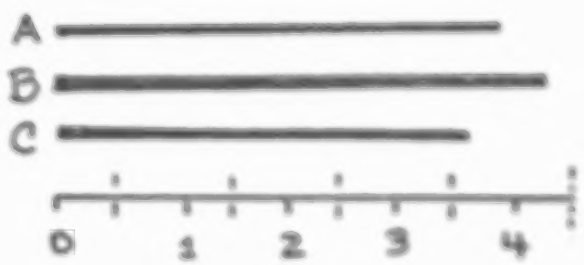


Provide the students with foot rulers (inches divided into 16ths), centimeter rulers, and a magnifying glass. 10:120-122



After the children have measured lines on a dittoed work paper, ask them if their measurements were exact. Let them look through a magnifying glass at the line and the ruler next to it. They can then see for themselves the error they have made in exactness.

It should be made clear that in a measurement stated to the nearest inch, the real length of the object may be as much as ½ inch more than, or ½ less than, the stated measurement.



A = 4 inches
B = 4 inches
C = 4 inches

Show that if a measure is made to the nearest half inch, the greatest possible error is ¼ inch.

And if it is made to the nearest 1/4 inch, the greatest possible error is 1/8 inch.

Part of understanding the nature of measurement is to know that the actual error cannot be determined and we settle on the greatest possible error by agreement. And our agreement is that "The greatest possible error is one half of the smallest divison on the scale used."

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